Cooperative Multi-Provider Routing Optimization and Income Distribution

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***Abstract*—We consider the problem of cooperative distributed routing optimization in multi-domain/multi-provider networks. The main object of our investigation are ASON/G-MPLS transport networks, still the results of our investigations could be extended to any multi-domain network where particular domains have limited mutual visibility of intra-domain resources. This paper refines a distributed decomposition mechanism for reliable cooperative optimization of flow reservation levels introduced in [1], by considering the fundamental issue of fair income distribution. The proposed idea of fair income distribution mechanism has been adopted from the theory of cooperative games (Shapley value). We show the benefits of adopting the proposed income distribution scheme by numerical simulations[1].**

I. Introduction

Automatically Switched Optical Networks (ASON) and Generalized Multi-Protocol Label Switched (G-MPLS) networks are equipped, with addition to the transport and to the management planes, with a control plane responsible for handling transport service calls as well as for handling the set-up process for the associated transport connections. As the control plane of a domain can access all the information on intra-domain topology and state, completion of intra-domain connections is (or at least, it could become) a relatively straightforward task – an operator is able to route intra-domain connections in an optimal way.

Inter-domain connections traverse multiple routing and/or administrative domains with limited mutual visibility of intra-domain resources. To route such connections, control planes of the involved domains have to cooperate – either *off-line* by publishing information on aggregated intra-domain topology or *on-line*, by active participation in step-by-step (better to say, domain-by-domain) process of routing for each connection request. For example, a practical implementation could require a "service level" above a Path Computation Element (PCE)-based multi-domain control-plane as proposed in [2] and [3], to manage service-related data and instantiate multi-provider connections. Despite the cooperation, eventually each control plane has to individually decide on how to route its part of the connection taking into account the corresponding intra-domain policies and known restrictions related to inter-domain links. As today's market forces implementation of QoS enabled services spanned over multiple administrative domains, locally optimized inter-domain routing decisions become increasingly inadequate. Instead, cooperative routing models and optimization goals are required.

The goal of this paper is to improve the distributed scheme for cooperative optimization of inter-domain traffic flow presented in [1] and [4]. These papers define an iterative distributed optimization process, run either in the control plane or in the management plane of the network, in which domains, cooperating in a coalition, calculate the optimal pattern of inter-domain traffic flow. The referred papers are mostly devoted to the description of an optimization process whose objective is to maximize the sum of incomes of individual domains under the assumption that the income of a domain depends linearly on the amount of inter-domain traffic the domain injects into the network. As the objective function might prefer that a domain should rather transit than inject traffic, such an implicit distribution rule could lead to unfair distribution of the total income. In fact, as a domain has no guarantees to gain any additional profit (in reality, it may even loose) there is no incentive to enter the coalition. This paper aims in closing that gap – it extends the model of distributed cooperative optimization with a mechanism of provably fair distribution of the coalition's income adopted from the theory of cooperative games.

The paper is organized as follows. Section II summarizes the distributed optimization model presented in [1] and [4] and introduces a necessary notation. Sections III and IV present a model for application income distribution mechanisms adopted from the theory of cooperative games, namely the notion of the Shapley value. Section V presents a method and the algorithm for computation of the Shapley values from the results of the distributed optimization process. Section VI presents numerical results illustrating the effect of the proposed distribution mechanism on the incomes of particular domains. Eventually, Section VII gives concluding remarks together with a sketch of the plan for further investigations.

II. Distributed routing optimization framework

The distributed method of cooperative optimization of inter-domain traffic flow was introduced in [1] and [4]. In [1] a generic multi-domain routing problem (consisting in

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optimization of bandwidth reservation levels on inter-domain links for traffic flows identified by traffic classes and traffic destinations) is formulated, and its possible decompositions are discussed. In [4] it is shown how to decompose the problem with respect to individual domains using sub-gradient optimization based on Lagrangean relaxation and it is demonstrated how to resolve an inter-domain routing optimization problem using a distributed process based on sub-gradient optimization combined with recovering of near-optimal bandwidth reservation levels. The original approach was further refined in a few papers (e.g., in [5]) where issues related to implementation architecture, computational efficiency and determination of reasonable stopping criteria for the distributed optimization process had been considered. Hereafter it is reminded the necessary formal notation together with base formulation of the original optimization problem. For further details please refer the original papers [1] and [4].

The considered model of the network consists of a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with the set of nodes $\mathcal{V}$ and the set of directed links $\mathcal{E}$ ($\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$). For a set of nodes $\mathcal{U} \subseteq \mathcal{V}$ we define the set $\delta^+(\mathcal{U})$ of links outgoing from set $\mathcal{U}$, and the set $\delta^-(\mathcal{U})$ of links incoming to set $\mathcal{U}$. More precisely, $\delta^+(\mathcal{U}) = \{e \in \mathcal{E} : a(e) \in \mathcal{U} \wedge b(e) \notin \mathcal{U}\}$ and $\delta^-(\mathcal{U}) = \{e \in \mathcal{E} : b(e) \in \mathcal{U} \wedge a(e) \notin \mathcal{U}\}$, where $a(e)$ and $b(e)$ denote the originating and terminating node, respectively, of link $e \in \mathcal{E}$. Besides, we shall write $\delta^\pm(v)$ instead of $\delta^\pm(\{v\})$, i.e., when $\mathcal{U} = \{v\}$ is a singleton.

$\mathcal{M}$ is the set of network domains. Each node $v \in \mathcal{V}$ belongs to exactly one domain denoted by $\mathcal{A}(v)$. Hence, set $\mathcal{V}$ is partitioned into subsets $\mathcal{V}^m = \{v \in \mathcal{V} : \mathcal{A}(v) = m\}, m \in \mathcal{M}$. For each domain $m \in \mathcal{M}$, $\mathcal{E}^m = \{e \in \mathcal{E} : a(e), b(e) \in \mathcal{V}^m\}$ is the set of *intra-domain* links between the nodes in the same domain m. The set of all intra-domain links is denoted by $\mathcal{E}_\mathcal{I} = \bigcup_{m \in \mathcal{M}} \mathcal{E}^m$. Further, the set of all *inter-domain* links is denoted by $\mathcal{E}_\mathcal{O}$, where $\mathcal{E}_\mathcal{O} = \{e \in \mathcal{E} : \mathcal{A}(a(e)) \neq \mathcal{A}(b(e))\} = \bigcup_{m \in \mathcal{M}} \delta^+(\mathcal{V}^m) = \bigcup_{m \in \mathcal{M}} \delta^-(\mathcal{V}^m)$. Clearly, the set of intra-domain links is disjoint with the set of inter-domain links. Finally, the capacity of link $e \in \mathcal{E}$ is denoted by c_e and expressed in units of bandwidth, for example in Mb/s.

