Demands Rescaling for Resource and Power Allocation in Cooperative Femtocell Networks

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Abstract—**Femtocell provisioning is emerging as a key technology to improve coverage and network capacity in indoor environments. When femtocells use different frequency bands than macrocells (i.e., split-spectrum approach), femto-to-femto interference remains the major issue. In particular, congestion cases in which femtocell demands exceed the available resources pose an important challenge. In this paper, we propose a joint resource and power allocation strategy for the management of interference in cooperative femtocell networks. We model the resource and power allocation problem as an operations research game, where imputations are deduced from cooperative game theory, namely the Shapley value and the Nucleolus, using utility components results of partial optimizations. The performance of the developed solutions is analyzed and extensive simulation results are presented to illustrate their potential advantages. In particular, we show that the Shapley value solution with power control offers the overall best performance in terms of throughput, fairness, and transmit power, compared to alternative solutions.**

I. Introduction

Femtocells have recently emerged as a promising technology to enable broadband connectivity in mobile access networks. Instead of redimensioning macrocells at the base station level, the modular installation of low-cost and low-power user-deployed units can provide multiple benefits. Indeed, it is expected that femtocells will enhance coverage indoors, deliver higher throughputs and off-load traffic from existing macro-cellular networks [1]. However, the deployment of Femtocell Access Points (FAPs) raises several technical issues among which interference management remains the most challenging. Interferences can occur with the macrocells as well as with neighboring FAPs, especially in suburban and urban environments. Under certain design choices, crosslayer interference with the macrocell is manageable, while co-layer interference among FAPs requires collaboration among neighboring cells. We can refer to this as collaborative femtocell networks since coordination or cooperation mechanisms are needed between independent femtocells to manage reciprocal interferences, power levels and resource allocation. Instead of unilaterally competing to access the radio resources, dissipating energy to provide higher speed communication to users, femtocells can cooperate under binding agreements in order to reduce interferences in a strategically acceptable way.

In this paper, we propose a game-theoretic approach for strategic resource and power allocation in collaborative femtocell networks. We formulate the problem as an operations research (OR) game in which the FAPs are modeled as players evaluating strategic coalitions between them so as to find power levels that maximize users' throughput and control interference. Based on these evaluations, femtocells' demands can be rescaled to strategically justified values. Finally, a power-level and throughput optimization using the rescaled demands is conducted. We evaluate game imputations based on two possible cooperative game theory methods, the Shapley value [2] and the Nucleolus [3]. The performance of the developed solutions is analyzed and extensive simulation results are presented to illustrate their potential advantages. In particular, we show that the Shapley value solution with power control offers the overall best performance in terms of throughput, fairness, and transmit power, compared to alternative solutions.

The reminder of this paper is organized as follows. Section II presents an overview of related works. In Section III, we describe the context of our work and formulate the problem as an OR game approach. Section IV presents our proposed game-theoretic approach, followed by a discussion of simulation results in Section V. Finally, Section VI concludes this paper.

II. Related Work

Interference management using power control has been extensively studied in the literature. [4] is a seminal work in this field, which approach converges to a Pareto-optimal solution whenever there exist power settings that meet the minimum signal-to-interference-plus-noise ratios (SINRs) constraint for all users.

In the context of femtocell networks, authors in [5] proposed a decentralized strategy to allocate Resource Blocks (RBs) and regulate femtocell's transmit powers depending on their distance from the underlying macrocell. In this case, distance information should be exchanged between femtocells and macrocells to calculate the minimum and maximum power allowed for transmission.

The authors in [6] study the power loading and resource allocation problem. They propose a water filling algorithm to mitigate interference from femtocells toward macrocells, but give higher priority to macrocells, which may results in a fairness problem, especially with the increasing number of indoor femtocell users.

A hybrid centralized/distributed approach is proposed in [7], in which the authors exploit cooperation among neighboring femtocells and improve resource allocation and throughput satisfaction via power optimization. First, femtocells are grouped in a distributed fashion into disjoint clusters with respect to interference maps. Then, within each cluster, a joint resource and power allocation is centralized at a cluster-head that periodically optimizes the throughput satisfaction, while minimizing the transmit power.

