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Discrete Optimization

Lot sizing with carbon emission constraints

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ABSTRACT

This paper introduces new environmental constraints, namely carbon emission constraints, in multi-sourcing lot-sizing problems. These constraints aim at limiting the carbon emission per unit of product supplied with different modes. A mode corresponds to the combination of a production facility and a transportation mode and is characterized by its economical costs and its unitary carbon emission. Four types of constraints are proposed and analyzed in the single-item uncapacitated lot-sizing problem. The periodic case is shown to be polynomially solvable, while the cumulative, global and rolling cases are *NP*-hard. Perspectives to extend this work are discussed.

1. Introduction

Legislation is evolving in order to enforce control on carbon emissions. This will probably be done by constraining companies to emit less than a given amount of carbon dioxide by product unit that is produced and transported. Along with legislation and norms, some companies may volunteer in this direction for marketing reasons and to get a competitive advantage. The amount of carbon emission will probably appear on item packages in the near future. Companies will face new constraints that will force them to reduce carbon emissions while still minimizing production and transportation costs. There are few papers addressing production planning and transportation problems that take into account environmental constraints. Generally, environmental issues are integrated as cost components in the objective function, and the resulting problems are solved using multi-criteria approaches (see [13,1]). Note that classical cost components (production and transportation costs) have the same behavior as environmental cost components (e.g. reducing the number of vehicles or the total distance).

One of the main objectives of green logistics is to evaluate the environmental impact of different distribution and production strategies to reduce the energy usage in logistics activities. Although the interest in green logistics has grown in the last decades, current logistics practice still rarely complies with environmental constraints. One of the objectives of the Kyoto protocol is to stabilize and then reduce greenhouse gas emissions in order to limit global warming. Carbon dioxide being one of the most important greenhouse gases, countries will have to reduce their carbon emissions, and will require companies to trim down their carbon emissions. A quota of carbon emission per company will probably be fixed (e.g. California). As a result, some companies have already started to monitor their carbon footprint to evaluate the environmental impact of their activities. The classical production and distribution models focus on the minimization of costs subject to operational constraints. Considering green logistics objectives and constraints will lead to new problems resulting in novel combinatorial optimization models.

Green supply chain management (see [23]) has been extended to include green inventory models that link inventory and ordering behavior and emissions. Sbihi and Eglese [21] present a short state-of-the-art on green logistics and combinatorial optimization. They describe some of the problems that arise in green logistics which can be formulated as combinatorial optimization problems. They focus on the topics of reverse logistics, waste management and vehicle routing and scheduling. Dekker et al. [9] provides a recent overview of issues and challenges in green logistics and operations research, focusing on supply chain management and design for transportation, inventory and production. Reverse logistics is an important part of Green supply chain management and a lot of research has been conducted in this field. Teunter et al. [24] address this issue for lot-sizing problems, and propose two models. In the first model, they assume that manufacturing and remanufacturing operations are carried out in the same factory. They model these operations with a joint setup cost. In the second model, they as-

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sume that manufacturing and remanufacturing operations are done on two separate lines. Several works on the integration of remanufacturing in a closed loop supply chain can be found in the literature [17,15,19].

Depending on the objectives of a company, the integration of carbon emission constraints can be considered at different decision levels (strategic, tactical and operational). At the strategic level, designing supply chain flows or locating a factory or a warehouse impact green constraints and objectives [8,26,18]. At the tactical level, carbon emissions can be considered in production and distribution planning decisions. Recently, some authors [14,6] study the integration of carbon emission constraints in classical inventory management models. At the operational level, carbon emission constraints can be related to vehicle routing or production scheduling decisions [3,11]. Our paper focuses on the tactical decision level of a supply chain.

There is little research addressing the introduction of carbon emission constraints in production and/or distribution planning models. In lot-sizing, we only found the work of Benjaafar et al. [4] that integrates carbon emission constraints. The authors insist on the potential impact of operational decisions on carbon emissions and the need for Operations Management research that incorporates carbon emission concerns. The authors also point out that the contribution of operational research in this area is almost absent. Benjaafar et al. [4] add a new capacity constraint that links and limits all carbon emissions related to production and storage over the planning horizon. The weakness of this constraint is that producers can create large carbon emissions at the beginning of the horizon by producing large quantities, and balance the total carbon emission by producing nothing at the end of the horizon.

As previously mentioned, the need of companies to monitor their carbon emissions is growing. This monitoring must be consistent with production and distribution planning models that must take into account carbon emission constraints. More precisely, if the monitoring of the carbon footprint is aggregated according to the type of vehicles and their consumption, there is no need to consider more detailed information on each vehicle in production and distribution planning models.

There are several methodologies to calculate carbon emissions (Greenhouse Gas protocol [12], ARTEMIS [2], EcoTransIT [10], etc.). Greenhouse Gas protocol is the most commonly used, since it is easy to use and its scope is worldwide. The unitary carbon emission of a product can be calculated using a linear function that depends on the distance traveled (in kilometers) and on the carbon emission of the vehicle used (in grams of CO_2 per kilometer). This carbon emission model is adopted in this paper where, for a given supplying mode, the carbon emission is proportional to the number of product units that are shipped. We define a supplying mode as a combination of a transportation mode (combining one or more types of vehicles) and a production facility.