Set $\mathcal{D}$ represents traffic demands between pairs of nodes The originating and terminating node of $d \in \mathcal{D}$ is denoted by $s(d)$ and $t(d)$, respectively, and h_d is the traffic volume of d, expressed in the same units of bandwidth as capacity of links. $\mathcal{D}(s,t) = \{d \in \mathcal{D} : s(d) = s \wedge t(d) = t\}$ denotes the set of all demands from node $s \in \mathcal{V}$ to node $t \in \mathcal{V}$ (note that there can be more than one demand between a given pair on nodes). In the sequel, z_d will denote the variable specifying the percentage of volume h_d actually handled in the network, i.e., $z_d h_d$ is the carried traffic of demand d. The set of all demands originating in domain m is denoted as $\mathcal{D}^m = \{d \in \mathcal{D} : s(d) \in \mathcal{V}^m\}$. The sets $\mathcal{D}^m = \{d \in \mathcal{D} : s(d) \in \mathcal{V}^m\}$, $m \in \mathcal{M}$, define a partition of $\mathcal{D}$.

Let x_{et} denote a variable specifying the amount of aggregated bandwidth (called *flow* in the sequel) reserved on intra-domain link $e \in \mathcal{E}_\mathcal{I}$ for the traffic destined for (a remote) node $t \in \mathcal{V}$. Then, for each inter-domain link $e \in \mathcal{E}_\mathcal{O}$ we introduce two flow variables: x^+_{et} and x^-_{et}. Variable x^+_{et} (respectively, x^-_{et}) denotes the amount of bandwidth reserved for traffic carried on e and destined for t that is reserved by domain $\mathcal{A}(a(e))$ (respectively, $\mathcal{A}(b(e))$) at which link e originates (respectively, terminates). Then for each domain $m \in \mathcal{M}$ we introduce the following flow vectors:

- $\boldsymbol{z}^m = (z_d : d \in \mathcal{D}^m)$
- $\boldsymbol{x}^m = (x_{et} : e \in \mathcal{E}^m, t \in \mathcal{V})$
- $\boldsymbol{x}^{m+} = (x^+_{et} : e \in \delta^+(\mathcal{V}^m), t \in \mathcal{V})$
- $\boldsymbol{x}^{m-} = (x^-_{et} : e \in \delta^-(\mathcal{V}^m), t \in \mathcal{V})$
- $\boldsymbol{X}^m = (\boldsymbol{z}^m, \boldsymbol{x}^m, \boldsymbol{x}^{m+}, \boldsymbol{x}^{m-})$.

The basic conditions that have to be fulfilled in each domain $m \in \mathcal{M}$ are flow conservation constraints

$$\begin{aligned} &\sum_{e \in \delta^+(v) \cap \mathcal{E}^m} x_{et} + \sum_{e \in \delta^+(v) \setminus \mathcal{E}^m} x^+_{et} \\ &- \sum_{e \in \delta^-(v) \cap \mathcal{E}^m} x_{et} - \sum_{e \in \delta^-(v) \setminus \mathcal{E}^m} x^-_{et} \\ &\qquad = \sum_{d \in \mathcal{D}(v,t)} z_d h_d, \quad t \in \mathcal{V}, v \in \mathcal{V}^m \setminus \{t\} \end{aligned} \tag{1a}$$

and capacity constraints

$$\sum_{t \in \mathcal{V}} x_{et} \leq c_e, \qquad e \in \mathcal{E}^m \tag{1b}$$

$$\sum_{t \in \mathcal{V}} x^+_{et} \leq c_e, \qquad e \in \delta^+(\mathcal{V}^m) \tag{1c}$$

$$\sum_{t \in \mathcal{V}} x^-_{et} \leq c_e, \qquad e \in \delta^-(\mathcal{V}^m). \tag{1d}$$

Let $\boldsymbol{\mathcal{X}}^m$ ($m \in \mathcal{M}$) denote the set of all vectors $\boldsymbol{X}^m$ satisfying constraints (1) and, possibly, certain extra domain-specific conditions. Such extra constraints can for example be implied by requirements for the weight-based shortest-path intra-domain routing (see Chapter 7 in [6]) or by QoS-type conditions such as $z_d \geq 1, d \in \mathcal{D}^m$. The routing optimization problem can now be stated as follows:

$$\max \; F(\boldsymbol{z}) = \sum_{m \in \mathcal{M}} \sum_{d \in \mathcal{D}^m} z_d h_d \tag{2a}$$

$$\text{s.t.} \quad \boldsymbol{X}^m \in \boldsymbol{\mathcal{X}}^m, \qquad m \in \mathcal{M} \tag{2b}$$

$$x^+_{et} \leq x^-_{et}, \qquad e \in \mathcal{E}_\mathcal{O}, \; t \in \mathcal{V}. \tag{2c}$$

Certainly, objective functions different from (2a) can also be considered.

