Received: 5 December 2017 | Revised: 7 June 2019 | Accepted: 21 June 2019
DOI: 10.1002/asjc.2223

REGULAR PAPER

WILEY

Sliding mode control of uncertain fractional-order systems: A reaching phase free approach

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Abstract

This paper proposes a sliding surface which renders the system dynamics to start directly from itself without a reaching phase. More specifically, the system dynamics is insensitive to matched disturbances/uncertainties throughout the entire system response. The controller design based on reduced-order subsystem is still preserved. It is different from integral sliding mode in which the design is based on the full order of the system to reach the same objective. The simulation results of its application to a fractional inverted pendulum system is demonstrated.

KEYWORDS

fractional-order system, reduced-order design, sliding mode control

1 | INTRODUCTION

The idea of fractional calculus was discussed for the first time over a letter from Leibniz to L'Hôpital in 1695. Fractional differential equations have been in use to model physical phenomena in the last couple of decades. The history of fractional-order calculus can be found in [1] [2]. The state space description is given in [3]. In [4], the authors instilled interest into the research community. New paths have been paved in the fractional calculus theory in [5]. Due to its wide advantages, in recent years, the study of fractional-order controllers has witnessed considerable interest [6,7]. The discussions on stability of fractional-order systems can be found in [8,9]. Some applications of fractional calculus have been given in [10–12].

The control under heavy uncertainties is one of the most challenging tasks. *Sliding Mode Control (SMC)* is one of the most efficient control strategies to deal with uncertainties [13]. Nowadays, it is used in control and observation of several classes of problems such as that related to power converters, vehicle motion control, etc.

The main objective of this class of controllers is to force the system states to stay in a predefined manifold (sliding surface) and maintain it there in spite of the presence of uncertainties in the system. Therefore, the sliding mode based design consists of two phases (i) *Reaching Phase* in which the system states are driven from the initial state to reach the sliding manifold in finite time and (ii) *Sliding Phase* in which the closed-loop system is induced into sliding motion. However, when the system reaches sliding phase, the consideration of robustness and order reduction come into picture which are the most important aspects of the sliding mode based design. It is worth noting that during the reaching phase, there is no guarantee of robustness [14]. In order to address robustness issue throughout the entire space, *Integral Sliding Mode Control (ISMC)* has been proposed in the SMC literature [14] but its design methodology has been based on full order of the system. However, the system exhibits a reduced-order dynamics after it has reached the sliding surface i.e. the system order gets reduced by one due to the introduction of the sliding variable, s such that $s = 0$ in finite time. As a consequence, the simplicity and flexibility of the design procedure which is provided by reduced-order subsystem in classical SMC is lost in ISMC. The motivation behind this work is to preserve the robustness in the system by eliminating the reaching phase such that the system remains on the sliding manifold from the very initial time.

For the fractional-order systems, sliding mode approach and its variants have been quite recently pursued in the literature (e.g. the work in [10,15–17] aims at finite-time stability and at the rejection of matched uncertainties/perturbations).

The main aim of the present paper is to address robustness from the very initial time and also maintain the design methodology based on order reduction for uncertain fractional-order systems. In order to achieve this, two different methodologies have been adopted:

- An integer reaching law approach is used proposing a sliding surface which eliminates the reaching phase and also, its stability is proved.
- Secondly, a sliding surface using fractional reaching law approach is proposed followed by the same procedure as in the case of integer reaching law approach.

The approach used in this work is based on Reimann-Liouville (R-L) and Caputo definitions of fractional derivatives. However, there are other definitions also. In [18], a frequency-distributed model is used which results into different transient response of the system. For a certain fractional-order system, if the model as described in [18] is used, then in that case the approach used in this paper may fail to give the desired results. In [19], non-smooth type control is used while adaptive SMC is used to stabilize the system in [20,21]. Adaptive SMC has its own beauty of reducing the magnitude of control which further decreases the amplitude of chattering. However, in each of the work, there is no guarantee of robustness in the reaching phase. In the approach used in this paper, the system remains on the sliding surface from time, $t = t_0$. So, robustness is achieved from the very initial time. Another advantage of this approach is that the control design based on reduced-order subsystem is preserved. The rest of this note is organized as follows. In Section 2, a brief summary of fractional-order calculus, fractional-order systems and the related Lyapunov stability extension for stability analysis are presented. A brief review of fractional-order sliding mode controller is introduced in Section 3. The main results of this paper are reported in Section 4. Section 5 discusses the simulation results followed by the concluding remarks in Section 6.

2 | PRELIMINARIES

2.1 | Fractional-Order Calculus

Fractional-order integration and differentiation constitute the fractional calculus. They are generalization of their integer-order counterparts. The theorems and rules in fractional-order calculus are applicable to their integer-order counterparts in a more generalized representation but not always in a straightforward manner. Two of the most common definitions of fractional-order calculus are the R-L definition and Caputo definition which are inspired by the definition of Cauchy generalized $n \in \mathbb{N}-$fold integral of function by replacing the factorial function by the more generalized Gamma function [5,22].

Definition 1. The α^{th}-order fractional integration of the function $f : (0, \infty) \to \mathbb{R}$ with respect to $t > 0$ and terminal value $t_0 > 0$ is given by

$$ {}_{t_0}I_t^{\alpha} f(t) := \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{(1-\alpha)}} d\tau, \tag{1}$$

where $\Gamma : (0, \infty) \to \mathbb{R}$ is the Euler's Gamma function:

$$\Gamma(\alpha) := \int_0^{\infty} x^{\alpha-1} e^{-x} dx$$

Definition 2. The R-L definition of the α^{th}-order fractional derivative is given by:

$$ {}_{t_0}^{RL}D_t^{\alpha} f(t) := \frac{1}{\Gamma(m-\alpha)} \frac{d^m}{dt^m} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{(\alpha-m+1)}} d\tau, \tag{2}$$

where $m \in \mathbb{N}$ such that $m \geq \lceil \alpha \rceil$, where $\lceil \alpha \rceil$ is the smallest integer greater than or equal to α where $0 < \alpha < 1$.