In this paper, we study multi-sourcing lot-sizing problems with carbon emission constraints. These new constraints are induced from a maximum allowed carbon dioxide emission coming from legislation, green taxes or the initiatives of companies. Contrary to Benjaafar et al. [4], where a global limit of carbon emission is studied, we consider a maximum environmental impact allowed on average per item. We study four types of carbon emission constraints: (1) a Periodic carbon emission constraint, (2) a Cumulative carbon emission constraint, (3) a Global carbon emission constraint and (4) a Rolling carbon emission constraint. The global carbon emission constraint has the same drawbacks than the constraint introduced in [4].

The main contribution of this paper is twofold. First, we propose new lot-sizing models that take into account different carbon emission constraints. Second, we determine the complexity status of these new models. We propose a polynomial dynamic programming algorithm for the problem with periodic carbon emission constraint, and show that the three other problems are *NP*-hard. The outline of the paper is as follows. In Section 2, we provide different mathematical formulations to model the four types of carbon emission constraints. In Section 3, we show that the uncapacitated multi-sourcing lot-sizing problem with the periodic carbon emission constraint can be solved using a polynomial dynamic programming algorithm. In Sections 4 and 5, we show that the uncapacitated multi-sourcing lot-sizing problem with the cumulative carbon emission constraint, global carbon emission constraint or rolling carbon emission constraint is *NP*-hard. We conclude and discuss some perspectives of this work in Section 6.

2. Mathematical programming models

Consider a multi-sourcing lot-sizing problem faced by a company that must determine, over a planning horizon of T periods, when, where and how much to produce of an item to satisfy a deterministic time-dependent demand. Different production locations and transportation modes are available to satisfy a given demand. Let us consider M different supplying modes, where a mode corresponds to the combination of a production facility and a transportation mode. To each mode are associated classical (fixed and variable) supplying costs together with an environmental impact modeling the carbon emission of the mode. As previously discussed, we assume that this carbon emission is proportional to the amount of units supplied with the mode, and can be expressed as a unitary environmental impact. We study four types of carbon emission constraints: (1) Periodic carbon emission constraint, (2) Cumulative carbon emission constraint, (3) Global carbon emission constraint and (4) Rolling carbon emission constraint. We shall see that the first and third constraint types are actually special cases of the fourth one. To model these new constraints, we define the following parameters and variables.

Parameters:	
d_t	Demand in period t,
	$h_t(s)$
	Cost of holding s units at the end of period t,
p_t^m	Unitary supplying cost of mode m in period t,
f_t^m	Supplying setup cost of mode m in period t,
e_t^m	Environmental impact (carbon emission) related to supplying one unit using mode m in period t,
	$E_t^{\max}$
	Maximum unitary environmental impact allowed in period t.
Variables:	
x_t^m	Quantity supplied in period t using mode m,
y_t^m	Binary variable which is equal to 1 if mode m is used in period t, and 0 otherwise,
s_t	Inventory carried from period t to period $t+1$.

Note that $E_t^{\max}$ depends on period t, since carbon emissions will probably be forced to be decreased by stages and not to a specific level right away. The variations of $E_t^{\max}$ depend on the level (strategic, tactical or operational) at which the lot-sizing models are used. We consider in the following either the stationary case or the general case without any assumption on the time dependency of $E^{\max}$. The classical formulation for the multi-sourcing lot-sizing problem, without carbon emission constraint, is given below:

$$\min \sum_{m=1}^{M}\sum_{t=1}^{T}(p_t^m x_t^m + f_t^m y_t^m) + \sum_{t=1}^{T} h_t(s_t) \quad (1)$$

$$\text{s.t.} \sum_{m=1}^{M} x_t^m - s_t + s_{t-1} = d_t \quad t = 1,\ldots,T \quad (2)$$

$$x_t^m \leqslant \left(\sum_{t'=t}^{T} d_{t'}\right) y_t^m \quad t = 1,\ldots,T,\ m = 1,\ldots,M \quad (3)$$

$$x_t^m \in \mathbb{R}^+, \quad y_t^m \in \{0,1\} \quad t = 1,\ldots,T,\ m = 1,\ldots,M \quad (4)$$
$$s_t \in \mathbb{R}^+, \quad t = 1,\ldots,T$$

The objective function (1) minimizes the fixed and variable production and transportation costs and holding costs. Constraint (2) models flow conservation, and Constraint (3) ensures that no item can be supplied with mode m in period t if this mode is not selected. We add to this formulation the carbon emission constraint (6), (7), (9) or (10), introduced in the following sections.

2.1. Periodic carbon emission constraint

This constraint is very tight, and assumes that the amount of carbon emission that is not used in a given period is lost. This constraint is useful if the company must ensure that it meets periodically its carbon emission objectives, and can be formulated as follows:

$$\frac{\sum_{m=1}^{M} e_t^m x_t^m}{\sum_{m=1}^{M} x_t^m} \leqslant E_t^{\max}, \quad t = 1,\ldots,T. \quad (5)$$

This constraint forces the average amount of carbon emission at any period t to be lower than or equal than the maximum unitary environmental impact allowed. This constraint can be rewritten:

$$\sum_{m=1}^{M}(e_t^m - E_t^{\max}) x_t^m \leqslant 0, \quad t = 1,\ldots,T. \quad (6)$$

At first sight, Constraint (6) looks like a capacity constraint but it is actually very different, since the coefficients $(e_t^m - E_t^{\max})$ of the x_t^m variables in Constraint (6) can be positive or negative. This creates a compensation phenomenon, with no limit on the quantity that can be ordered. Note that, in any feasible solution, at least one mode with a non-positive coefficient must be chosen if a positive quantity is ordered. To ensure feasibility, we assume that, for a first demand occurring in period t, there is at least one mode m such that $e_{t'}^m \leqslant E_{t'}^{\max}$ in some period $t' \leqslant t$.