Let $\boldsymbol{\lambda} = (\lambda_{et} : e \in \mathcal{E}_\mathcal{O}, t \in \mathcal{V})$ be a vector of (non-negative) multipliers associated with constraints (2c). The Lagrangean function $L(\boldsymbol{\lambda}; \boldsymbol{X}), \boldsymbol{\lambda} \geq \boldsymbol{0}, \boldsymbol{X} = (\boldsymbol{X}^m : m \in \mathcal{M}) \in \boldsymbol{\mathcal{X}} = \bigotimes_{m \in \mathcal{M}} \boldsymbol{\mathcal{X}}^m$ associated with problem (2) is of the following decomposed form:

$$L(\boldsymbol{\lambda}; \boldsymbol{X}) = \sum_{m \in \mathcal{M}} L^m(\boldsymbol{\lambda}^m; \boldsymbol{X}^m). \tag{3}$$

In (3), $\boldsymbol{\lambda}^m = (\lambda_{et} : e \in \delta^-(\mathcal{V}^m) \cup \delta^+(\mathcal{V}^m), t \in \mathcal{V})$ is the sub-vector of $\boldsymbol{\lambda}$ composed of the values λ_{et} for all inter-domain links e originating or terminating in domain $m \in \mathcal{M}$, and $L^m(\boldsymbol{\lambda}^m;\boldsymbol{X}^m)$ denotes the partial Lagrangean corresponding to domain $m \in \mathcal{M}$ equal to

$$\sum_{d\in\mathcal{D}^m} z_d h_d + \sum_{t\in\mathcal{V}} \Big(\sum_{e\in\delta^-(\mathcal{V}^m)} \lambda_{et} x^-_{et} - \sum_{e\in\delta^+(\mathcal{V}^m)} \lambda_{et} x^+_{et} \Big), \quad (4)$$

where $\boldsymbol{\lambda}^m \geq \mathbf{0}$ and $\boldsymbol{X}^m \in \boldsymbol{\mathcal{X}}^m$. The problem dual to (2) (see for example [7]) becomes as follows:

$$w^* = \min_{\boldsymbol{\lambda}\geq\mathbf{0}} w(\boldsymbol{\lambda}). \quad (5)$$

The (non-empty) set of optimal solutions of problem (5) will be denoted by $\boldsymbol{\Lambda}^*$. The dual function w is defined as $w(\boldsymbol{\lambda}) = \sum_{m\in\mathcal{M}} w^m(\boldsymbol{\lambda}^m)$ and is computed through resolving separate subproblems:

$$w^m(\boldsymbol{\lambda}^m) = \max_{\boldsymbol{X}^m\in\boldsymbol{\mathcal{X}}^m} L^m(\boldsymbol{\lambda}^m;\boldsymbol{X}^m),\ m \in \mathcal{M}. \quad (6)$$

For any $\boldsymbol{\lambda} \geq \mathbf{0}$, $\boldsymbol{X}(\boldsymbol{\lambda}) \in \boldsymbol{\mathcal{X}}$ will denote the so called *maximizer* of the Lagrangean function (3), i.e., any optimal solution of the Lagrange problem:

$$\boldsymbol{X}(\boldsymbol{\lambda}) = \arg\max_{\boldsymbol{X}\in\boldsymbol{\mathcal{X}}} L(\boldsymbol{\lambda};\boldsymbol{X}). \quad (7)$$

Any maximizer $\boldsymbol{X}(\boldsymbol{\lambda}) = (\boldsymbol{X}^m(\boldsymbol{\lambda}^m) : m \in \mathcal{M})$ is computed through solving independent subproblems (6):

$$\boldsymbol{X}^m(\boldsymbol{\lambda}^m) = \arg\max_{\boldsymbol{X}^m\in\boldsymbol{\mathcal{X}}^m} L^m(\boldsymbol{\lambda}^m;\boldsymbol{X}^m),\ m \in \mathcal{M}. \quad (8)$$

In the sequel the quantity $\nabla w(\boldsymbol{\lambda})$ will denote a subgradient of the dual function w at point $\boldsymbol{\lambda}$. Subgradients are obtained as a by-product of the (distributed) computation of the values of $w(\boldsymbol{\lambda})$: if $\boldsymbol{X}(\boldsymbol{\lambda})$ is a maximizer of the Lagrangean function (3) for a given $\boldsymbol{\lambda}$, then the corresponding subgradient $\nabla w(\boldsymbol{\lambda})$ is as follows ([7]):

$$\nabla w(\boldsymbol{\lambda}) = (x(\boldsymbol{\lambda})^-_{et} - x(\boldsymbol{\lambda})^+_{et} : e \in \mathcal{E}_{\mathcal{O}}, t \in \mathcal{V}). \quad (9)$$

The dual problem can be resolved using general subgradient minimization (SM) techniques (see for example [8] and [9]). However, this is not sufficient because as discussed in [5] an optimal dual solution $\boldsymbol{\lambda}^* \in \boldsymbol{\Lambda}^*$ of problem (5) does not in general yield an optimal (nor even feasible) primal solution $\boldsymbol{X}^*$, i.e., an optimal solution of problem (2). What we only know for sure is that any optimal $\boldsymbol{\lambda}^*$ gives the optimal value F^* of the primal objective function (2a): $F^* = w(\boldsymbol{\lambda}^*) = w^*$. We note here that although any optimal primal solution $\boldsymbol{X}^*$ is a maximizer for any $\boldsymbol{\lambda}^* \in \boldsymbol{\Lambda}^*$, the converse is not true. In fact, in general a maximizer $\boldsymbol{X}(\boldsymbol{\lambda}^*)$ can be primal infeasible.

SM can be combined with recovering a (near-) optimal primal solution X^*, leading to a class of algorithms referred to as SM-PR (subgradient minimization—primal recovery). It turns out that convergence of the divergent series SM-PR algorithms can be slow even when applied to medium-size instances of (2). Certain improvement can be achieved when using deflected subgradients in Step 1(a) (see Section 3 in [10]). Still, it seems that one of the best available SM-PR methods are the so called proximal bundle methods, see [11], [12], and [13]. Therefore, in our numerical experiments discussed in Section VI we also used a SM-PR algorithm of the proximal bundle type (called ConicBundle 0.1) implemented in [14] (for a brief theoretical introduction see [15]).

III. Rationales

We are thus interested in an interconnection scenario in which domains collaborate in a sort of multi-domain alliance. Multiple domains interact to optimize link capacity reservation levels, thus to improve their inter-domain routing, escaping a solution guided by unilateral and selfish choices toward a more effective global solution. If this interconnection principle is undoubtedly attractive, and the proposed approach shall be suitable, the incentives are still not obvious. In the following, we investigate this aspect and propose a game theoretical multi-domain income distribution scheme based on the Shapley-value concept [16] that shall motivate the adoption of the proposed multi-domain routing optimization framework.