Recently, there has been significant interest in applying game theory to the analysis of collaborative communication networks, with the aim to identify rational strategic solutions for multiple decision-maker situations. As opposed to mono-decision maker problems which can be solved with centralized approaches, game-theoretic solutions adopt a multi-agent approach to account for different objective functions and/or counter objections to rationally non justified solutions [8]. When the collaboration among network agents does not imply binding agreements and need just coordination, non-cooperative game theory can identify strategic solutions as a function of various types of game equilibria [9]. Some proposals in this direction are [10] – [12], where authors investigate several power control games that converge to the Nash equilibrium. When instead binding agreements are required to motivate cooperation, cooperative game theory allows solutions with the desirable properties of efficiency and rationality [13]. Specifically, authors in [14] model the femtocell spectrum sharing problem as a coalitional game in partition function form using an utility function that captures the costs in terms of transmit power.

In this paper, differently than in [14], our objective is to define cyclic spectrum and power allocation rules. Rather than partitioning the femtocell network topology in disjoint clusters as in [7], we allow femtocells to negotiate both resources and transmit powers in multiple femtocell groups, where groups are locally detected as function of interferer femtocell neighbors. We target a solution in which the joint resource and transmit power allocation is periodically pre-computed based on changing femtocell resource demands and interference maps. In particular, we consider dense environment situations in which the overall demands exceed the available resources. We investigate two solution concepts in cooperative game theory: the well-known Shapley value [2] and the less-known Nucleolus [3].

III. Context and Problem Formulation

We consider an OFDMA (e.g., LTE) femtocell's network consisting of several FAPs representing residential or enterprise networks. In such system, the frame structure relies on time-frequency RBs, also called tiles. In our study, we focus on co-layer interference mitigation as in [7], [14], and we study the case of downlink communications. Each FAP serves a number of users. User demands represent the required bandwidth, then expressed in number of required tiles.

As already mentioned, in urban dense environment, we expect that the overall demand of femtocells is often higher than the available resources. Therefore, our objective is to find, for such congestion situations, a strategic resource and power allocation that guarantee throughput expectations while controlling the interference between femto-femto users. In the following, we first present notations used in our analysis, then we present the corresponding (mono decision-maker) optimization problem.

A. Notations

- $\mathcal{F} = \{F_1, ..., F_N\}$ is the set of FAPs, where N is the total number of femtocells deployed in the network.
- $\mathcal{I}_n$ denotes the interference set of $F_n \in \mathcal{F}$, which corresponds to the set of femtocells composed of F_n and the femtocells causing interference to F_n. Note that interference is not symmetric since it depends on user positions.
- $\mathcal{U}_n$ is the set of users attached to the FAP F_n.
- d_u^n denotes the demand of user $u \in \mathcal{U}_n$.
- $\mathcal{K} = \{1, ..., K\}$ is the set of available tiles.
- $\Delta_{n,u}^k$ is the binary resource allocation variable for user $u \in \mathcal{U}_n$, which is set to 1 if the tile k is used, and 0 otherwise.
- $P_{n,u}^k$ is the transmit power allocated from FAP F_n to its user u on tile k, where $P_{n,u}^k \geq P_{min}$ if the tile k is used by user u, or $P_{n,u}^k = 0$ otherwise.
- P_{min} is the minimum required transmit power per tile for a successful transmission.
- P_{max} is the total power constraint per FAP.
- $\Gamma_{u,k}$ is the required SINR for user u on tile k.

B. Related Optimization Problem

For the sake of comparison with common resource and power allocation (RPA) problem approaches, between *non-independent* femtocell networks, let us first show how RPA could be formulated as a mono decision-maker optimization problem, i.e., as in the QP-FCRA approach [7] mentioned in Section II.

If femtocells are not independent, a centralized node (i.e., the cluster-head in the case of QP-FCRA) may solve the RPA problem as shown in Problem 1.

Problem 1 RPA problem formulation

$$\min \sum_{F_n \in \mathcal{F}} \sum_{u \in \mathcal{U}_n} \sum_{k=1}^{K} \alpha \;\; P_{n,u}^k - (1-\alpha)\Delta_{n,u}^k$$