2.2. Cumulative carbon emission constraint

Constraint (7) below is weaker than Constraint (6). The amount of unused carbon emission of a given period can be used in future periods without exceeding the cumulative capacities.

$$\sum_{t'=1}^{t}\sum_{m=1}^{M}(e_{t'}^m - E_{t'}^{\max}) x_{t'}^m \leqslant 0, \quad t = 1,\ldots,T. \quad (7)$$

This constraint can be modeled using inventory variables J_t (with $J_t \geqslant 0$ and $J_0 = 0$), representing the amount of unused carbon emission that can be used in future periods.

$$J_t = J_{t-1} - \sum_{m=1}^{M}(e_t^m - E_t^{\max}) x_t^m, \quad t = 1,\ldots,T. \quad (8)$$

2.3. Global carbon emission constraint

This constraint extends Constraint (7) on the whole horizon, and is thus weaker. In Constraint (9), the unitary carbon emission over the whole horizon cannot be larger than the maximum unitary environmental impact allowed. In this case, the maximum unitary environmental impact $E_t^{\max}$ no longer depends on the horizon and is stationary, i.e. $E_t^{\max} = E^{\max}\ \forall t = 1,\ldots,T$.

$$\sum_{t=1}^{T}\sum_{m=1}^{M}(e_t^m - E^{\max}) x_t^m \leqslant 0. \quad (9)$$

One of the main limitations of Constraint (9) is that the solution strongly depends on the horizon length T. Generally speaking, this constraint has similar drawbacks, discussed in Section 1, to those of the constraint introduced in [4].

2.4. Rolling carbon emission constraint

Constraint (7) assumes, at each period t, that the horizon from 1 to t can be used to compensate carbon emissions between periods. In Constraint (10) below, we suppose that this is only possible on a rolling horizon of R periods. This seems more realistic, and makes the problem less dependent on the planning horizon T. Note that the periodic carbon emission constraint (6) is equivalent to Constraint (10) when $R = 1$, and that the global emission constraint (9) is equivalent to Constraint (10) when $R = T$.

$$\sum_{t'=t-R+1}^{t}\sum_{m=1}^{M}(e_{t'}^m - E_{t'}^{\max}) x_{t'}^m \leqslant 0, \quad t = R,\ldots,T. \quad (10)$$

Inventory variables J_t can still be used, but constraints on perishable inventories must be introduced, since unused carbon emission in period t cannot compensate carbon emission in periods after $t + R$.

3. The uncapacitated single-item lot-sizing problem with the periodic carbon emission constraint

We want to establish that the multi-sourcing Uncapacitated Lot-Sizing problem with the Periodic Carbon emission constraint (ULS-PC) problem can be solved in polynomial time. More precisely, we show that we can reformulate the ULS-PC problem as a standard lot-sizing problem, i.e. without carbon emission constraints, using a pre-computation step in $O(M^2T)$. Thus, standard lot-sizing combinatorial algorithms can be used to solve the problem.

3.1. ULS-PC problem analysis

Recall that the periodic carbon emission constraint (6) ensures that, in each period t, the average amount of carbon emission per product ordered does not exceed the impact limit $E_t^{\max}$. For the sake of conciseness, we denote by $\bar{e}_t^m$ the value $(e_t^m - E_t^{\max})$, and a mode m is called *ecological* in period t if $\bar{e}_t^m \leqslant 0$. For a period t, we denote by $F_t = \{m \in 1,\ldots,M | \bar{e}_t^m \leqslant 0\}$ the subset of its ecological modes. The following dominance properties can be stated.

Property 1. *Consider a solution of the ULS-PC problem using two modes m_1 and m_2 in a given period t. If $p_t^{m_1} \leqslant p_t^{m_2}$ and $e_t^{m_1} \leqslant e_t^{m_2}$, then mode m_1 dominates mode m_2 (i.e. a solution using modes m_1 and m_2 can be improved by removing the quantity x_t^{m2} produced using mode m_2 and by producing x_t^{m2} using mode m_1).*

Property 2. *Any solution of the ULS-PC problem uses at least one ecological mode in each period with an order, i.e. $\sum_{m=1}^{T} x_t^m > 0 \Rightarrow \sum_{m \in F_t} x_t^m > 0$.*

Proof. The proof is straightforward, since a solution with an order in a given period t, i.e. $x_t^m > 0$, and which does not contain a mode m' such that $e_t^{m'} \leqslant E_t^{\max}$ cannot be feasible since Constraint (6) is violated. □

Theorem 1. *There exists an optimal solution for the ULS-PC problem that uses at most two modes in each period: One ecological mode and possibly one non-ecological mode.*

