# Solving the Binary Quadratic Problem with convexification and spectral-gauge cuts

## Keywords

Master 2 internship or Master Thesis.
Integer programming, Semidefinite programming relaxations, Branch-and-bound

## Supervision

- Sourour Elloumi from UMA, Ensta, Palaiseau, and
- Sophie Demassey, Antonio Sasaki, Valentina Sessa from UMA, PSL, Mines de Paris, Sophia-Antipolis

## Contact

sourour.elloumi(@)ensta.fr, sophie.demassey(@)minesparis.psl.eu

## Objectives

Optimizing a nonconvex quadratic function on binary variables

$$(BQP): \ \min x^\top Q x : x \in \{0,1\}^n, \ \text{with } Q \not\succeq 0$$

is the simplest form of global optimization, but not the easiest problem to solve.

The semidefinite programming (SDP) relaxation provides tight lower bounds but solvers for it are hard to integrate into a branch-and-bound and they do not scale well. A second option is to reformulate the continuous relaxation as a linear program based on McCormick constraints but the bound is weaker and this reformulation grows quadratically with $n$. The object of this internship is to experiment on two other alternative approaches, involving the novel spectral-gauge cuts [1].

The spectral-gauge cuts provide a generic framework of a linear cutting-plane algorithm for solving SDPs with general theoretic guarantees and promising practical results due to their easy computation. We aim to assess and improve the framework in the context of solving large instances of the Binary Quadratic Problem, using a branch-and-cut algorithm and two different bounding techniques.

1. In the first approach, we propose to replace the solution of the SDP relaxation at each node of the branch-and-bound, with the solution of the LP relaxation augmented with spectral-gauge cuts.

2. In the second approach, we also propose to approximate the SDP relaxation using such a linear relaxation, but only at the root node. Following the QCR method [2], the dual solution allows us to formulate another good – continuous convex quadratic – relaxation of BQP. We can then use this relaxation for bounding in the branch-and-bound, and keep generating cuts at each node to still improve the bound.

This numerical study will be conducted using the Cplex or Gurobi branch-and-bound and callbacks to implement the cut generation. It will explore the impact of different parametrizations of the cut framework: limits on the number of cuts, choices of the gauge and of the rank for generating the cuts, ways to enforce cut sparsity, etc.

Additional adaptations of the framework to the context of BQP could then be envisaged, e.g. tightening the cuts regarding to the objective, or disaggregating the positive semidefinite constraint regarding its sparsity.

## References


[1] A. Sasaki, S. Demassey, V. Sessa (2026) Spectral-gauge cuts for semidefinite programming. https://arxiv.org/abs/2607.05935

[2] A. Billionnet, S. Elloumi (2007). Using a mixed integer quadratic programming solver for the unconstrained quadratic 0-1 problem. Mathematical programming, 109(1), 55-68.