Definition 3. The Caputo definition of the α^{th}-order fractional derivative of the m times continuously differentiable function $f : (0, \infty) \to \mathbb{R}$ or $f \in C^m((0, \infty), \mathbb{R})$ is given by:

$$ {}_{t_0}^{c}D_t^{\alpha} f(t) := \frac{1}{\Gamma(m-\alpha)} \int_{t_0}^{t} \frac{f^{(m)}(\tau)}{(t-\tau)^{(\alpha-m+1)}} d\tau. \tag{3}$$

A few important properties of fractional derivatives and integrals are as follows [23] :

- For $\alpha = n$, where n is an integer, the operation ${}_{t_0}^{c}D_t^{\alpha} f(t)$ gives the same result as the classical differentiation of integer order n.
- For $\alpha = 0$, the operation ${}_{t_0}^{c}D_t^{\alpha} f(t)$ is the identity operation:

$$ {}_{t_0}^{c}D_t^{\alpha} f(t) = f(t). \tag{4}$$

- Fractional differentiation is a linear operation:

$$ {}_{t_0}^{c}D_t^{\alpha}(af(t) + bg(t)) = a\,{}_{t_0}^{c}D_t^{\alpha} f(t) + {}_{t_0}^{c}D_t^{\alpha} g(t). \tag{5}$$

- The additive index law (semigroup property)

$$ {}_{t_0}^{c}D_t^{\alpha}\, {}_{t_0}^{c}D_t^{\beta} f(t) = {}_{t_0}^{c}D_t^{\beta}\, {}_{t_0}^{c}D_t^{\alpha} f(t) = {}_{t_0}^{c}D_t^{\alpha+\beta} f(t), \quad (6) $$

holds for $f(t) \in C^1[0,T]$ for some $T > 0$ where, $\alpha, \beta \in \mathbb{R}^+$ and $\alpha + \beta \leq 1$ [24].

Remark 1. Caputo derivative and R-L are the two mostly used definitions in fractional calculus [22]. Since the initial value of fractional differential equation with Caputo derivative is the same as the initial value of integer differential equation, it is the most acceptable one. For example, the initial value of the fractional differential equation ${}_{t_0}^{c}D_t^{\alpha}x(t) = f(t,x)$ with $\alpha \in (0,1), t > 0$ is assumed as $x(0) \equiv x_0$.

However, for the same fractional differential equation with R-L in place of Caputo, ${}_{0}^{RL}D_t^{\alpha}x(t) = f(t,x)$ with $\alpha \in (0,1), t > 0$, the initial value of $x(t)$ involves fractional integral (and/or derivative). Here, the initial condition is given as $[{}_{0}^{RL}D_t^{\alpha-1}x(t)]_{t=0} = x_0'$.

On the other hand there is a limitation in case of Caputo definition. It is not able to capture the exact physical behavior of the system as illustrated in [25]. When the initial condition is non-zero, the system trajectories generated by Caputo definition differ from the actual ones. An account of physical and geometrical interpretations for initial condition value and fractional derivatives can be found in [25,26] and [27].

Given a control system, the first and the most important question is whether it is stable, because an unstable control system is typically useless and potentially dangerous. Qualitatively, a system is described as stable if by starting the system somewhere near its desired operating point, it will stay around the point ever after. The most useful and general approach for studying the stability of linear and nonlinear control systems is the theory introduced by Lyapunov. In the next subsection we are going to review the fractional extension of Lyapunov stability which has been recently proposed in [8,28].

2.2 | Fractional Extension of Lyapunov Stability

Using Caputo definition, an $n-$dimensional fractional-order system can be defined as,

$$ {}_{t_0}^{c}D_t^{\alpha}x(t) = f(x,t); \ \forall t \geq t_0 \quad (7) $$

where, $\alpha \in (0,1)$ and $f(x,t)$ is locally bounded in x and piecewise continuous in t for all $t \geq t_0$ and $x \in \mathbb{D}$, where $\mathbb{D} \subset \mathbb{R}^n$ is a domain that contains the origin $x = 0$. For stability analysis of systems in (7), a fractional extension of Lyapunov's direct method was proposed in [8] which is based on the following definition:

Definition 4. A continuous function $\gamma : [0, \mathrm{t}) \rightarrow [0, \infty)$ is a class-$\mathcal{K}$ function if it is strictly increasing and $\gamma(0) = 0$.

Theorem 1. *Let $x = 0$ be an equilibrium point for the non-autonomous fractional-order system i.e., $f(x,t) = 0, \forall t \geq t_0$. If there exists a Lyapunov function $V(t,x(t)) : [t_0, \infty) \times \mathbb{D} \rightarrow \mathbb{R}$ and a class-$\mathcal{K}$ function $\gamma_i (i = 1,2,3)$ such that, $\gamma_1(||x||) \leq V(t,x(t)) \leq \gamma_2(||x||)$ and ${}_{t_0}^{c}D_t^{\alpha}V(t,x(t)) \leq -\gamma_3(||x||)$ where, $\alpha \in (0,1)$ then, the system (7) is asymptotically stable.*

Theorem 2. *Let $x \in \mathbb{R}^n$ be a continuously differentiable vector-valued function. Then, for any time instant $t \geq t_0$ and $\forall \alpha \in (0,1)$,*

$$ \frac{1}{2}\, {}_{t_0}^{c}D_t^{\alpha}x^{\mathrm{T}}(t)x(t) \leq x^{\mathrm{T}}(t)\, {}_{t_0}^{c}D_t^{\alpha}x(t). \quad (8) $$

It will be used in the later sections for the Lyapunov analysis of fractional-order systems with the proposed control input. Since this result was derived using Caputo derivatives, the same definition will be used throughout this paper unless mentioned otherwise.

3 | FRACTIONAL-ORDER SLIDING MODE CONTROLLER

Consider a controllable*commensurate fractional-order linear time-invariant system given by,

$$ {}_{t_0}^{c}D_t^{\alpha}\bar{x}(t) = \bar{A}\bar{x}(t) + \bar{B}(u(t) + d(t)) \quad (9) $$

where, $\bar{x}(\cdot) \in \mathbb{R}^n$, $\bar{A} \in \mathbb{R}^{n\times n}$, $\bar{B} \in \mathbb{R}^{n\times m}$, $u(\cdot) \in \mathbb{R}^m$, $d(\cdot) \in \mathbb{R}^m$ are pseudo states,† system matrix, input matrix, control input, disturbance input respectively. It is assumed that exact evolution of disturbance with respect to time is not known but it is bounded.