Proof. Let us introduce variables $X_t = \sum_m x_t^m$, representing the total amount of products ordered in period t in a solution. The ULS-PC problem decomposes, using a Bender's approach, into a master problem (*MP*) and *T* independent subproblems $IP_t(X_t)$, with:

$$(MP)\begin{cases}\min & \sum_{t=1}^{T} z_t^*(X_t) + \sum_{t=1}^{T} h_t(s_t) \\ \text{s.t.} & X_t - s_t + s_{t-1} = d_t \qquad t = 1,\ldots,T \\ & X_t = 0 \qquad t = 1,\ldots,T \text{ such that } F_t = \emptyset \\ & X_t \in \mathbb{R}^+ \qquad t = 1,\ldots,T \end{cases}$$

where $z_t^*(X_t)$ is the optimal value of the problem below:

$$(IP_t(X_t))\begin{cases}\min & \sum_{m=1}^{M}(p_t^m x_t^m + f_t^m y_t^m) \\ \text{s.t.} & \sum_{m=1}^{M} x_t^m = X_t \\ & \sum_{m=1}^{M} \bar{e}_t^m x_t^m \leqslant 0 \\ & x_t^m \leqslant X_t y_t^m \qquad m = 1,\ldots,M \\ & x_t^m \in \mathbb{R}^+,\ y_t^m \in \{0,1\} \quad m = 1,\ldots,M \end{cases}$$

$IP_t(X_t)$ is a single-period ULS-PC problem, which consists in supplying X_t products in period t at minimum cost, while satisfying a carbon emission constraint. Note that, because of Property 2, $IP_t(X_t)$ is feasible if and only if at least one ecological mode is available. In the Master Program (*MP*), this is ensured through Constraints ($F_t = \emptyset \Rightarrow X_t = 0$).

First, consider the relaxation of $IP_t(X_t)$ where the last constraint $x_t^m \leqslant X_t y_t^m$ is removed. This corresponds to the special case with no setup cost. The problem then reduces to a linear program on variables x^m with only two constraints. From elementary LP theory, at most two variables are non-zero in a basic solution, and the first assertion of the theorem follows. For the second assertion, at least one ecological mode m_1 must be used to obtain a feasible solution. If another mode m_2 is used in the basic solution, then both constraints are tight, which implies that either $\bar{e}_t^{m_1} = \bar{e}_t^{m_2} = 0$, and then $p_t^{m_1} = p_t^{m_2}$, i.e. both modes are identical, or $\bar{e}_t^{m_1} < 0$ and $\bar{e}_t^{m_2} > 0$.

Now consider the general case with setup costs. Let $\hat{\pi} = (\hat{x}, \hat{y})$ be a feasible policy and let us denote by $\widehat{\mathcal{M}}_t = \{m | \hat{y}_t^m = 1\}$ the subset of modes used in period t. We can transform $\hat{\pi}$ into a feasible policy of lower cost using at most two modes in each period: Given the subset $\widehat{\mathcal{M}}_t$, we determine the optimal quantities $\bar{x}_t^m$ to order according to these modes to fulfill the quantities $\widehat{X}_t = \sum_m \hat{x}_t^m$, i.e. we solve problem $IP_t(\widehat{X}_t)$ where binary variables y_t are fixed to $\hat{y}_t$. This problem is a linear program similar to the previous relaxation with a restricted subset of modes $\widehat{\mathcal{M}}_t$. Thus, in an optimal basic solution $\bar{x}_t$, at most 2 variables are not equal to zero, one corresponding to an ecological mode and the other to a non-ecological mode. Using only these modes in period t provides a solution with a cost lower than or equal to $\hat{\pi}$, due to the optimality of $\bar{x}_t$, which concludes the proof. □

From the dominance property of Theorem 1, we derive the next results. We show that we can eliminate the carbon emission constraint by considering a quadratic number of modes.

Theorem 2. *The ULS-PC problem can be reformulated as an uncapacitated multi-sourcing lot-sizing problem with M^2 modes using $O(M^2T)$ operations.*

Proof. Theorem 1 enables us to focus on policies using at most two modes per period. Assume that, in period t, an optimal policy uses the pair of modes (m_1, m_2), with m_1 the ecological mode, to order a quantity $\widehat{X}_t$. The ordering cost is thus: $z_t^*(\widehat{X}_t) = f_t^{m_1} + f_t^{m_2} + p_t^{m_1}\hat{x}_t^{m_1} + p_t^{m_2}\hat{x}_t^{m_2}$. From the decomposition used in the proof of Theorem 1, variables $\hat{x}$ are the optimal basic solution of the following LP:

$$(R_t(\widehat{X}_t))\begin{cases}\min & p_t^{m_1}x_t^{m_1} + p_t^{m_2}x_t^{m_2} \\ \text{s.t.} & x_t^{m_1} + x_t^{m_2} = \widehat{X}_t \\ & \bar{e}_t^{m_1}x_t^{m_1} + \bar{e}_t^{m_2}x_t^{m_2} \leqslant 0 \\ & x_t^{m_1}, x_t^{m_2} \in \mathbb{R}^+ \end{cases}$$

The key observation is that the proportion of products ordered according to each mode does not depend on the quantity $\widehat{X}_t$. Indeed, the second constraint has a zero right-hand side, thus multiplying $\widehat{X}_t$ by λ scales the entire polyhedron by λ. Let $p_t^{m_1m_2}$ be the optimal value of $R_t(1)$, i.e. the optimal cost to supply one unit of product, and let us denote by $f_t^{m_1m_2}$ the quantity $f_{m_1} + f_{m_2}$. By convention, $p_t^{m_1m_1}$ refers to the solution where only ecological mode m_1 is used, and we let $f_t^{m_1m_1} = f_t^{m_1}$. From the previous discussion, we have $z_t^*(\widehat{X}_t) = f_t^{m_1m_2} + p_t^{m_1m_2}\widehat{X}_t$.

