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# Graph Learning with $\ell_0$-norm & SLOPE Penalties

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**Contacts**: {arnaud.breloy, amelie.lambert} @lecnam.net , zacharie.ales@ensta.fr
**Location**: 2 rue Conté, Paris.

The graphical Lasso is a prominent method to estimate the conditional correlation graph in Gaussian graphical models (GGM). This allows us to obtain a graph topology from raw signals, observed at each nodes, as presented in Figure 1. The obtained graph can then be used in many algorithms (graph clustering, prior structure for a GNN) and applications (notably time-series analysis). This algorithm aims at solving the following optimization problem

$$\underset{\boldsymbol{\Theta}\in\mathcal{S}_p^{++}}{\text{minimize}} \quad \text{Tr}(\mathbf{S}\boldsymbol{\Theta}) - \log|\boldsymbol{\Theta}| + \lambda||\boldsymbol{\Theta}||_{1,\text{off}}, \tag{1}$$

in which the $\ell_1$-norm penalty promotes a sparse structure into the precision matrix of a Gaussian model.

In this project, we will investigate the use of alternative penalties, such as:

- SLOPE, that can be addressed with the algorithm in [1, 2],
- $\ell_0$-norm, that can be addressed with the toolbox [3],

that could improve the graph learning process. The algorithms will be validated on simulated data, and time series from various sources (sensor networks, S&P500 dataset), see e.g. examples in [4, 5, 6].

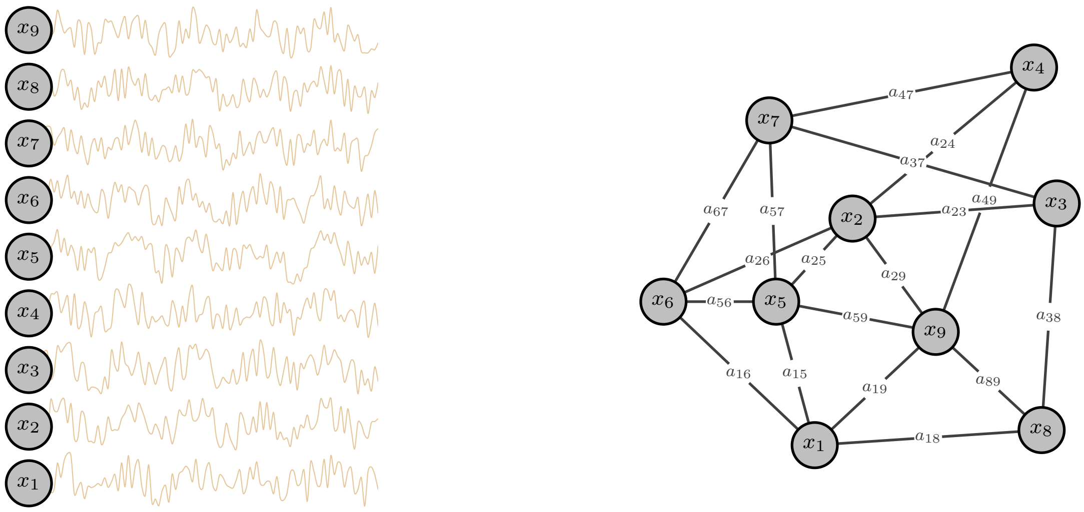


Figure 1: Signals at each nodes (data matrix $\mathbf{X}$), and estimated conditional correlation graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathbf{A})$.

## References

[1] J. Larsson, Q. Klopfenstein, M. Massias, and J. Wallin, "Coordinate descent for slope," in *International Conference on Artificial Intelligence and Statistics*. PMLR, 2023, pp. 4802–4821.

[2] J. Larsson, M. Bogdan, K. Grzesiak, M. Massias, and J. Wallin, "Efficient solvers for slope in r, python, julia, and c++," *arXiv preprint arXiv:2511.02430*, 2025.

[3] T. Guyard, C. Herzet, and C. Elvira, "El0ps: An exact l0-regularized problems solver," 2025.

[4] S. Kumar, J. Ying, J. V. d. M. Cardoso, and D. P. Palomar, "A unified framework for structured graph learning via spectral constraints," *Journal of Machine Learning Research*, vol. 21, no. 22, pp. 1–60, 2020.

[5] J. Ying, J. V. de Miranda Cardoso, and D. Palomar, "Nonconvex sparse graph learning under Laplacian constrained graphical model," *Advances in Neural Information Processing Systems*, vol. 33, pp. 7101–7113, 2020.

[6] C. Pouliquen, P. Gonçalves, T. Vayer, and M. Massias, "En quête de précision: Un benchmark open-source et un solveur polyvalent pour le graphical lasso," in *GRETSI 2025-XXXème Colloque Francophone de Traitement du Signal et des Images*, 2025.