`subject to:`

$(a)\;\; \forall k,\;\; \forall F_n \in \mathcal{F},\;\; \forall u \in \mathcal{U}_n :$

$$P_{n,u}^k \geq \Gamma_{u,k} \times pl(u,n) \times (\sum_{m \neq n} P_{m,u'}^k / pl(u,m) + \sigma^2)$$
$$-(1-\Delta_{n,u}^k) \times M \times P_{max}.$$

$$(b)\;\; \forall k,\;\; \forall F_n \in \mathcal{F},\;\; \forall\;\; u,v \in \mathcal{U}_n : \Delta_{n,u}^k + \Delta_{n,v}^k \leq 1$$

$$(c)\;\; \forall F_n \in \mathcal{F},\;\; \forall u \in \mathcal{U}_n :\;\; \sum_{k=1}^{K} \Delta_{n,u}^k \leq d_u^n$$

$$(d)\;\; \forall F_n \in \mathcal{F} :\;\; \sum_{u \in \mathcal{U}_n} \sum_{k=1}^{K} P_{n,u}^k \leq P_{max}$$

$$(e)\;\; \forall k,\;\; \forall F_n \in \mathcal{F},\;\; \forall u \in \mathcal{U}_n :\;\; P_{n,u}^k \geq \Delta_{n,u}^k \times P_{min}$$

$$(f)\;\; \forall k,\;\; \forall F_n \in \mathcal{F},\;\; \forall u \in \mathcal{U}_n :\;\; \Delta_{n,u}^k \in \{0,1\}$$

In this problem, $pl(u,n)$ denotes the path loss between user u and its FAP F_n, $w_{u,k} = \sum_{m \neq n} P_{m,u'}^k / pl(u,m)$ represents the interference suffered by user u on the tile k, and σ is the noise density. Note that in our case, the path loss is modeled based on A1-type generalized path loss models in the frequency range $2-6$ GHz developed in WINNER [15].

Condition (a) denotes that the transmit power on tile k should guarantee the required SINR. The second term on the right hand of the inequality ensures that $P_{n,u}^k = 0$ if $\Delta_{n,u}^k = 0$, where M is a carefully chosen very high value. If the tile is in use ($\Delta_{n,u}^k = 1$), then the second part of the inequality turns to zero and the $P_{n,u}^k$ gets the required value. Condition (b) ensures that two users attached to the same FAP cannot use the same tile. Condition (c) indicates that a user can not obtain more than what he demands. Conditions (d) and (e) refer to the power constraints, and finally condition (f) indicates that $\Delta_{n,u}^{k}$ is a binary variable.

Later, we compare our proposal to such QP-FCRA [7] solution arising the interest in strategic approaches and stressing tradeoffs between them. It is worth noting that QP-FCRA is used as baseline for comparison since Problem 1 is NP-hard, hence a complete centralized solution is not possible.

IV. Proposed Game Theoretic Approach

As mentioned earlier, in urban environments, a dense deployment of femtocells is expected, so that situations in which the overall resource claim (i.e., sum of the demands) overcomes the amount of available tiles (K) in the shared spectrum. In such situations, we cannot ensure that the resource assignment (i.e., tiles as well as the corresponding transmit power allocation), while resolving the RPA problem as in [7], is strategically done and that users are faithfully and equally treated. More clearly, the objective of Problem 1 is to optimize the total transmit power, without adapting the allocation to each femtocell claim and interference situation. For example, we have to avoid penalizing femtocells presenting low interference degree and those with lower demands.

This suggests to resolve the RPA problem via collaboration among neighboring femtocells, under an adequate binding agreement fixing common rules on shared information and allocation scheme. In the following, we detail the proposed game-theoretic approach, which is composed of two main phases: an Interference Set Detection phase, and an Operations Research Game Iteration phase.

A. Interference Set Detection

Upon each significant change in demands or in network topology, each femtocell determines the set of interferer femtocells that cause interference to its users based on the minimum required SINR. FAPs are able to share their interference set with other FAPs in the network. Next, the list of interference sets are sorted, first according to their cardinality, and then according to the overall demands, both in a decreasing manner (i.e., first the largest sets with highest overall demands).

B. Operations Research Game Iteration

In the second phase, resources as well as transmit powers are eventually allocated, proceeding with solving an operations research (OR) game, i.e., a cooperative game with coalitional values computed as a result of partial optimizations (detailed here-after) for each interference set, and following the order in the sorted list from the first phase (the rational is that we first solve the most critical situations). Strategically, in this way we do not penalize FAPs that interfere less compared to FAPs that interfere more, as well as FAPs that claim little resources compared to FAPs that claim a lot.

Let us now focus on the OR game modeling within each iteration (i.e., within an interference set). We distinguish here between two steps: (i) Demands rescaling, and (ii) Tiles and transmit power assignment.