Adopting the distributed multi-domain optimization approach presented in Sect. II, the global routing solution is likely to improve with respect to multi-domain throughput and load distribution efficiency. Nevertheless it is likely that the corresponding optimal global solution arises disparities among domains. Still acceptable when the involved domains belong to a same provider network, such routing disparities would decrease the reciprocal trust among individual providers (called Autonomous Systems, ASs, AS carriers or carrier providers in the Internet architecture). Eventually, the alliance agreement may not be settled for the lack of fair incentive schemes.

Under the current Internet business model, AS carriers provide IP connectivity to customer ASs in the form of paid "transit agreements", selling bandwidth and Quality of Service (QoS). The traffic received from customer ASs is then routed across the Internet using either other customers' networks, or the network of other providers. In this last case, another paid transit agreement may settle the routing policy. Alternatively, a free-transit agreement, also called "peering agreement", can settle it. In a peering agreement, two carrier providers agree for free reciprocal transit (between their clients' networks only) if they can both get mutual operational and economical benefits from it. A review of Internet agreements can be found in [17].

Under this standpoint, the framework proposed in [1] and [4] can be considered as a sort of "extended peering agreement" from which providers obtain mutual benefit without side payments. However, we claim that for such frameworks the agreement shall rely on side payments since the multi-provider optimization can arise disparities. In order to reserve bandwidth for external connections for which no direct earning is obtained, a provider may need a form of economical incentive. It is indeed possible that, by reserving bandwidth for external connections, a provider grants earnings to its "peers" bigger than the earnings related to its own services. Instead of "extended peering agreement" it seems more appropriate to refer to an economical "alliance of providers" that wish to cooperate for multi-provider connection-oriented (ASON/GMPLS) services, sharing the related incomes (besides the costs of an integrated technical architecture such as proposed in [2]).

It is thus needed to define a fair scheme for multi-provider income distribution that rewards a provider in a way that is not solely based on the generated traffic (Content provider behavior, see [20]) or absorbed traffic (Eyeball provider behavior, see [20]) but also accordingly to its *alliance transit contribution*, i.e., that takes into account how much a provider supports the services of the other providers allocating its network's resources. The originally conceived scheme in [1] and [4] relies on the current Internet business model, with transit agreements settled between domains. Hence, a provider income derives from the Internet connections provided to its direct clients.

With a far-sighted standpoint, in [19] it is proven that, if part of the profits due to inter-provider services were shared, the Internet providers would behave less selfishly, yielding better global routing with lower routing cost than under the current practice. Using the Shapley value concept from *cooperative games*, in [19] it is argued that profits and costs may be fairly imputed considering the importance of each AS in the interconnected "coalition" composed of ASs routing "common" inter-AS flows [20]. In this way, it is proven that ASs have incentive to better route yielding to a common inter-domain routing cost lower than with the current practice, besides than interconnection cost savings.

IV. A Shapley Value Perspective for Income Distribution

The Shapley Value concept is a game theoretic solution for value imputation problems that offers interesting properties recalled below [16]. For this reason, it has been applied to very diverse fields [21]. In game theory, interacting agents are modeled as players that take decisions rationally following the utility functions of all the players. In cooperative games, since some players may contribute more than others for the collaboration, the value imputation problem consists in how to distribute a global value (or revenue) among the players. How important is each player to the coalition, and what payoff can be reasonably expected, are questions to which cooperative coalitional game theory answers with many theoretical concepts - not worth being all reviewed here (for a review consider [18]). Among these concepts, the Shapley value considers the strategic weight (importance) of each player in the alliance to share the alliance value.

The Shapley value is calculated as follows:

- consider all the possible permutations of the players,
- for each permutation and each player, calculate the marginal contribution that the player grants if he joins the coalition formed by the predecessor players,
- for each player, calculate the average of its marginal contributions on all the permutations.

For example, assume that we have a set of players $\{1,2,3\}$ (so possible permutations are $\{1,2,3\}$, $\{1,3,2\}$, $\{2,1,3\}$, $\{2,3,1\}$, $\{3,1,2\}$, $\{3,2,1\}$). Thus marginal contributions of particular players in case of permutation $\{3,1,2\}$ would be $\mu(\{1,3\})-\mu(\{3\})$ (of player 1), $\mu(\{1,2,3\})-\mu(\{1,3\})$ (of player 2) and $\mu(\{3\})-\mu(\emptyset)=\mu(\{3\})$ (of player 3);

The Shapley value is thus equal to zero for null players, which do not offer any marginal contribution to a coalition in any case, and equal to the single-player payoff for dummy players, which are indifferent in staying in the coalition or not.

A. Mathematical formulation

The Shapley value can be used to assign the payoff (income) of a player as function of his marginal contribution to the coalition. Given that the marginal contribution that a player brings to a coalition (i.e. the alliance income related to its connection services) varies as function of the players that already form the coalition, it is essential considering the order in which the player enters the coalition (or would enter if a coalition evaluates the opportunity of joining the new player).

Mathematically, we use the formulation of a coalitional game. Let $\mathcal{N}$ denote a set of players. We start with a function $\mu : \mathcal{P}(\mathcal{N}) \rightarrow \mathbb{R}$, that goes from subsets of players $\mathcal{S} \subseteq \mathcal{N}$ to reals, called the "worth function", with the properties:

i) $\mu(\emptyset) = 0$;
ii) $\mu(S \cup T) \geq \mu(S) + \mu(T)$, $\forall S, T \subseteq \mathcal{N} \mid S \cap T = \emptyset$.

The computation of μ will be explicited in the next section. The interpretation of the function μ is as follows: if S is a coalition of players which agree to cooperate, then $\mu(S)$ describes the total expected gain from this cooperation, independent of what the actors outside of S do. ii) is the "superadditivity" condition, hypothesis of classical cooperative game theory, which expresses the fact that collaboration can only help, and never hurts. A shapley value imputation ω_i can thus be calculated for each player $i \in \mathcal{N}$ as function of μ:

$$\omega_i(\mu) = \sum_{S \subseteq \mathcal{N} \setminus \{i\}} \frac{|S|!(n-|S|-1)!}{n!} (\mu(S \cup \{i\}) - \mu(S)) \quad (10)$$

where the sum extends over all subsets S of $\mathcal{N}$ not containing player i. The formula can be justified if one imagines the coalition being formed one player at a time, with each player demanding its contribution $\mu(S \setminus \{i\}) - \mu(S)$ as a fair compensation, and then averaging over the possible different permutations in which the coalition can be formed.