Clearly, we do not know a priori which pair of modes (u, v) is used in an optimal policy but, due to Theorem 1, we can replace program $IP_t(X_t)$ by the following optimization problem:

$$z_t^*(\widehat{X}_t) = \min\left\{f_t^{uv} + p_t^{uv}\widehat{X}_t | u, v = 1,\ldots,M \text{ and } \bar{e}_t^u \leqslant 0\right\}$$

This is exactly the supplying cost of the uncapacitated multi-sourcing lot-sizing problem where, at each period, $O(M^2)$ modes are available. Each mode corresponds to a pair (u, v) with a setup cost f_t^{uv} and a unitary production cost p_t^{uv}. The holding cost function h remains unchanged. This reformulation requires the computation of all costs p_t^{uv}, which can be done in time $O(M^2)$ for each period. The theorem follows. □

As a corollary of Theorem 2, the lot-sizing problem with the periodic carbon emission constraint is polynomial if and only if the corresponding lot-sizing problem without the periodic carbon emission constraint is polynomial. Hence, Constraint (6) does not modify the complexity status of the problem, but only increases (in a reasonable amount) the computation time, due to the precomputation step and the increase of the number of modes. Roughly speaking, the algorithmic complexity of the lot-sizing problem is increased by a factor M^2 due to Constraint (6). However, note that our result is restricted to linear supplying costs. We show, in the following sections, that the situation is different for the other carbon emission constraints, as the corresponding ULS problems become *NP*-hard.

3.2. A dynamic programming for the USL-PC problem

In this section, we assume that the holding costs h_t are linear, i.e. $h_t(s_t) = h_t s_t$ in the objective function (1). We explicitly show how the dynamic programming algorithm of Wagelmans et al. [25] can be adapted to solve the ULS-PC problem in $O(TM^2 \log M + T^2)$. To formalize the algorithm, the following theorem is required, which follows from Theorem 2 since the multi-sourcing lot-sizing problem satisfies the zero inventory ordering (ZIO) policy (i.e. $\widehat{I}_{t-1} \cdot \sum_{m=1}^{M} \hat{x}_t^m = 0$ for $t = 1, \ldots, T$).

Theorem 3. *Zero inventory ordering (ZIO) policies are dominant for the ULS-PC problem.*

Using the previous properties, we derive a dynamic programming algorithm to solve the ULS-PC problem. The rationale is mainly based on the following:

- Each demand is entirely produced in a single period (see Theorem 3).
- At each period t and for each pair of modes (m_1, m_2), a dominant solution $X_t^{m_1} + X_t^{m_2}$ must cover a demand of type $d_{tt'} = \sum_{k=t}^{t'} d_k$ (see Theorem 3).
- At most two modes are used in the same period (see Theorem 1).

In the following, a backward dynamic programming algorithm is proposed to solve the ULS-PC problem based on recursion formulas that use the principles described above and the dynamic programming proposed by Wagelmans et al. [25]. The dynamic programming algorithm uses the following new cost and parameter definitions.

- Let $d_{tt'}$ define the cumulative demand such that $d_{tt'} = \sum_{k=t}^{t'} d_k$.
- Inventory variables can be eliminated from the initial formulation since they can be expressed using production variables and demands $\left(s_t = \sum_{t'=1}^{t}\sum_{m=1}^{M} x_{t'}^m - \sum_{t'=1}^{t} d_{t'}\right)$. The objective function is reformulated as follows: $\sum_{m=1}^{M}\sum_{t=1}^{T}(c_t^m x_t^m + f_t^m y_t^m) + \sum_{t=1}^{T} h_t \sum_{t'=1}^{t} d_t$ with $c_t^m = p_t^m + \sum_{t'=1}^{t} h_{t'}$. This reformulation is useful because holding costs can be ignored.
- Let $G(t)$ be the value of an optimal solution to the instance of ULS-PC with a planning horizon from t to T with $t = 1, \ldots, T$. $G(T+1)$ is equal to zero.
- Let $H(t,t')$ be the function that provides the best total cost (fixed and variable costs) for producing $d_{tt'}$ in period t. According to Theorem 1, at most two modes can be used to produce $d_{tt'}$ in period t.

Because of the ZIO policy, the following recursion holds:

$$G(t) = \begin{cases} \min\limits_{t<t'\leqslant T+1}\{G(t') + H(t,t'-1)\}, & \text{if } d_t > 0 \\ \min\{G(t+1), \min\limits_{t+1<t'\leqslant T+1}\{G(t') + H(t,t'-1)\}\}, & \text{if } d_t = 0 \end{cases} \tag{11}$$

The optimal value is given by $G(1)$.