There always exists an invertible non-singular matrix $T \in \mathbb{R}^{n\times n}$ such that using a linear transformation $z(t) = Tx(t)$, (9) can be transformed into the regular form,

*Controllability test for the commensurate fractional- order linear time-invariant system (LTI) is the same as for the integer-order LTI system [15].

†For a fractional order system, the knowledge of $x(t_0)$ (t_0 being the initial time) is not sufficient to determine the future behavior of the system. Consequently, the collection of physical variables in a vector x does not strictly represent the state of the system. This is why the term, "Pseudo State" is coined in the literature in order to represent the physical variables of the fractional order systems [18,29]. In this manuscript same terminology has been used.

$$
\begin{aligned}
{}_{t_0}^{c}D_t^{\alpha} z_1(t) &= A_{11}z_1(t) + A_{12}z_2(t) \\
{}_{t_0}^{c}D_t^{\alpha} z_2(t) &= A_{21}z_1(t) + A_{22}z_2(t) + B_2(u(t) + d(t))
\end{aligned} \tag{10}
$$

where, $\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = z(t), z_1(\cdot) \in \mathbb{R}^{n-m}, z_2(\cdot) \in \mathbb{R}^m$.

Since the pair $(\bar{A}, \bar{B})$ is controllable, the pair (A_{11}, A_{12}) will also be controllable. The above system of equations can be represented as,

$$
{}_{t_0}^{c}D_t^{\alpha} z(t) = Az(t) + B(u(t) + d(t)) \tag{11}
$$

where, $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}$.

Now, the main aim is to design SMC such that sliding takes place from the very initial time $t \geq t_0$ and $z(t)$ approaches towards the origin as $t \to \infty$ in the presence of bounded matched disturbance $d(t)$. So, $x(t)$ approaches asymptotically towards the origin as T is invertible. For the simplicity of presentation, it is assumed that $u \in \mathbb{R}$ and $B_2 \in \mathbb{R}$. However, similar results can be extended for the multi-input case in a straightforward manner. It is important to mention that the present work is based on the following assumption:

Assumption 1. For a non-smooth controller, the existence and uniqueness of solutions of the system are defined in the Filippov sense [15] i.e., letting x denote the pseudo states of the entire system ${}_{t_0}^{c}D_t^{\alpha} x(t) = f(x(t), d(t))$, $\alpha > 0$, disturbance $d \in \mathbb{R}^m$ and assuming $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ to be locally bounded, then the solutions are defined with the differential inclusion,

$$
{}_{t_0}^{c}D_t^{\alpha} x(t) \in \bigcap_{\delta>0} \bigcap_{\mu N=0} cl(co(\zeta(B_\delta(x) \setminus N)))
$$

where, cl and co denote the closure and the convex hull respectively. $B_\delta(x)$ is the unit ball and the sets N are all sets of zero Lebesgue measure. Here, δ represents the small ball around the discontinuity point on the state trajectories when the system is on the sliding surface. μN represents the factor by which the trajectories are scaled so that the deviation of the sliding variable from $s = 0$ is minimized.

In [15], the sliding surface has been designed using fractional reaching law and integer reaching law. The sliding surface for (10) using integer reaching law is,

$$
s(z, t) = {}_{t_0}I_t^{1-\alpha}(c_1 z_1(t) + z_2(t)) \tag{12}
$$

where, $s : \mathbb{R}^n \times (t_0, \infty) \to \mathbb{R}$, $c_1 \in \mathbb{R}^{1\times(n-1)}$ and for,

$$
\begin{aligned}
u(t) = B_2^{-1}(&v - c_1\{(A_{11} - A_{12}c_1)z_1(t)\} + A_{12}\, {}_{t_0}I_t^{1-\alpha} s \\
&- A_{21}z_1(t) - A_{22}z_2(t))
\end{aligned}
$$

where, $v = -k_1 \mathrm{sign}(s)$, it has been proved in [15] that for s to be *zero* in finite time, $k_1 > |B_2||d|$.

In case of fractional reaching law,

$$
s(z, t) = c_1 z_1(t) + z_2(t) \tag{13}
$$

Using fractional derivative of s in (13), (10) becomes,

$$
\begin{aligned}
{}_{t_0}^{c}D_t^{\alpha} z_1(t) &= (A_{11} - A_{12}c_1)z_1(t) + A_{12}s \\
{}_{t_0}^{c}D_t^{\alpha} s &= c_1\, {}_{t_0}^{c}D_t^{\alpha} z_1(t) + {}_{t_0}^{c}D_t^{\alpha} z_2(t) \\
&= c_1\{(A_{11} - A_{12}c_1)z_1(t) + A_{12}s\} \\
&\quad + A_{21}z_1(t) + A_{22}z_2(t) + B_2u(t) + B_2 d
\end{aligned}
$$

Here, the control $u(t)$ is chosen as, $u(t) = B_2^{-1}(v - c_1(A_{11} - A_{12}c_1)z_1(t) + A_{12}s - A_{21}z_1(t) - A_{22}z_2(t))$ where, $v = -k_1 \mathrm{sign}(s)$. After applying the control, the closed-loop system becomes,

$$
\begin{aligned}
{}_{t_0}^{c}D_t^{\alpha} z_1(t) &= (A_{11} - A_{12}c_1)z_1(t) + A_{12}s, \\
{}_{t_0}^{c}D_t^{\alpha} s &= -k_1 \mathrm{sign}(s) + B_2 d
\end{aligned} \tag{14}
$$

Now, the following theorem is important:

Theorem 3. *The sliding surface s in (13) becomes zero in finite time if $k_1 > |B_2||d|$.*

Proof. The Lyapunov function is chosen as $V = \frac{1}{2}s^2$. Then, ${}_{t_0}^{c}D_t^{\alpha} V = \frac{1}{2}\, {}_{t_0}^{c}D_t^{\alpha} s^2$. Using (8),

$$
\begin{aligned}
{}_{t_0}^{c}D_t^{\alpha} V &\leq s\, {}_{t_0}^{c}D_t^{\alpha} s = s(-k_1 \mathrm{sign}(s) + B_2 d) \\
&\leq -k_1|s| + |s||B_2 d| = -|s|(k_1 - |B_2 d|) \\
&= -(2V)^{\frac{1}{2}}(k_1 - |B_2 d|) \leq -\eta(2V)^{\frac{1}{2}}
\end{aligned}
$$