1) Demands Rescaling: First, within an interference set, demands of each FAP are rescaled in order to allocate rational resources to each player (i.e., FAP) without exceeding the available resources. Indeed, assuming that femtocells belonging to the same interference set share information about respective demands, the interaction can be modeled as a cooperative game. In essence, FAPs that are member of the same coalition S, within the same given interference set $\mathcal{I}_n$, cooperate in order to determine the allocated tiles as well as the required transmit power, so as to avoid interference among themselves. However, FAPs that are members of a given coalition $S \subseteq \mathcal{I}_n$ are still affected by the transmit power of FAPs outside S, i.e., the FAPs in $\mathcal{I}_n \backslash S$. In our case, we assume a worst case conflicting scenario, where FAPs outside S transmit with the maximum allowed power P_{max}. Indeed, FAPs outside S do not cooperate, and hence can use the maximum allowed power to satisfy at maximum their users.

Thus, the game starts by performing local optimizations within each coalition S using Problem 1 defined above, i.e., $\mathcal{F} \equiv S$. For each coalition $S \subseteq \mathcal{I}_n$, we define the profit $v(S)$ reflecting the total benefit, in terms of resources, when FAPs form the coalition S, as follows.

$$v(S) = max\Big(0,\ |S| \times K - \sum_{F_n \in S} \sum_{u \in \mathcal{U}_n} x_u^n\Big) \tag{1}$$

where $x_u^n = d_u^n - \sum_{k=1}^{K} \Delta_{n,u}^{k}$ indicates the resources that are not available for user u, due to indeed that femtocells outside the coalition transmit at maximum power. Recall that this parameter is obtained by resolving the above-mentioned optimization problem.

Once $v(S)$ is computed for each $S \subseteq \mathcal{I}_n$, both the Shapley value [2] and the Nucleolus [3] can be used to strategically determine the resources each player (i.e., FAP) should have. Each FAP then updates its demands according to the new computed value: demands are thus rescaled with values that are strategically justified and rationally acceptable by all competing femtocells, since they have been computed while accounting for all possible strategic situations (the coalitions).

Finally, using the rescaled demands, a global optimization can be performed to assign resources (i.e., tiles) as well as the final transmit power on each tile to users. This is the aim of the second step.

2) Tiles and Transmit Power Assignment: Knowing now the exact amount of resources that each FAP within the given interference set $\mathcal{I}_n$ should have, a global optimization is performed to assign, for each FAP within $\mathcal{I}_n$, the dedicated resources along with the final corresponding transmit power. To this end, Problem 1 defined above is solved again such that $\mathcal{F} \equiv \mathcal{I}_n$ in this case, and taking as input the rescaled demands computed in the previous step.

It is worth noting that the above two steps are repeated for all interference sets following the order in the sorted list from the first phase. Since a FAP can belong to many interfering sets, if it has already participated to a game in a previous game iteration, it is excluded from the next game iteration in which it appears. However, we note that the interference that may be produced by the corresponding FAP is taken into account. Indeed, in Problem 1, $P_{m,u'}^{k}$, which corresponds to the transmit power of interferer femtocells, is either equal to P_{max} if $F_m \in \mathcal{I}_n \backslash S$ and has not yet participated in a previous game iteration or adjusted to its already computed value, otherwise.

V. Performance Evaluation

In this section, we evaluate the performance of the proposed OR game approach using two game-theoretic imputation solutions for demands rescaling, i.e., the Shapley value and

TABLE I
SIMULATION PARAMETERS

N	200	σ^2	$-121.45\ dBm$
K	100	P_{max}	$20\ mW$
α	10^{-3}	P_{min}	$0.1\ mW$

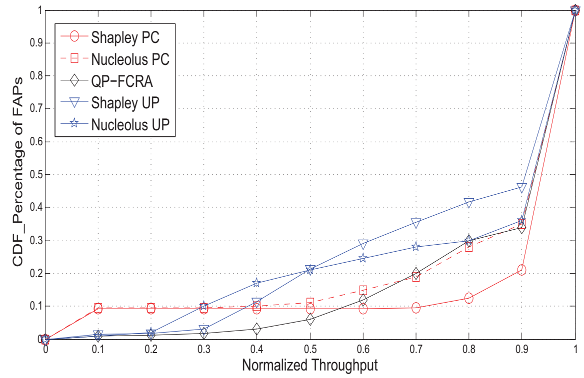
Nucleolus. We compare the benefits of our approaches with respect to the case where the transmit power is uniformly distributed among all allocated tiles (i.e., Shapley value and Nucleolus with uniformly distributed power, as in our previous work [16]), as well as the semi-centralized optimization approach, as in QP-FCRA [7]. Note that the corresponding optimization problems are solved using the solver "IBM ilog cplex" [17].