We first want to analyze the time complexity for calculating $H(t,t')$ for any t and t'. Let $a_l + b_l X$ be the cost function associated to l, where l denotes either one ecological mode or a combination of two modes. Parameter a_l corresponds to f_t^{uv} and parameter b_l depends on c_t^u, c_t^v, $\bar{e}_t^u$ and $\bar{e}_t^v$. Let $z_t^*(X)$ be the optimal cost to supply a quantity X in period t. As observed in the proof of Theorem 2, function z_t^* is the lower envelope of the set of cost functions $a_l + b_l X$, and is thus concave and piecewise linear. Hence, z_t^* can be defined by a series of breakpoints and slopes associated to the allowed combinations of modes. Since there are at most M^2 possible combinations, there are at most $M^2 - 1$ breakpoints.

First, note that if these breakpoints and slopes are known and sorted, computing all values $H(t,t')$ for a fixed period t can be done in $O(T + M^2)$. Indeed to evaluate $z_t^*(X)$ for a quantity X, it is sufficient to find an interval of two consecutive breakpoints to which it belongs. For a given period t, we must evaluate $z_t^*(X)$ for at most T different values of X, namely $X_t = \{d_{t,t}, d_{t,t+1}, \ldots, d_{t,T}\}$. Since both the values of X_t and the breakpoints are sorted, we can use a merge procedure to go only once through the set of breakpoints and thus compute all $z_t^*(d_{tt'})$ in time proportional to the number of breakpoints and the number of values to evaluate. Holding costs can be computed by accumulation during the merge, without increasing the time complexity, due to the linearity of h.

Hence, before starting the dynamic program, we propose to use a pretreatment procedure to determine the breakpoints and slopes for each period t. This corresponds to finding the extreme points of the 2-dimensional polyhedron defined by the inequalities $Y \leqslant a_l + b_l X$. The problem of finding the extreme points of a polyhedron defined by s linear constraints in the plane can be solved in time $O(s \log s)$ using the result of Shamos and Hoey [22]. As a consequence for each period t, we can find all the breakpoints and their optimal costs in time $O(M^2 \log M)$, and thus determine all values of $H(t,t')$ for $t \leqslant t' \leqslant T$ in time complexity $O(TM^2 \log M + T(T + M^2))$ which is in $O(TM^2 \log M + T^2)$.

Finally let us analyze the time complexity of the dynamic programming algorithm. The values $H(t,t')$ being calculated, quantity $G(t)$ can be determined in time $O(T)$ for each period t. Thus determining $G(1)$ can be done in time $O(T^2)$. The overall complexity of the dynamic programming algorithm, including precomputations, is thus in $O(T\,M^2 \log M + T^2)$. Note that this complexity cannot be easily reduced using the geometric techniques described in Wagelmans et al. [25] since the production cost $(H(t,t'))$ is concave and piecewise linear. A key element of the proof in Wagelmans et al. [25] is that the production cost is linear in t.

4. The uncapacitated single-item lot-sizing problem with the cumulative carbon emission constraint

In this section, we study the multi-sourcing Uncapacitated Lot-Sizing problem with the Cumulative Carbon emission constraint (ULS-CC): For each period t, the average amount of carbon emission per product ordered from the first period up to t should not exceed an impact limit $E_t^{\max}$. As in the case of the periodic carbon emission constraint, it is dominant to use at most two modes per period.

Theorem 4. *There exists an optimal solution for the ULS-CC problem that uses at most two modes in each period: One ecological mode and possibly one non-ecological mode.*

Proof. As in Theorem 1, consider variables $X_t = \sum_m x_t^m$ and let us introduce variables $\bar{E}_t = \sum_m \bar{e}_t^m x_t^m$, which represent the algebraic carbon impact of period t. The mathematical formulation of ULS-CC then decomposes into one master problem (MP') and T independent subproblems $IP'_t(X_t, \bar{E}_t)$, consisting in supplying quantity X_t at the cheapest cost while ensuring that the emission impact does not exceed $\bar{E}_t$. Program $IP_t(X_t)$ corresponds to the special case $\bar{E}_t = 0$. Program $IP'_t(X_t, \bar{E}_t)$ still has only two constraints, and Theorem 4 follows in a way analogous to Theorem 1. □

We apparently are in a situation very similar to the periodic carbon emission constraint. It turns out that problem ULS-CC is far more difficult to solve than the ULS-PC problem. Contrary to Theorem 3, we first show that the ZIO property is not dominant for the ULS-CC problem, and that the best ZIO policy may perform arbitrarily badly.

Lemma 1. *For the ULS-CC problem, the cost of the best ZIO policy may be arbitrarily large compared to the cost of an optimal policy.*

Proof. The proof is based on the construction on an instance with two periods and two modes u and v. Setup costs and holding costs are zero, while the other parameters are given in Table 1. The optimal policy consists in ordering 2 units in period 1 using mode u, and $2D$ units in period 2 using mode v. The resulting cost is 2. Observe that any finite cost policy must order 2 units using mode u in period 1. Thus the only finite cost ZIO policy consists in ordering all the demands in period 1 using mode u, for a total cost of $2D + 2$, which concludes the proof. □

We now show that the ULS-CC problem is NP-hard. We reduce from a special version of the Subset Sum problem with an additional cardinality constraint on the size of the selected set: There

Table 1
Parameters of the instance with a non-ZIO optimal solution.