□

where, $\eta = k_1 - |B_2||d| > 0$. Using the above inequality, $s = 0$ results in finite time [15] which can be derived as follows:

Putting $t_0 = 0$ in (14), ${}_{0}^{c}D_t^{\alpha} s = -k_1 \mathrm{sign}(s) + B_2 d$. Taking fractional integral of order α on both sides,

$$
{}_0I_t^{\alpha}\ {}_{0}^{c}D_t^{\alpha} s = k_1\ {}_0I_t^{\alpha} \mathrm{sign}(s) + B_2\ {}_0I_t^{\alpha} d \tag{15}
$$

Since, ${}_0I_t^{\alpha}\ {}_{0}^{c}D_t^{\alpha} s = s(t) - {}_{0}^{c}D_t^{\alpha-1} s(0)\frac{t^{\alpha-1}}{\Gamma(\alpha)}$ and ${}_0I_t^{\alpha} c = c\frac{t^{\alpha}}{\Gamma(\alpha+1)}$, Equation (15) becomes after finite time $t = T$,

$$
\begin{aligned}
s(T) - {}_{0}^{c}D_t^{\alpha-1} s(0)\frac{t^{\alpha-1}}{\Gamma(\alpha)} &= -k_1 \mathrm{sign}(s(0))\frac{T^{\alpha}}{\Gamma(\alpha+1)} \\
&\quad + B_2\ {}_0I_t^{\alpha} d
\end{aligned}
$$

Multiplying with $\mathrm{sign}(s(0))$ and using $s(T) = 0$,

$$
\begin{aligned}
-{}_{0}^{c}D_t^{\alpha-1} s(0)\mathrm{sign}(s(0))\frac{T^{\alpha-1}}{\Gamma(\alpha)} &= -k_1\frac{T^{\alpha}}{\Gamma(\alpha+1)} \\
+ B_2\ {}_0I_t^{\alpha}(\mathrm{sign}(s(0))d)
\end{aligned}
$$

Using the inequality,

$$
{}_0I_t^{\alpha}(\mathrm{sign}(s(0))d) \leq {}_{t_0}I_t^{\alpha}|d| \leq {}_0I_t^{\alpha} d_0 = d_0\frac{T^{\alpha}}{\Gamma(\alpha+1)}
$$

Equation (16) becomes,

$$
-{}_{0}^{c}D_t^{\alpha-1} s(0)\mathrm{sign}(s(0))\frac{T^{\alpha-1}}{\Gamma(\alpha)} \leq -(k_1 - B_2 d_0)\frac{T^{\alpha}}{\Gamma(\alpha+1)}
$$

which further results into,

$$T \leq \frac{\Gamma(\alpha+1)\,{}_0^cD_t^{\alpha-1}s(0)\mathrm{sign}(s(0))}{\Gamma(\alpha)(k_1 - B_2 d_0)} \tag{16}$$

which is always finite.

Remark 2. It is clear that sliding mode has taken place after a finite time $t \geq T$ where, T is such that $s(z, T) = 0$. Further, a modified sliding surface is proposed in which sliding mode starts from $t \geq t_0$ such that the reduced-order design methodology is preserved.

4 | MAIN RESULTS

Consider the same system as in Equation (10). The sliding surface for integer reaching law is,

$$s = {}_{t_0}I_t^{1-\alpha}\left\{(c_1 z_1(t) + z_2(t)) - (c_1 z_1(t_0) + z_2(t_0))\,e^{-\lambda(t-t_0)}\right\}$$

where $\lambda > 0$ and $c_1 \in \mathbb{R}^{1\times n-1}$ are the design parameters. Note that $s = 0$ at initial time $t = t_0$. Then, the system (10) is transformed as,

$$\begin{aligned}
{}_{t_0}^cD_t^{\alpha} z_1(t) &= (A_{11} - A_{12}c_1)z_1(t) \\
&\quad + A_{12}\left\{{}_{t_0}^cD_t^{1-\alpha}s + (c_1 z_1(t_0) + z_2(t_0))\,e^{-\lambda(t-t_0)}\right\} \\
\dot{s} &= {}_{t_0}^c D_t^{\alpha}\left\{(c_1 z_1(t) + z_2(t)) - (c_1 z_1(t_0) + z_2(t_0))\,e^{-\lambda(t-t_0)}\right\} \\
&= c_1\left[(A_{11} - A_{12}c_1)z_1(t) + A_{12}\left\{{}_{t_0}^cD_t^{1-\alpha}s + (c_1 z_1(t_0) + z_2(t_0))\,e^{-\lambda(t-t_0)}\right\}\right] \\
&\quad + A_{21}z_1(t) + A_{22}z_1(t) + B_2u(t) + B_2 f \\
&\quad - (-\lambda)^{\alpha}(c_1 z_1(t_0) + z_2(t_0))\,e^{-\lambda(t-t_0)}
\end{aligned}$$

The control input is selected as,

$$\begin{aligned}
u(t) = B_2^{-1}&\left[v - c_1\left\{(A_{11} - A_{12}c_1)z_1(t) + A_{12} \times\left\{{}_{t_0}^cD_t^{1-\alpha}s + (c_1 z_1(t_0) + z_2(t_0))e^{-\lambda(t-t_0)}\right\}\right\}\right] \\
&- B_2^{-1}(A_{21}z_1(t) + A_{22}z_1(t))
\end{aligned} \tag{17}$$

where, $v = -k_1\mathrm{sign}(s)$. Hence, $\dot{s} = -k_1\mathrm{sign}(s) + B_2 d + \Xi$, where $\Xi = B_2^{-1}[(-\lambda)^{\alpha}((c_1 z_1(t_0) + z_2(t_0))e^{-\lambda(t-t_0)}]$. It is important to note that $|\Xi|$ is always bounded for any initial condition $z(t_0)$.

Further, it is proved that the trajectories remain on the sliding surface s once they start from it at $t = t_0$ and then, asymptotically converge to $z_1(t) = z_2(t) = 0$.