We simulated several scenarios with a dense network size of 200 FAPs where, for each simulation, FAPs are randomly distributed in a 2-D 400m×400m area. We considered two interference level scenarios, a low-level one and a high-level one, based on two SINR thresholds, 10 and 25 dB, to show the impact of the interference degree on the performance metrics. Based on the SINR, the path loss model of WINNER [15], and with static user positions; each FAP determines the set of its interferer femtocells depending on the received signal strength. Users are uniformly distributed within the FAPs with a maximum number of four users per FAP. Each user uniformly generates its traffic demand that can be directly translated to a certain number of tiles, with a maximum value of 25 tiles per user. The analysis is achieved using a typical OFDMA frame (downlink LTE frame) consisting of $K = 100$ tiles. The simulation parameters are reported in Table I. We focus on the comparison among the different strategies based on the offered normalized throughput, the allocation fairness, as well as the transmit power.

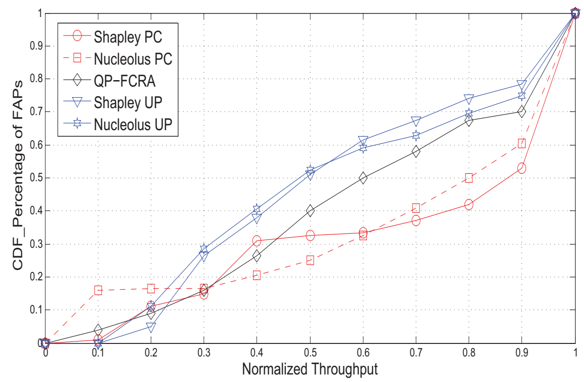
A. Throughput analysis

Fig. 1 reports the mean normalized throughput (i.e., mean ratio of the number of allocated tiles to the total initial demands; in the following referred to as throughput) for the two interference level scenarios. We can observe that the game-theoretic approaches with power control (referred to as Shapley PC and Nucleolus PC in the figure) outperform the other schemes, especially in high interference level [see Fig. 1(b)]. In particular, we can observe that:

- The median throughput is always higher for the Shapley PC; e.g., in the high interference case, 87% for the Shapley PC, meaning that 50% of femtocells have a throughput of 87% or more, 80% for Nucleolus PC, 60% for QP-FCRA, and 50% for both Shapley and Nucleolus with uniformly distributed power (referred to as Shapley UP and Nucleolus UP in the figure).
- At medium and high throughputs, our game-theoretic approaches with power control outperforms the remaining schemes; e.g., in the high interference case, Shapley PC allows 48% of FAPs with throughput greater than 90%, compared to 40% for Nucleolus PC, 30% for QP-FCRA, 25% for Nucleolus UP, and 20% for Shapley UP.
- Among the game-theoretic approaches, the Shapley value persistently outperforms the Nucleolus, with relevant differences at high throughputs.
- At low throughput and low interference level [see Fig. 1(a)], both QP-FCRA and game-theoretic approaches with uniformly distributed power offer good performance as they ensure that only 2% of FAPs obtain a throughput less than 10%, compared to 10% for game-theoretic solutions with power control.



(a) SINR = 10 dB



(b) SINR = 25 dB

Fig. 1. Throughput Cumulative Distribution Function (CDF)

The latter point can be explained by the fact that, our approaches strategically allocate low transmit powers for users, even if they are located at the cell edge, to control interference. In low interference scenario, this results in lower throughput compared to the other schemes, which use higher transmit power. However, such agreement between FAPs maximizes the throughput for the majority of femtocells, as shown in Fig. 1(a), where Shapley PC allows 80% of femtocells with throughput greater than 90%, compared to 65% for both Nucleolus PC/UP and QP-FCRA, and 50% for Shapley UP.

All in all, the Shapley value with power control seems the most appropriate approach with respect to the offered throughput, especially in high interference scenario, as in urban environments with a dense deployment of femtocells.