Parameters	Demand	e^u	e^v	$E^{\max}$	p^u	p^v
Period 1	1	0	$D+1$	D	1	∞
Period 2	$2D+1$	0	$D+1$	D	∞	0

are n items, each one associated with a weight w_i, together with a knapsack capacity W and an integer k. The problem consists in deciding if a subset of exactly k objects exists, allowing multiple copies of items, such that the total weight equals W. That is, is there a n-integer vector $(\alpha_1,\ldots,\alpha_n)$ such that $\sum_i w_i\alpha_i = W$ and $\sum_i \alpha_i = k$? Without the cardinality constraint on the number k of objects to select, the problem is known as the Money Changing Problem (MCP) which is NP-hard (see [16,5,20]). Adding the cardinality constraint on the number of objects, Caprara et al. [7] introduce the problem in its {0,1} version, i.e. when each item can be picked at most once, under the name *Exact k-item Subset Sum Problem* (E-kSSP), and show that it is *NP*-hard. By analogy, we call our problem Exact-kMCP for *Exact k-item Money Changing Problem*. We establish below that Exact-kMCP is NP-complete.

Lemma 2. *The Exact-kMCP problem is NP-complete.*

Proof. First notice that the problem k-MCP, where the cardinality constraint is relaxed to select *at most* k objects, is NP-complete, since problem MCP is a special case (set $k \geqslant W/\min_i w_i$). We use the same argument as Caprara et al. [7] to reduce problem Exact-kMCP from k-MCP. Consider any instance I of k-MCP, with n items of weights w_i, a value W and an integer k. We transform instance I into an instance I' of Exact-kMCP in the following way. Let us consider $n + k$ items:

- The first n items (original items) have weights $w'_i = kw_i + 1$,
- The last k items (dummy items) have unit weights.

The question is whether there exists k objects summing exactly to $W' = k(W + 1)$.

We show that there exists a solution for instance I if and only if there exists a solution for instance I'. Instance I has a positive answer if and only if there exists $(\alpha_i)_{1,n}$ such that $\sum_{i=1}^n w_i\alpha_i = W$ and $\sum_{i=1}^n \alpha_i \leqslant k$. Denoting by $k' = \sum_{i=1}^n \alpha_i$, we can extend α to a $(n + k)$-vector α' by setting to one the first $k - k'$ dummy items such that $\sum_{i=1}^{n+k}\alpha'_i = k$. Similarly, from a $(n + k)$-vector α' whose components sum up to k, we can define a n-vector α by keeping only original items. We have:

$$\sum_{i=1}^n w_i\alpha_i = W \iff k\sum_{i=1}^n w_i\alpha_i + k' = kW + k' \iff \sum_{i=1}^n (kw_i + 1)\alpha_i$$
$$= kW + k' \iff \sum_{i=1}^n w'_i\alpha'_i + \sum_{i=n+1}^{n+k}\alpha'_i$$
$$= kW + k' + (k - k') \iff \sum_{i=1}^{n+k} w'_i\alpha'_i = W'$$

The result follows. $\square$

Theorem 5. *The ULS-CC Problem is NP-hard, even on stationary instances with unit demands.*

Proof. We perform the reduction from the Exact-kMCP problem using the result of Lemma 1. We can restrict to instances with $k \geqslant 2$ and thus we can assume that $W > w_i$ for all items. Without loss of generality, we can also assume that $w_i \geqslant 1$ for all items (otherwise add 1 to all weights and increase W by k). We transform an instance I of the Exact-kMCP into a stationary instance of ULS-CC in the following way:

- There are $M = n + 1$ different modes. Modes 1 to n are associated to the Exact-kMCP items and have algebraic impacts $\bar{e}^i = W - w_i$ and setup costs $f^i = w_i$. Mode M is the only ecological mode, with $\bar{e}^M = -(k-1)W$ and setup cost $f^M = W + 1$.
- There are $T = k + 1$ periods, each one with a unit demand to satisfy.
- The holding cost is set to $h = kW$ and the production costs are zero for all modes.
- It is asked if a solution of cost at most $2W + 1$ exists.

Assume first that instance I has a positive answer: Let $S = (i_1,\ldots,i_k)$ be a valid list of items. We can build a valid solution for the ULS-CC instance by ordering one unit in period 1 using mode M, and then ordering one unit in period t using mode i_{t-1}. The cost of this solution is $(W + 1) + w(S) = 2W + 1$ ($w(S)$ is the cost incurred by items in S). This solution is feasible since $\sum_{i\in S}\bar{e}^i = kW - W = -\bar{e}^M$.