B. Properties

The Shapley Value satisfies desirable properties of individual fairness, efficiency, symmetry, additivity and null player modeling (for a detailed characterization see [21]). In fact, the vector of Shapley values is the only payoff vector - defined on the class of all superadditive games - that satisfies these five properties. Namely, under a Shapley value distribution, in our framework every provider gets at least as much as it would have got without any collaboration, and two strategically equivalent providers obtain the same value. Moreover, the Shapley value distribution supports anonymity. That is, the labeling of the players doesn't play a role in the assignment of their payoffs, i.e., if i and j are two players, and μ^1 is the worth function that acts just like μ^2 except that the roles of i and j have been exchanged, then $\omega_i(\mu^1) = \omega_j(\mu^2)$. Finally, the Shapley value is the single imputation rule that supports

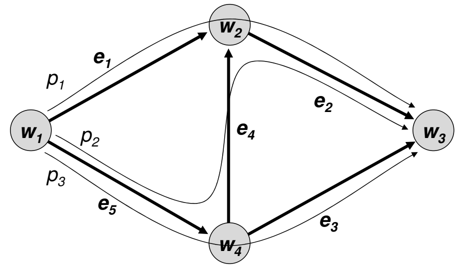


Fig. 1. Connectivity graph of an exemplary multi-domain network

marginality, i.e., which uses only the marginal contributions of a player as argument [21].

V. Worth function

Let us consider a network model consisting of a directed graph $\mathcal{H} = (\mathcal{W},\mathcal{F})$ with the set of nodes $\mathcal{W}$ and the set of edges $\mathcal{F}$. Let $\boldsymbol{\mathcal{W}} = \{\mathcal{W}_i : i \in \mathcal{I}\}$ where $\mathcal{I} = \{1, 2, ..., N\}$ and $N = 2^{|\mathcal{W}|}$. Each subset $\mathcal{W}_i \in \boldsymbol{\mathcal{W}}$ induces a subgraph $\mathcal{H}_i = (\mathcal{W}_i, \mathcal{F}_i)$ of graph $\mathcal{H}$ where $\mathcal{F}_i = (\mathcal{W}_i \times \mathcal{W}_i) \cap \mathcal{F}$.

Set $\mathcal{D} \subseteq (\mathcal{W} \times \mathcal{W})$ represents traffic demands; let us recall that the originating and terminating node of $d \in \mathcal{D}$ is denoted by $s(d)$ and $t(d)$, respectively, and h_d is the traffic volume of d. Set $\mathcal{P}_d$ denotes the set of paths that realize demand $d \in \mathcal{D}$ and y_{dp} is a fraction of volume h_d sent over path $p \in \mathcal{P}_d$ ($\sum_{p\in\mathcal{P}_d} y_{dp} = h_d$). Let $\mathcal{P} = \bigcup_{d\in\mathcal{D}} \mathcal{P}_d$ denote the set of all paths and the set $\mathcal{P}_i = \{p \in \mathcal{P} : \forall_{e\in p}\, e \in \mathcal{F}_i\}$ denote the set of paths that survive in subnetwork $\mathcal{H}_i$.

The total amount of traffic that subnetwork $\mathcal{H}_i$ delivers to its destination nodes can be expressed as $\mathcal{T}(\mathcal{H}_i) = \sum_{d\in\mathcal{D}} \sum_{p\in\mathcal{P}_i\cap\mathcal{P}_d} y_{dp}$. Then, with an additional assumption that the income that subnetwork $\mathcal{H}_i$ earns depends linearly on the value of $\mathcal{T}(\mathcal{H}_i)$, the worth function of subnetwork $\mathcal{H}_i$ may be defined as

$$\mu(\mathcal{H}_i) = \alpha\mathcal{T}(\mathcal{H}_i). \tag{11}$$

Without loss of generality, we hereafter assume that $\alpha = 1$. Let us consider a simple network presented in Figure 1. The network realizes a single demand d_1 ($\mathcal{D} = \{d_1\}$) of volume $h_{d_1} = 1$ using three paths $\mathcal{P} = \mathcal{P}_d = \{p_1, p_2, p_3\}$ with respective loads $y_{d_1p_1} = 0.5$, $y_{d_1p_2} = 0.25$, $y_{d_1p_3} = 0.25$. We observe that the value of the worth function $\mu(\mathcal{H}_i)$ is equal to zero for every subnetwork $\mathcal{H}_i$ such that $\mathcal{P}_i = \emptyset$. Hence, the only meaningful subnetworks are those induced by subsets $\mathcal{W}_1 = \{w_1, w_2, w_3, w_4\}, \mathcal{W}_2 = \{w_1, w_2, w_3\}$ and $\mathcal{W}_3 = \{w_1, w_4, w_3\}$. Their respective worth function values are $\mu(\mathcal{H}_1) = 1$, $\mu(\mathcal{H}_2) = 0.5$, $\mu(\mathcal{H}_3) = 0.25$. Worth function (11) can be used to compute the Shapley value imputation of nodes in $\mathcal{W}$ (cf. Table I). Set Π is a family of all sequences (permutations) of nodes in $\mathcal{W}$ while $\pi \in \Pi$ denotes a single permutation. Then $\mathcal{H}(\pi, w)$ denotes a subnetwork induced by the set of nodes that precede node w in permutation π. The Shapley value of node $w \in \mathcal{W}$ is an average over all $\pi \in \Pi$ of its marginal contribution to every subnetwork $\mathcal{H}(\pi, w)$ (cf. Section III).