Lemma 1. *If $k_1 > |B_2 d| + |\Xi|$, then the trajectories are maintained on the sliding surface $s = 0, \forall t \geq t_0$.*

Proof. Consider the Lyapunov function, $V = \frac{1}{2}s^2$. By taking the time derivative of Lyapunov function along closed-loop subsystem $\dot{s} = -k_1\mathrm{sign}(s) + B_2 d + \Xi$,

$$\begin{aligned}
\dot{V} = s\dot{s} &= s(-k_1\mathrm{sign}(s) + B_2 d + \Xi) \\
&= -k_1|s| + sB_2 d + s\Xi \\
&\leq -k_1|s| + |s|\,|B_2 d| + |s|\,|\Xi| \\
&= -(2V)^{\frac{1}{2}}(k_1 - |B_2 d| - |\Xi|) \\
&\leq -\eta(2V)^{\frac{1}{2}}
\end{aligned}$$

□

where, $\eta = k_1 - |B_2 d| - \Xi$. When $\eta = k_1 - |B_2 d| - \Xi > 0$, Lyapunov stability theory ($V = 0$ and $\dot{V} \leq 0$) $\Rightarrow V = 0, \forall t \geq t_0$ implies $s = 0, \forall t \geq t_0$. This completes the proof. The expression for the finite time, T can be derived as follows:

$$\begin{aligned}
\frac{dV}{dt} &\leq -\eta\sqrt{2}V^{1/2} \\
\int_0^T dt &\leq -\int_{V_0}^0 \frac{dV}{\eta\sqrt{2}(V)^{1/2}} \\
T &\leq -\int_{V_0}^0 \frac{dV}{\eta\sqrt{2}(V)^{1/2}} = \frac{\sqrt{2V(0)}}{\eta}
\end{aligned}$$

Lemma 2. *If the matrix $(A_{11} - A_{12}c_1)$ is negative definite,*[‡] *then the closed-loop system is asymptotically stable.*

Proof. Take the Lyapunov function, $V = \frac{1}{2}z_1^{\top}(t)z_1(t)$. Then, ${}_{t_0}^cD_t^{\alpha}V = \frac{1}{2}{}_{t_0}^cD_t^{\alpha}z_1^{\top}(t)z_1(t)$. Using (8),

$$\begin{aligned}
{}_{t_0}^cD_t^{\alpha}V \leq z_1^{\top}(t)\,{}_{t_0}^cD_t^{\alpha}z_1(t) &\leq z_1^{\top}(t)(A_{11} - A_{12}c_1)z_1(t) \\
&\quad + z_1^{\top}(t)A_{12}\left\{{}_{t_0}^cD_t^{1-\alpha}s + (c_1 z_1(t_0) + z_2(t_0))\,e^{-\lambda(t-t_0)}\right\}
\end{aligned}$$

□

As $s = 0$ from time $t = t_0$, the term $(c_1 z_1(t_0) + z_2(t_0))e^{-\lambda(t-t_0)} \to 0$ as $t \to \infty$, $z_1(t)$ and hence, the system is asymptotically stable if the matrix $(A_{11} - A_{12}c_1)$ is negative definite. This completes the proof.

Remark 3. It is important to note that if we select $v = -\lambda|s|^{\frac{1}{2}}\,\mathrm{sign}(s) - \alpha\int_{t_0}^{t}\mathrm{sign}(s)d\tau$, *where* $\alpha = 1.1\Delta$ *and* $\lambda = 1.5\sqrt{\Delta}$ *such that* $B_2|\dot{d}(t)| \leq \Delta$, *where* Δ *is some a priori known constant, then the proposed control* (17) *generates continuous signal and it is also better for the chattering minimization problem, which is commonly encountered during the practical implementation of discontinuous control. The above suggested controller is known as Super-Twisting in the literature. Again,*

[‡]For negative definiteness, the leading principle minors of $-(A_{11} - A_{12}c_1)$ should be positive i.e. the leading principle minors of $(A_{11} - A_{12}c_1)$ should have alternating signs, with the odd-numbered minors being negative and the even-numbered minors being positive [31].

the trajectories once start from the sliding surface, will remain there for the subsequent time (for more detailed explanation, see [30] *and the references cited therein).*

Now, using fractional reaching law approach, the sliding surface is defined as,

$$s = c_1 z_1(t) + z_2(t) - (c_1 z_1(t_0) + z_2(t_0))e^{-\lambda(t-t_0)} \tag{18}$$

Note that $s = 0$ when $t = t_0$. Using (18), (10) becomes,

$$\begin{aligned}
{}_{t_0}^{c}D_t^{\alpha} z_1(t) &= (A_{11} - A_{12}c_1)z_1(t) \\
&\quad + A_{12}\left\{s + (c_1 z_1(t_0) + z_2(t_0))e^{-\lambda(t-t_0)}\right\} \\
{}_{t_0}^{c}D_t^{\alpha} s &= {}_{t_0}^{c}D_t^{\alpha} z_1(t) + {}_{t_0}^{c}D_t^{\alpha} z_2(t) \\
&\quad - (c_1 z_1(t_0) + z_2(t_0)) {}_{t_0}^{c}D_t^{\alpha}(e^{-\lambda(t-t_0)}) \\
&= c_1\left[(A_{11} - A_{12}c_1)z_1(t)\right. \\
&\quad \left. + A_{12}\left\{s + (c_1 z_1(t_0) + z_2(t_0))e^{-\lambda(t-t_0)}\right\}\right] \\
&\quad + A_{21}z_1(t) + A_{22}z_2(t) + B_2(u(t) + d(t)) \\
&\quad - (-\lambda)^{\alpha}((c_1 z_1(t_0) + z_2(t_0))e^{-\lambda(t-t_0)}
\end{aligned} \tag{19}$$

The control input is designed as,

$$\begin{aligned}
u(t) &= B_2^{-1}\left[v - c_1\left\{(A_{11} - A_{12}c_1)z_1(t) + A_{12}\right.\right. \\
&\quad \left.\left. \times \left(s + (c_1 z_1(t_0) + z_2(t_0))e^{-\lambda(t-t_0)}\right)\right\}\right] \\
&\quad - B_2^{-1}(A_{21}z_1(t) + A_{22}z_2(t))
\end{aligned} \tag{20}$$

where, $v = -k_1 \text{sign}(s)$. From (19) and (20), ${}_{t_0}^{c}D_t^{\alpha} s = -k_1 \text{sign}(s) + B_2 d + \Xi$ where, $\Xi = B_2^{-1}(-\lambda)^{\alpha}((c_1 z_1(t_0) + z_2(t_0))e^{-\lambda(t-t_0)}$. Agian, the trajectories remain on the sliding surface $s = 0$ from the very initial time $t = t_0$, provided $k_1 > |B_2||d| + |\Xi|$. This can be shown as follows:

