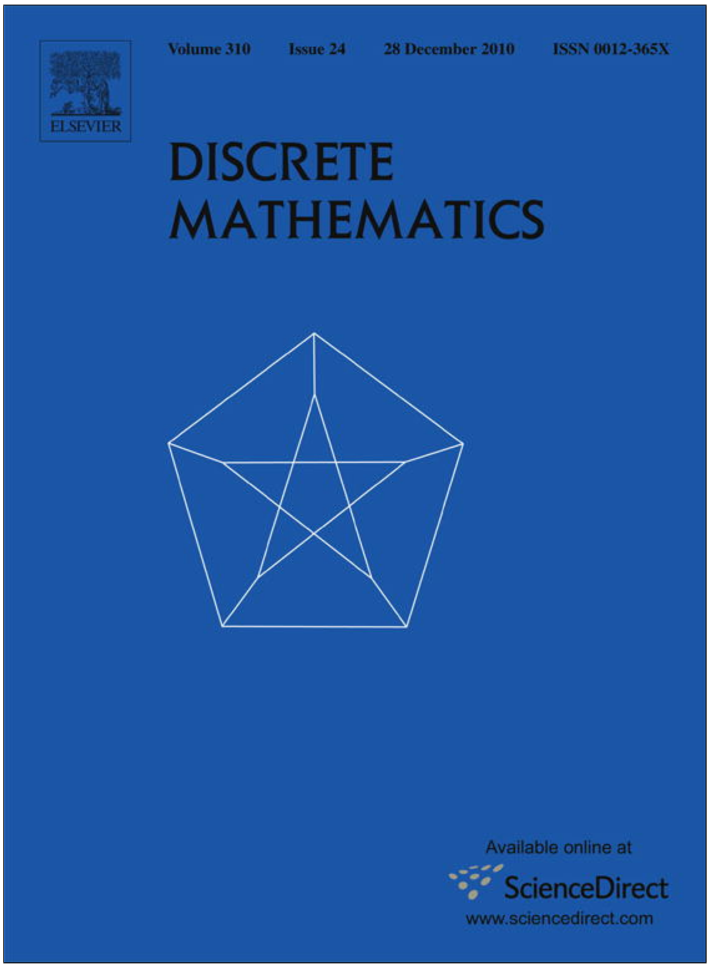
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Note

A note on a conjecture on maximum matching in almost regular graphs

C. Picouleau

Conservatoire National des Arts et Métiers, CEDRIC Laboratory, Paris, France

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ABSTRACT

Mkrtchyan, Petrosyan, and Vardanyan made the following conjecture: Every graph G with $\Delta(G) - \delta(G) \leq 1$ has a maximum matching whose unsaturated vertices do not have a common neighbor. We disprove this conjecture.

Mkrtchyan et al. [1] conjectured that every nearly regular graph G (that is, with $\Delta(G) - \delta(G) \leq 1$) has a maximum matching whose unsaturated vertices do not have a common neighbor. As usual, $\Delta(G)$ and $\delta(G)$ denote the maximum and minimum degrees of vertices in G. We present an example disproving this conjecture; it is the graph G in Fig. 1.

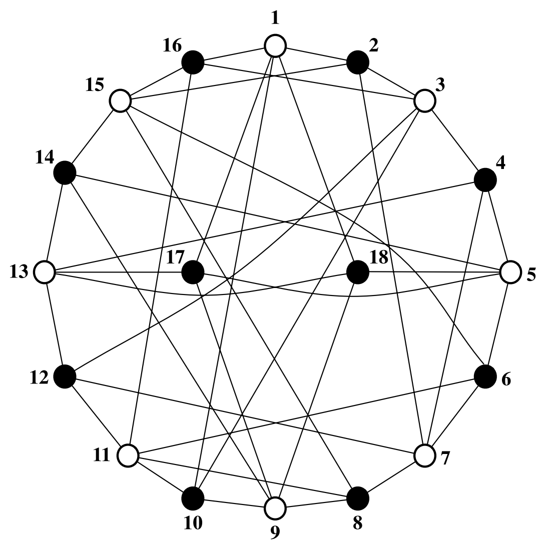

Fig. 1. The graph G.

Observe that G is bipartite, with eight white vertices and ten black vertices. Each black vertex has degree 4, and each white vertex has degree 5. Since G is unbalanced, there is no perfect matching, and the edges matching $2i - 1$ to $2i$ for $1 \leq i \leq 8$ form a maximum matching. Note that vertices 17 and 18 have the same neighborhood: $\{1, 5, 9, 13\}$.

E-mail address: christophe.picouleau@cnam.fr.

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We show that for each maximum matching M of G, the two unsaturated vertices have a common neighbor. Let x and y be the two unsaturated vertices; note that x and y must both be black vertices. In fact, we show that every two black vertices have a common neighbor, so we can ignore the matchings. We list the cases:

1. $\{x, y\} = \{17, 18\}$: x and y have four common neighbors.
2. $x \in \{17, 18\}$, $y \notin \{17, 18\}$: y is adjacent to 1, 5, 9, or 13.
3. $x, y \notin \{17, 18\}$: If x and y differ by 2 (modulo 16), then the vertex between them on the outer cycle is a common neighbor. We list the other pairs as (x, y, z), where z is a common neighbor of x and y, in increasing order of the distance between the labels of x and y: (2, 6, 15), (4, 8, 7), (6, 10, 11), (8, 12, 11), (10, 14, 9), (12, 16, 11), (14, 2, 15), (16, 4, 3), (2, 8, 15), (4, 10, 3), (6, 12, 7), (8, 14, 9), (10, 16, 11), (12, 2, 3), (14, 4, 5), (16, 6, 11), (2, 10, 3), (4, 12, 3), (6, 14, 15), (8, 16, 15).

Hence the conjecture is false when G is bipartite with maximum degree 5 and minimum degree 4. The question remains open for 4-regular graphs with $r \geq 4$. For bipartite regular graphs, the statement holds trivially, since every such graph has a perfect matching.

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References

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