Conversely, assume that all the demands can be satisfied at cost at most $2W + 1$. First, observe that it is necessary to use mode M at the first period to satisfy the constraint carbon emission for $t = 1$. Since the total cost cannot exceed $(2W + 1)$, a valid policy orders using mode M exactly once. Secondly, the value of the holding cost imposes that $\varepsilon = \sum_t s_t$ does not exceed $1/k < 1$. It implies that a valid policy has to order in every period to satisfy the unit demands. Thus, since production costs are zero and only mode M is ecological, exactly one mode is used in each period. Let S be the list of the modes used from period 2 up to period $T = k + 1$. We claim that S is a valid solution for instance I. From the previous discussion, S exactly contains k elements. Its total weight is equal to the total setup cost from period 2 to period T. Thus we have $f_M + h\varepsilon + w(S) \leqslant 2W + 1$, which implies that $w(S) \leqslant W - h\varepsilon = W(1 - k\varepsilon)$.

To conclude, we now show that a valid policy does not carry any inventory ($\varepsilon = 0$), which will imply that $w(S) = W$. Observe that the only reason to supply more than 1 unit in a period using mode m is that m has a small carbon emission impact. By an interchange argument, it is possible to see that it is dominant to carry all the inventory ε from period 1 to 2. If x_t is the amount of products supplied at each period, then we have $x_1 = 1 + \varepsilon$ and $\sum_2^T x_t = k - \varepsilon$. It results that:

$$w(S) = \sum_{t=2}^T w_{i_t} = \sum_{t=2}^T w_{i_t}x_t + \sum_{t=2}^T w_{i_t}(1 - x_t) \geqslant \sum_{t=2}^T w_{i_t}x_t + \sum_{t=2}^T (1 - x_t)$$
$$\geqslant \sum_{t=2}^T w_{i_t}x_t + \varepsilon$$

Satisfying the carbon emission constraint in period T implies that $-(1+\varepsilon)(k-1)W + \sum_{t=2}^T \bar{e}_{i_t}x_t \leqslant 0$. Replacing $\bar{e}_{i_t}$ by its value $W - w_{i_t}$, we obtain the inequality $\sum_{t=2}^T w_{i_t}x_t \geqslant \sum_{t=2}^T x_t W - (1+\varepsilon)(k-1)W$. It follows that:

$$w(S) \geqslant \sum_{t=2}^T w_{i_t}x_t + \varepsilon \geqslant [(k - \varepsilon) - (1 + \varepsilon)(k - 1)]W + \varepsilon$$
$$\geqslant (1 - k\varepsilon)W + \varepsilon$$

The second inequality is due to the fact that $\sum_2^T x_t = k - \varepsilon$. As we established that $w(S) \leqslant W(1 - k\varepsilon)$, we necessarily have $\varepsilon = 0$: A valid policy orders one unit at each period. The previous inequality immediately implies that $w(S) = W$, which concludes the proof. $\square$

5. The uncapacitated single-item lot-sizing problem with the global carbon emission constraint or rolling carbon emission constraint

The multi-sourcing Uncapacitated Lot-Sizing problem with the Global Carbon emission constraint (ULS-GC) is a relaxation of the ULS-CC problem, where $(T-1)$ constraints are removed to only keep Constraint (9). Although the ULS-GC problem is simpler, it remains *NP*-hard. The proof uses the same reduction as Theorem 5 and is omitted.

Recall that Constraint (10) imposes a maximum per unit carbon emission E_t^{max} on every interval of R consecutive periods. It is still dominant to use at most two modes per period since, in the decomposition, the subproblems $IP_t(X_t, E_t^{max})$ are unchanged. The special case $R = 1$ of the Uncapacitated Lot-Sizing problem with the Rolling Carbon emission constraint (ULS-RC) corresponds to the ULS-PC problem, which is polynomial. On the contrary, the case $R = T$ of the ULS-RC problem is equivalent to the ULS-GC problem, and thus the ULS-RC problem is also *NP*-hard.

6. Conclusion and further research directions

We believe the integration of carbon emission constraints in lot-sizing problems leads to relevant and original problems. This paper is a first step to model such problems from which several new lot-sizing problems could arise. We tried to define and categorize these new constraints. We proposed and studied four types of carbon emission constraints: (1) periodic carbon emission constraint, (2) cumulative carbon emission constraint, (3) global carbon emission constraint and (4) rolling carbon emission constraint. We showed that the multi-sourcing uncapacitated lot-sizing problem (ULS) with periodic carbon emission can be solved optimally using a dynamic programming algorithm. We also proved that, when considering one of the other three constraints, the ULS problem becomes *NP*-hard. For future research, it would be interesting to propose exact methods to solve the *NP*-hard problems or approximation algorithms for some special cases.

Different sensitivity analysis can also be conducted by different actors. In fact, the impact of introducing carbon emission constraints in a supply chain can be analyzed from different points of view. Local authorities or governments could be interested in conducting some analysis to determine the best way to introduce a new legislation, possibly in several steps, on carbon emission without excessively penalizing manufacturers. Companies could also be interested in conducting some analysis to find out which carbon emission constraint is more relevant for them. They may be interested in displaying carbon footprints on their products while keeping their competitiveness.

In this paper, carbon emissions are aggregated in each supplying mode, which is a combination of a production location and a transportation mode (requiring one or more types of vehicles). The carbon emission of each supplying mode is modeled using a linear function of the quantity delivered. This model could be made more detailed by considering fixed and variable carbon emissions. The variable carbon emission would still be a linear function of the delivered quantity and the fixed carbon emission could depend on the number of required "vehicles" (e.g. containers). It would be interesting to study the added value of a more detailed carbon emission constraint and the complexity of the resulting problems.

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