permutation	marginal contribution to subnetwork $\mathcal{H}(\pi, w)$			
$\boldsymbol{\pi} \in \Pi$	$w = w_1$	$w = w_2$	$w = w_3$	$w = w_4$
$\{w_1, w_2, w_3, w_4\}$	0	0	.5	.5
$\{w_1, w_2, w_4, w_3\}$	0	0	1	0
$\{w_1, w_4, w_2, w_3\}$	0	0	1	0
$\{w_1, w_4, w_3, w_2\}$	0	.75	.25	0
$\{w_1, w_3, w_4, w_2\}$	0	.75	0	.25
$\{w_1, w_3, w_2, w_4\}$	0	.5	0	.5
$\{w_2, w_3, w_4, w_1\}$	1	0	0	0
$\{w_2, w_3, w_1, w_4\}$	.5	0	0	.5
$\{w_2, w_1, w_3, w_4\}$	0	0	.5	.5
$\{w_2, w_1, w_4, w_3\}$	0	0	1	0
$\{w_2, w_4, w_1, w_3\}$	0	0	1	0
$\{w_2, w_4, w_3, w_1\}$	1	0	0	0
$\{w_3, w_4, w_1, w_2\}$	.25	.75	0	0
$\{w_3, w_4, w_2, w_1\}$	1	0	0	0
$\{w_3, w_2, w_4, w_1\}$	1	0	0	0
$\{w_3, w_2, w_1, w_4\}$	.5	0	0	.5
$\{w_3, w_1, w_2, w_4\}$	0	.5	0	.5
$\{w_3, w_1, w_4, w_2\}$	0	.75	0	.25
$\{w_4, w_1, w_2, w_3\}$	0	0	1	0
$\{w_4, w_1, w_3, w_2\}$	0	.75	.25	0
$\{w_4, w_3, w_1, w_2\}$	.25	.75	0	0
$\{w_4, w_3, w_2, w_1\}$	1	0	0	0
$\{w_4, w_2, w_3, w_1\}$	1	0	0	0
$\{w_4, w_2, w_1, w_3\}$	0	0	1	0
Shapley values	**.3125**	**.229167**	**.3125**	**.14833**

TABLE I
Intermediate and the final Shapley values

To compute Shapley value imputation that the domains of the original multi-domain network receive, induced by the optimal routing solution (cf. Section II), we first construct the domain connectivity graph $\mathcal{H}(\mathcal{W}, \mathcal{F})$ of the original multi-domain network (cf. Section II) graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Each node $w_m \in \mathcal{W}, m \in \mathcal{M}$ represents domain m of the original network while each link $f = \{w_{m'}, w_{m''}\}, m', m'' \in \mathcal{M}$ represents the set $\mathcal{L}(f)$ of all directed inter-domain links between domains m' and m''.

Then we define the vector of reservations $\boldsymbol{Y} = \{y_{ft} : f \in \mathcal{F}, t \in \mathcal{W}\}$ where a single reservation y_{ft} denotes the amount of capacity on link f that is reserved for traffic destined to node t. Observe that value of $y_{fw}, w = w_m$ can be computed using either the formula $\sum_{e\in\mathcal{L}(f)} x^+_{et}$ or $\sum_{e\in\mathcal{L}(f)} x^-_{et}$ where $m = \mathcal{A}(t)$ is the domain node t belongs to.

We assume that the subset of edges $\mathcal{F}^t \subseteq \mathcal{F}$ used by flow of traffic from every node $w \in \mathcal{W}$ to an arbitrary destination node $t \in \mathcal{W}$ induces the acyclic graph $\mathcal{H}^t = (\mathcal{W}^t, \mathcal{F}^t)$ (in fact vector $\boldsymbol{Y}$ generates flows that are near-acyclic still they can be made acyclic by means of a preprocessing).

Then we consider the full set of demands $D = (\mathcal{W} \times \mathcal{W})$. Observe that volume of demand $d \in \mathcal{D}$ from node $s(d)$ to node $t(d)$ can be computed using $h_d = \sum_{e\in\delta^+(w)} y_{et} - \sum_{f\in\delta-(w)} y_{ft}$. As traffic belonging to demand d is undistinguishable (vector $\boldsymbol{Y}$ defines only reservations that are aggregated over destination nodes) it is reasonable to assume that the set of paths $\mathcal{P}_d, d \in \mathcal{D}$ of demand d contains all paths in graph $\mathcal{H}^t$ leading from node $s(d)$ to node $t(d)$. Let $\phi^t(w)$ denote the total volume of traffic destined to node t in node w and let $\psi^t(f)$ denote a percentage of that volume that each link $f \in \delta^+(w) \cap \mathcal{F}^t$ carries. Observe that the a rule of splitting of the volume of every demand d such that $t(d) = t$ in every node of graph $\mathcal{H}_t$ is defined by

$$\psi^t(f) = y_{ft} / \sum_{e \in \delta^+(w) \cap \mathcal{F}^t} y_e t. \tag{12}$$

Using formula (12) we can compute loads y_{pd} on every path $p \in \mathcal{P}_d$ of every demand $d \in \mathcal{D}$ and finally, by following the pattern used in the example, also to compute Shapley value imputation that domains eventually receive.

Algorithm V.1 computes value $\mu^t(\mathcal{H}_i)$ which denotes a component of the worth function of subnetwork $\mathcal{H}_i$ for the traffic to destination node $t \in \mathcal{W}_i$. It takes advantage of the observation that the value of this component is equal to the volume $\phi_i^t(t)$ of node t, i.e., $\mu^t(H_i) = \phi_i^t(t)$ and also of the assumption that graph $\mathcal{H}^t$ is acyclic. Procedure *NODEVOL* computes the value of volume $\phi_i^t(w)$ of node $w \in \mathcal{W}_i$. Procedure *LINKVOL* computes the the amount of traffic to node t that loads link $f \in \mathcal{F}_i$ in graph $\mathcal{H}_t$.

