Consider a convex quadratic constraint ($Q \succeq \mathbf{0}$):

$$\mathbf{x}^\top Q\mathbf{x} + 2\mathbf{q}^\top\mathbf{x} + r \leq 0 \tag{*}$$

Here are two ways to cast it into a second order cone constraint used in SOCP.

Method 1

Write $Q = LL^T$ using either the eigen-decomposition or the square root one (or Cholesky). Let us consider the SOCP constraint:

$$\left\| \begin{matrix} 1 + 2\mathbf{q}^\top\mathbf{x} + r \\ 2L^\top\mathbf{x} \end{matrix} \right\|_2^2 \leq \left(1 - (2\mathbf{q}^\top\mathbf{x} + r)\right)^2$$

This SOCP inequality boils down to:

$$\begin{aligned} (2L^\top\mathbf{x})^\top(2L^\top\mathbf{x}) + (1 + (2\mathbf{q}^\top\mathbf{x} + r))^2 &\leq \left(1 - (2\mathbf{q}^\top\mathbf{x} + r)\right)^2 \\ 4\mathbf{x}^\top LL^\top\mathbf{x} + (1 + (2\mathbf{q}^\top\mathbf{x} + r))^2 &\leq \left(1 - (2\mathbf{q}^\top\mathbf{x} + r)\right)^2 \\ 4\mathbf{x}^\top Q\mathbf{x} + 4 \cdot (2\mathbf{q}^\top\mathbf{x} + r) &\leq 0, \end{aligned}$$

which is (*).

Method 2

We still write $Q = LL^\top$, but we will see that we need to take L to be the square root matrix of *positive definite* Q, so that $L = L^\top$.

Let us consider the SOCP constraint:

$$\left\| L^T x + {L^{-1}}^T\mathbf{q} \right\|_2^2 \leq -r + \mathbf{q}^T Q^{-1}\mathbf{q}$$

This SOCP inequality boils down to:

$$\begin{aligned} \mathbf{x}^\top LL^\top\mathbf{x} + \mathbf{q}^\top \underbrace{L^{-1}{L^{-1}}^\top}_{=Q^{-1} \text{ iff } L=L^\top} q + 2\mathbf{q}^T \underbrace{L^{-1}L^\top}_{=I \text{ iff } L=L^\top} \mathbf{x} &\leq -r + \mathbf{q}^T Q^{-1}\mathbf{q} \\ \mathbf{x}^T Q\ x + \mathbf{q}^T Q^{-1}\mathbf{q} + 2\mathbf{q}^T\mathbf{x} &\leq -r + \mathbf{q}^T Q^{-1}\mathbf{q} \end{aligned}$$

which is (*). The underbraces show that we need $L = L^\top$. Since we use Q^-1, we also need Q to be strictly positive definite.