Consider the Lyapunov function, $V = \frac{1}{2}z_1^{\top}(t)z_1(t)$. Taking the fractional derivative,

$$\begin{aligned}
{}_{t_0}^{c}D_t^{\alpha} V \leq z_1^{\top}(t) {}_{t_0}^{c}D_t^{\alpha} z_1(t) &\leq z_1^{\top}(t)(A_{11} - A_{12}c_1)z_1(t) \\
&\quad + z_1^{\top}(t)A_{12}\left\{s + (c_1 z_1(t_0) + z_2(t_0))e^{-\lambda(t-t_0)}\right\}
\end{aligned}$$

As $s = 0$ from time $t = t_0$, the term $(c_1 z_1(t_0) + z_2(t_0))e^{-\lambda(t-t_0)} \to 0$ as $t \to \infty$. Further, $z_1(t)$ and hence the system is asymptotically stable if $(A_{11} - A_{12}c_1)$ is negative definite.

5 | ILLUSTRATIVE EXAMPLE

A commensurate fractional-order uncertain system is considered to illustrate the theoretical results obtained in the paper. The example of a fractional inverted pendulum system is taken. In this system, an inverted pendulum is mounted on the top of a cart such that the pendulum is attached to an extension immersed in a viscoelastic solution [32]. The cart is able to move back and forth. The whole system can be represented by,

$$\ddot{x} = \frac{1}{(m_c + m_p)}\left(\frac{1}{2}m_p l(\ddot{\theta}cos\theta - (\dot{\theta})^2 sin\theta) - f\dot{x} + F\right)$$

$$\ddot{\theta} = \frac{1}{(J + \frac{1}{4}m_p l^2)}\left(\frac{1}{2}m_p l(\ddot{x}cos\theta + gsin\theta) + \tau\right)$$

$$\frac{d^{\alpha}\tau}{dt^{\alpha}} = -\omega_l^{\alpha}\tau - k\omega_l^{\alpha}\dot{\theta} - k\left(\frac{\omega_l}{\omega_h}\right)^{\alpha}\frac{d^{(\alpha+1)}\theta}{dt^{(\alpha+1)}}$$

where, x is the position of the cart, θ is the angle of deflection of the pendulum, m_c is the mass of the cart, m_p is the mass of the pendulum, f is the friction coefficient of the cart, τ is the applied torque, k is the damping coefficient of the viscoelastic solution, α is the derivation order of the damper, ω_l and ω_h are the lower and higher frequencies of the bandwidth of the fractional derivative. The state vector is chosen as,

$$X = \left[x\ \frac{d^{0.5}x}{dt^{0.5}}\ \frac{dx}{dt}\ \frac{d^{1.5}x}{dt^{1.5}}\ \theta\ \frac{d^{0.5}\theta}{dt^{0.5}}\ \frac{d\theta}{dt}\ \frac{d^{1.5}\theta}{dt^{1.5}}\ \tau\right]^T$$

The above equations can be linearized about the equilibrium point of the system resulting in pseudo state-space form having commensurate order, $\alpha = 0.5$,

$$\frac{d^{\alpha}X(t)}{dt^{\alpha}} = AX(t) + B(u(t) + d(t)) \tag{21}$$

where,

$$A = \alpha\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a_{43} & 0 & a_{45} & 0 & 0 & 0 & a_{49} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & a_{83} & 0 & a_{85} & 0 & 0 & 0 & a_{89} \\
0 & 0 & 0 & 0 & 0 & 0 & a_{97} & a_{98} & a_{99}
\end{bmatrix}$$

$$B = \alpha\left[0\ 0\ 0\ 0.116\ 0\ 0\ 0\ 0.338\ 0\right]^T$$

where, $\alpha = \frac{1}{4J(m_c+m_p)+m_c m_p l^2}$, $a_{43} = -4fJ - fm_p l^2$, $a_{45} = m_p^2 l^2 g$, $a_{49} = 2m_p l$, $a_{83} = -2fm_p l$, $a_{85} = 2m_p gl(m_c + m_p)$, $a_{89} = 4(m_c + m_p)$, $a_{97} = -k\alpha(\omega_l)^{0.5}$, $a_{98} = -k\alpha(\frac{\omega_l}{\omega_h})^{0.5}$, $a_{99} = -\alpha(\omega)^{0.5}$, $b_4 = J + m_p l^2$, $b_8 = 2m_p l$. Here, J is the moment of inertia of the pendulum and l is its length. The values taken are $m_p = 0.53$ kg, $m_c = 3.2$ kg, $l = 0.36$ m, $f = 6.2$ kg.sec^{-1}, $J = 0.065$ kg.m^2, $k = 0.1$ N.m.sec.$^{\alpha}$rad^{-1}, $\omega_l = 0.1$ rad.sec.$^{-1}$, $\omega_h = 10$ rad.sec.$^{-1}$, $g = 9.81$ m.sec.$^{-2}$, $d(t) = 0.1\sin(t)$. The sliding surface is chosen as,

$$\begin{aligned}
s(t) &= ([c_1\, c_2\, c_3\, c_3\, c_4\, c_5\, c_6\, c_7\, c_8\, 1]\, X(t)) \\
&\quad - ([c_1\, c_2\, c_3\, c_3\, c_4\, c_5\, c_6\, c_7\, c_8\, 1]\, X_0)e^{-\lambda(t-t_0)}
\end{aligned}$$

where, c_1 to c_8 are the gain values selected such that the reduced-order dynamics is stable. The controller parameter k_1 has to be selected such that $k_1 > |B_2 d| + |\Xi|$. We know that $|B_2 d| = 0.1$ and $|\Xi|$ is also small. Hence, we choose $k_1 = 10$ and $\lambda = 0.4$. The evolution of states, sliding surface and control input with time are shown in Figure 1, Figure 2 and Figure 3 respectively. In Figure 1, the state trajectories are shown to converge to the equilibrium point in finite time in the presence of matched disturbance/uncertainties. From Figure 2, it is clear that starting from $t = 0$, the trajectories are always maintained on the sliding surface.