Algorithm V.1: WORTHCOMPONENT($i \in \mathcal{I}, t \in \mathcal{W}_i$)

procedure NODEVOL(w)
if not $visited(w)$
then $\begin{cases} \textbf{for each } e \in \{\delta^-(w) \cap \mathcal{F}_i \cap \mathcal{H}^t\} \\ \quad \textbf{do } vol(w) \leftarrow vol(w) + \text{LINKVOL}(e) \\ visited(w) \leftarrow true \end{cases}$
return $(vol(w))$

procedure LINKVOL(e)
if NODEVOL($a(e)$)$\psi^t(e) < y_{et}$
then return (NODEVOL($a(e)$)$\psi^t(e)$)
else return (y_{et})

main
for each $w \in \mathcal{W}_i$
do $\begin{cases} vol(w) \leftarrow 0 \\ visited(w) \leftarrow \textbf{false} \end{cases}$
return (NODEVOL(t))

Finally, the worth function of the subnetwork $\mathcal{H}_i$ is computed as

$$\mu(\mathcal{H}_i) = \sum_{t \in \mathcal{W}_i} \mu^t(\mathcal{H}_i) \tag{13}$$

Note that the worth function $\mu(\mathcal{H}_i)$ of subnetwork $\mathcal{H}_i, i \in \mathcal{I}$ is computed using the single vector of reservations $\boldsymbol{Y}$. This can be considered as heuristic since the worth value of a subnetwork is not computed w.r.t. the optimal vector of reservations that would be obtained from a restriction of Problem 2. Nevertheless, it is pragmatical since a provider has no final choice to enter or to leave the coalition imposed by business agreements that take into account not only direct profits but also other issues – e.g., the possibility for extending customer base.

edge $f \in \mathcal{F}$		destination node $t \in \mathcal{W}$						
$a(f)$	$b(f)$	m_1	m_6	m_0	m_5	m_4	m_3	m_2
m_0	m_4	155	0	0	0	3987	174	0
m_0	m_5	132	0	0	2694	0	1053	0
m_0	m_6	563	11174	0	0	0	0	1031
m_1	m_2	0	724	809	0	0	0	6384
m_1	m_3	0	0	0	495	625	1619	0
m_2	m_1	7669	0	0	0	0	0	0
m_2	m_3	0	0	0	0	0	2675	0
m_2	m_6	0	7092	1822	542	651	0	0
m_3	m_1	5596	0	0	0	0	0	0
m_3	m_2	0	246	0	0	0	0	5168
m_3	m_4	0	0	202	0	4992	0	0
m_3	m_5	0	794	1070	2274	0	0	0
m_4	m_0	0	0	2158	0	0	0	0
m_4	m_3	772	0	0	0	0	1265	495
m_4	m_5	0	0	0	9418	0	0	0
m_4	m_6	0	4862	0	0	0	0	214
m_5	m_0	0	0	7756	0	0	0	0
m_5	m_3	624	0	0	0	0	7273	393
m_5	m_4	0	0	0	0	7870	0	0
m_5	m_6	0	3413	0	0	0	0	212
m_6	m_0	0	0	8903	0	0	0	0
m_6	m_2	1280	0	0	0	0	773	9921
m_6	m_4	0	0	0	0	2467	0	0
m_6	m_5	0	0	0	1550	0	272	0

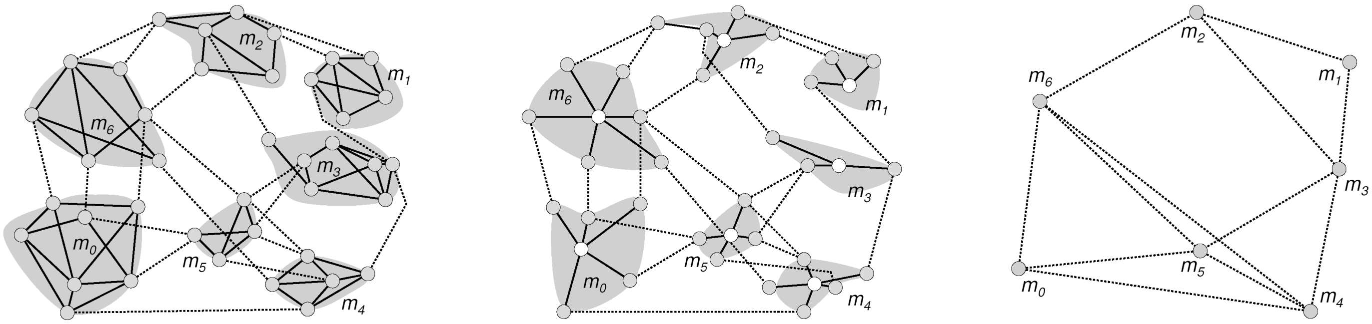
TABLE II
COMPONENTS OF VECTOR $\boldsymbol{Y}$

VI. NUMERICAL EXPERIMENTS

In our experiments we compared imputation of the income of a multi-domain network generated by the proposed Shapley value based distribution with that generated by the original distribution (cf. either [1] or [4]), where the whole income related to demand d is attributed to a domain that injects this demand into the network, and no domain receives income for transiting nor terminating the traffic. Note that value of the income has been defined (cf. Section V) as equal to (depending linearly on) the total volume of demands that the coalition serves.

We did the investigations in a context of a single multi-domain network consisting of seven domains (the original topology of the network is presented in Figure 2a, where thick lines represent intra-domain links and thin lines represent inter-domain links; independently from the type of the link, a single line represents a pair of oppositely directed unidirectional links of equal capacity). The considered traffic matrix is random.

First, to reduce the complexity of the original network, a *star-aggregation* of intra-domain networks was applied (according to the methodology described in [5]). Then, for the aggregated network (in Figure 2b) the distributed optimization process had been run. Finally, the domain connectivity graph $\mathcal{H} = (\mathcal{W}, \mathcal{F})$ (in Figure 2c) was constructed and the optimal routing solution of Problem 2 was projected onto that graph (the vector of reservations $\boldsymbol{Y}$ induced by the optimal routing solution is presented in Table II). As in general case a node receives sum of share components induced by flows to many different destination nodes, the differences between imputations generated by two considered distributions may be difficult to depict. To



(a) The original network (b) The network after *star aggregation* (c) The domain connectivity graph $\mathcal{H}$

Fig. 2. Network topology abstraction schemes

avoid that problem, we decided to restrict our consideration to just a single flow directed to an arbitrary (say, m_3) destination node. This flow is presented in Figure 3, where number beside a link $f \in \mathcal{F}$ denote the value of reservation $y_{ft}, t = m_3$, i.e., the amount of bandwidth reserved on link f for traffic to node m_3. Considering these reservations, one can then easily compute the amount of traffic to node m_3 that every node either injects into the network, terminates or transits (cf. Table III) hence (following the method introduced in Section V) he can compute also Shapley values attributed to a particular domain. Table IV shows imputations generated by the two distributions (Shapley based and the original one) in context of the considered flow. Columns i, t and tr of the table show income components related to traffic that a domain injects and terminates, respectively. The last column $\sum$ denotes total imputation that a domain receives.