B. Fairness analysis

We evaluate the fairness of the solutions using two aspects. (i) With respect to the Jain's fairness index, defined as:

$$FI = \left(\sum_{n=1}^{N} \sum_{u \in \mathcal{U}_n} (\beta_u^n / d_u^n) \right)^2 \Big/ \left(N \times \sum_{n=1}^{N} \sum_{u \in \mathcal{U}_n} \left(\beta_u^n / d_u^n \right)^2 \right) \tag{2}$$

where $\beta_u^n = \sum_{k=1}^{K} \Delta_{n,u}^k$ indicates the allocated resources to user u. The fairness indexes are reported in Table II. We can notice that the Shapley value with power control gives the highest fairness, thanks to the strategic constraints that avoid

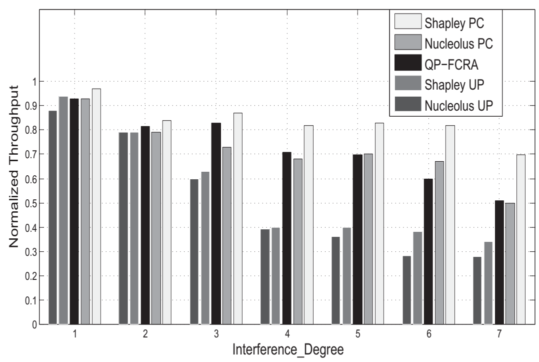


Fig. 2. Throughput distribution as a function of the interference degree, SINR = 10 dB.

TABLE II
MEAN FAIRNESS INDEXES

SINR	Nucle. PC	Shap. PC	Nucle. UP	Shap. UP	QP-FCRA
10 dB	0.9129	0.9318	0.6927	0.7201	0.9167
25 dB	0.7609	0.7891	0.5852	0.6239	0.7742

penalizing femtocells presenting low interference degree (see Fig. 2). On the other hand, the performance of Nucleolus PC is slightly lower than the QP-FCRA approach, but remains a far better than the case with uniformly distributed power.

(ii) Fig. 2 further investigates how femtocell interference degree is taken into account, illustrating the mean normalized throughput as a function of the interference degree. We can clearly notice that the Shapley value with power control always outperforms the other methods. It is appropriate to conclude that the interference degree is taken into account in a significantly different way with the Shapley value, showing an interesting fairness performance.

C. Transmit Power analysis

Finally, Fig. 3 shows the transmit power per user as function of the distance to the served FAP. We can notice that game-theoretic approaches with power control clearly minimize the allocated transmit power compared to the other schemes, while achieving a higher throughput, as shown in the previous figures. In addition, we can observe that the transmit power increases with the distance to the served FAP. This is simply because users far away from their FAP need more power to reach them, so that more transmit power is needed.

VI. CONCLUSION

In this paper, we have presented a novel approach based on cooperative game theory to address the problem of interference mitigation in femtocell networks. Specifically, we presented a game-theoretic approach for strategic resource and power allocation in cooperative femtocell OFDMA networks. Upon detection of interference maps, the proposed approach iterates operations research games from the largest interference set with highest demand to the lower sets. Within each iteration, femtocells' demands are first rescaled by performing local optimizations within the formed strategic coalitions, then a global optimization problem using the rescaled demands as input is solved to assign resources as well as transmit power to femto users. We adopted solutions from coalitional game theory, the Nucleolus and the Shapley value, and analyzed the performance of the developed schemes. Compared to two alternative solutions without demand rescaling, one based on game theory but with fixed femtocell transmit powers, and one based on semi-centralized computations, our proposed approach achieves better performance. In particular, the Shapley value solution with power control is strictly superior to all the others in terms of throughput, fairness, and transmit power. This approach represents therefore a promising solution for resource and power allocation in future femtocell network deployments.

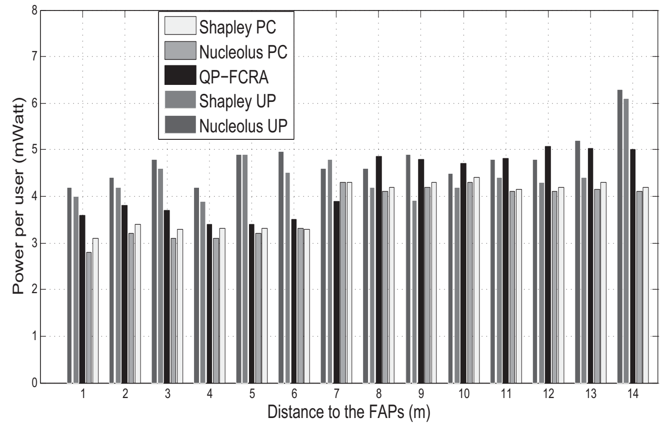


Fig. 3. Transmit power per user as a function of the distance, SINR = 25 dB.

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