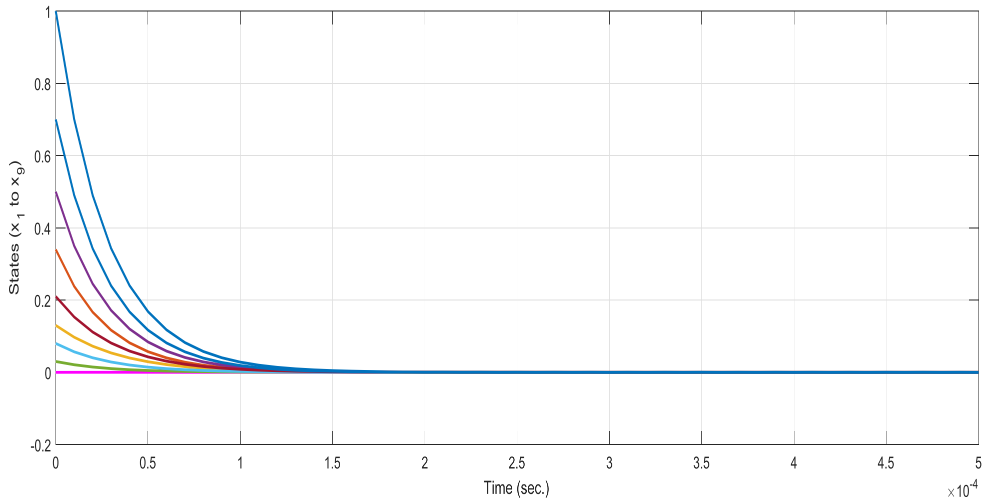

FIGURE 1 Evolution of States (x_1 to x_9) w.r.t. time [Color figure can be viewed at wileyonlinelibrary.com]

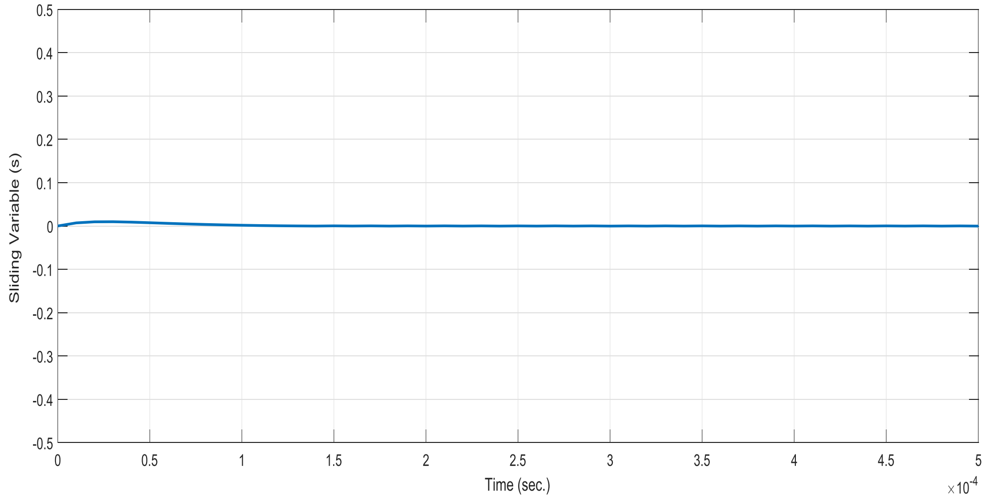

FIGURE 2 Evolution of Sliding Surface (s) w.r.t. time [Color figure can be viewed at wileyonlinelibrary.com]

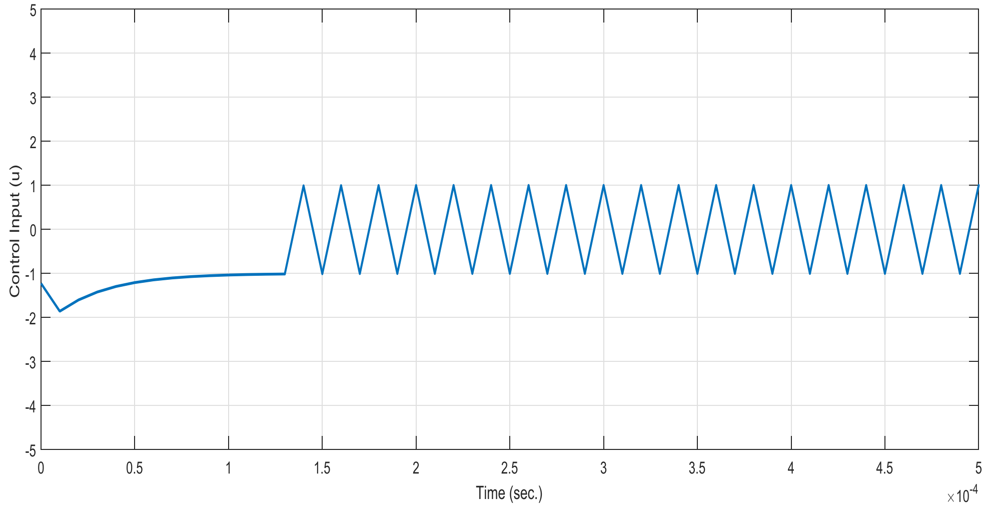

FIGURE 3 Evolution of Control Input (u) w.r.t. time [Color figure can be viewed at wileyonlinelibrary.com]

6 | CONCLUSION

The work presented in the paper proposes a new sliding surface based controller for uncertain fractional-order systems. Two different control schemes, one based on integer reaching law and the other on fractional reaching law have been used in order to maintain the trajectories on the sliding surface from the very initial time preserving robustness. The simplicity of the technique lies in the control design being based on the reduced order subsystem as in the case of classical sliding mode control. The effectiveness of the proposed approach is verified through numerical simulation for the case of a fractional inverted pendulum system.