Observing Tables III and IV one can conclude that the original distribution is unfair – as there are significant unpaid volumes of traffic terminated by domain m_3 and transited through domains m_2, m_4 and m_5. The second part of Table IV shows that the proposed Shapley value distribution schemes offers significantly fairer results, as domains are awarded for every type of their contribution in the total income (the 'x's mean that the Shapley value attributed to transit domain cannot be easily divided into components related to injecting and transiting of traffic). Namely, the Shapley scheme assigns to m_3 the biggest share while the original scheme would assign a null income: without m_3 12833 units of traffic (c.f. the total ingress traffic at m_3 in Figure 3) could not be provided, so the corresponding revenue is distributed fairly also to m_3 recognizing to it an income share of 6010. Or, m_0 and m_6 not reserving bandwidth for any external connection, receive roughly one third of the original share.

In this study case we can appreciate the application of the concept initially proposed by the authors of [20]. They claim that nowadays the Internet is characterized by "Content providers" (e.g., Youtube) that delivers traffic to "Eyeball providers" (e.g., Polish Telecom) that connect large communities of customers. Since the Content providers get revenue by selling services to customers of Eyeball providers using the network of "Carrier providers" (e.g., Opentransit), they propose to share the corresponding income among all the providers in the delivery chain. In our restricted study case, m_0, m_1 and m_6 can be seen as Content providers simply injecting traffic toward m_3, the Eyeball provider, crossing m_2, m_4 and m_5 that act as Carrier providers, as well as Content providers in turn.

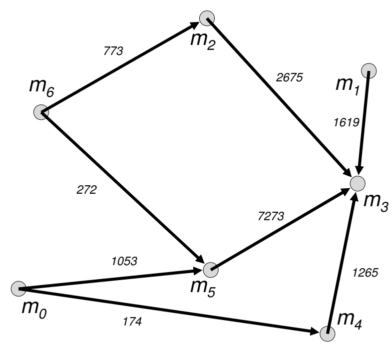


Fig. 3. Flow to domain m_3

domain	injects	terminates	transits
m_0	1225	0	0
m_1	1619	0	0
m_2	1902	0	773
m_3	0	12833	0
m_4	1091	0	174
m_5	5948	0	1325
m_6	1045	0	0

TABLE III
FLOW m_3 COMPONENTS

In the general case, by contemplating a mixed Content/Eyeball/Carrier behavior for each domain, our framework somehow adopts and extends the concept proposed in [20], by coupling a routing decomposition optimization framework that deals with multiple connections, with a fair income distribution policy. For the general case, in Figure 4 we compare the original distribution to the final Shapley values, which are computed summing all the contributions due to *all the flows* and *all the destinations*.

	original distribution				Shapley distribution			
	i	t	tr	$\sum$	i	t	tr	$\sum$
m_0	1225	0	0	1225	408	0	0	408
m_1	1619	0	0	1619	809	0	0	809
m_2	1902	0	0	1902	x	0	x	1188
m_3	0	0	0	0	0	6010	0	6010
m_4	1091	0	0	1091	x	0	x	602
m_5	5948	0	0	5948	x	0	x	3408
m_6	1045	0	0	1045	323	0	0	323

TABLE IV
FLOW m_3 RELATED INCOME DISTRIBUTION

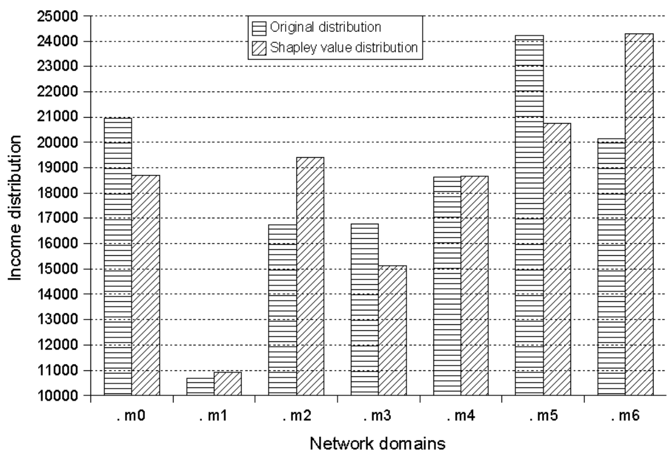


Fig. 4. Income distribution schemes comparison

We can better appreciate the global effect of the proposed distribution scheme. For each domain, the result is a fair weighting of the traffic injected, terminated and transited, following the Shapley imputation rule (10).

Those domains better interconnected, i.e. with more neighbors and more intra-domain availability (c.f. Figure 2), are able to transit traffic (c.f. the reservation levels in Table II) and get a higher share. This is the case for example of m_6 and m_2 that increase their share of 29% and 16%, respectively. Those domains that inject a lot of traffic but still offer an adequate transit for alliance connections, e.g. m_4, maintain a similar share. Instead, for those whose injected traffic volume is not sufficiently compensated with transit contribution, e.g. m_5, m_3 and m_0, the share decreases (of roughly 10%).

VII. CONCLUDING REMARKS

In this paper we have presented a cooperative multi-provider routing optimization framework in which providers cooperate to the resource reservation for inter-provider connections. We discussed under which circumstances this might result economically feasible. In order to support the adoption of such a multi-provider routing optimization framework, we proposed a fair income distribution scheme relying on the Shapley Value concept from cooperative game theory, showing how the complex issue of computing the Shapley values using decomposition result parameters can be solved heuristically.

By comparison with the original implicit income distribution policy, we show the benefits of the adoption of the Shapley value distribution scheme. Those domains that attract large volumes of traffic can receive an income for such a contribution. Those providers that do not balance their injected traffic volume with bandwidth reserved for external connection transit, see their income share decreased. Those domains that do not offer transit at all are fairly penalized. Our approach is a further step (after a few others such as [20]) toward the definition of feasible cooperative routing frameworks and acceptable business models for the future Internet.

As a further work we aim to refine the optimization decomposition method so as to allow a pro-active integration of the Shapley values. The idea is to control the amount of traffic volume a provider is allowed to inject within the alliance. It might be desirable to allow rewarding a provider's transit contribution directly with intra-alliance traffic injection ability by bounding the inter-provider throughput.

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