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REFERENCES

1. B. Ross, *The development of fractional calculus 1695-1900*, Hist. Mater. **4** (1977), no. 1, 75–89.
2. J. T. Machado, V. Kiryakova, and F. Mainardi, *Recent history of fractional calculus*, Commun. Nonlinear Sci. Numer. Simul. **16** (2011), no. 3, 1140–1153.
3. J. Sabatier, C. Farges, and J. C. Trigeassou, *Fractional systems state space description: some wrong ideas and proposed solutions*, J. Vib. Control. **20** (2014), no. 7, 1076–1084.
4. K. B. Oldham and J. Spanier, *The fractional calculus: Theory and applications of differentiation and integration to arbitrary order*, Academic Press, New York, 1974, pp. 111.
5. I. Podlubny, *Fractional Differential Equations*, Academic Press, 1999.
6. S. Kamal, A. Raman, and B. Bandyopadhyay, *Finite time stabilization of fractional order uncertain chain of integrator: A sliding mode approach*, IEEE International Conference on Industrial Technology (ICIT), Athens, Greece, 2012, pp. 1132–1135.
7. D. Xue and Y. Chen, *A comparative introduction of four fractional order controllers*, Shanghai, China, 2002, pp. 3228–3235.
8. Y. Li, Y. Chen, and I. Podlubny, *Stability of fractional order nonlinear dynamic systems: Lyapunov direct method and generalized Mittag-leffler stability*, Comput. Math. Appl. **59** (2010), no. 5, 1810–1821.
9. C. P. Li and F. R. Zhang, *A survey on the stability of fractional differential equations*, Eur. Phys. J. Spec Top **193** (2011), no. 1, 27–47.
10. D. Baleanu, Z. B. Güven, and J. A. T. Machado, *New Trends in Nanotechnology and Fractional Calculus Applications*, CRC Press, 2010.
11. D. Chen et al., *Control of a class of fractional-order chaotic systems via sliding mode*, Nonlinear Dyn. **67** (2012), no. 1, 893–901.
12. D. Chen et al., *Fractional order Lyapunov stability theorem and its applications in synchronization of complex dynamical networks*, Commun. Nonlinear Sci. Numer. Simul. **19** (2014), no. 12, 4105–4121.
13. C. Edwards and S. Spurgeon, *Sliding mode control: Theory and applications*, CRC Press, 1998.
14. V. Utkin and J. Shi, *Integral Sliding Mode in Systems Operating under Uncertainty Conditions*, Proceedings of the 35th IEEE Conference on Decision and Control, Kobe, Japan, 1996, pp. 4591–4596.
15. B. Bandyopadhyay and S. Kamal, *Stabilization and Control of Fractional Order Systems: A Sliding Mode Approach*, Vol. **317**, Springer, Lecture Notes in Electrical Engineering, 2014.
16. M. P. Aghababa, *A switching fractional calculus-based controller for normal non-linear dynamical systems*, Nonlinear Dyn. **75** (2014), no. 3, 577–588.
17. M. P. Aghababa, *Control of Fractional-Order Systems Using Chatter-Free Sliding Mode Approach*, J. Comput. Nonlinear Dyn. **9** (2014), no. 3, 031003.
18. J. C. Trigeassou et al., *State variables and transients of fractional order differential systems*, Comput. Math. Appl. **64** (2012), no. 10, 3117–3140.
19. M. P. Aghababa, *Stabilization of a class of fractional-order chaotic systems using a non-smooth control methodology*, Nonlinear Dyn. **89** (2017), no. 2, 1357–1370.
20. M. P. Aghababa, *Adaptive switching control of uncertain fractional systems: Application to Chua's circuit*, Int. J. Adapt Control Signal Process. **32** (2018), 1206–1221.
21. S. Mirzajani, M. P. Aghababa, and A. Heydari, *Adaptive T-S fuzzy control design for fractional-order systems with parametric uncertainty and input constraint*, Fuzzy Set. Syst. **365** (2019), 22–39.
22. C. Li, D. Qian, and Y. Chen, *On Riemann-Liouville and Caputo derivatives*, Discrete Dynamics in Nature and Society (2011), Article ID 562494, 15 pages. https://doi.org/10.1155/2011/562494
23. Y. Chen, I. Petras, and D. Xue, *Fractional order control- A tutorial*, American Control Conference, St. Louis, MO, USA, 2009, pp. 1397–1411.
24. C. Li and W. Dang, *Remarks on fractional derivatives*, Appl. Math. Comput. **187** (2007), no. 2, 777–784.
25. J. Sabatier et al., *How to impose physically coherent initial conditions to a fractional system?* Commun. Nonlinear Sci. Numer. Simul. **15** (2010), no. 5, 1318–1326.
26. N. Heymans and I. Podlubny, *Physical interpretation of initial conditions for fractional differential equations with Riemann-Liouville fractional derivatives*, Rheol. Acta **45** (2006), no. 5, 765–771.
27. S. Zhang, *Monotone iterative method for initial value problem involving Riemann-Liouville fractional derivatives*, Nonlinear Anal. Theory Methods Appl. **71** (2009), no. 5-6, 2087–2093.
28. N. A. -Camacho, M. A. D. -Mermoud, and J. A. Gallegos, *Lyapunov functions for fractional order systems*, Commun. Nonlinear Sci. Numer. Simul. **19** (2014), no. 9, 2951–2957.
29. J. Sabatier, C. Farges, and J. C. Trigeassou, *On state space description of fractional systems*, 5th Symposium on Fractional Differentiation and its Applications (FDA), Nanjing, China, 2012.
30. A. Chalanga, S. Kamal, and B. Bandyopadhyay, *A New Algorithm for Continuous Sliding Mode Control with Implementation to Industrial Emulator Setup*, IEEE/ASME Trans. Mechatron. **20** (2015), no. 5, 2194–2204.
31. H. K. Khalil, *Nonlinear systems*, 2002. Prentice Hall.

32. C. Farges, M. Moze, and J. Sabatier, *Pseudo-state feedback stabilization of commensurate fractional order systems*, Automatica **46** (2010), no. 10, 1730–1734.

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How to cite this article: Kamal S, Sharma RK, Dinh TN, MS H, Bandyopadhyay B. Sliding mode control of uncertain fractional-order systems: A reaching phase free approach. *Asian J Control*. 2019;1–10. https://doi.org/10.1002/asjc.2223