Interval estimation of switched Takagi-Sugeno systems with unmeasurable premise variables

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Abstract: This paper deals with interval observers design for nonlinear switched systems. The nonlinear modes are represented by the Takagi-Sugeno (T-S) fuzzy models with premise variables depending on unmeasurable terms, e.g. the state vector. This T-S structure can be used to represent exactly a nonlinear switched system in a compact set of the state space. The introduced method in this paper allows to compute the lower and upper bounds of the system state under the assumption that the disturbances as well as the measurement noises are unknown but bounded. First, the stability conditions of the proposed T-S interval observers are developed via Linear Matrix Inequality (LMI) formulations to ensure the convergence of the nonnegative observation error dynamics. Then, changes of coordinates are employed to relax the restrictive requirement of nonnegativity constraints. Theoretical results are applied to a numerical example to illustrate the effectiveness of the proposed method.

Keywords: Nonlinear switched system, Takagi-Sugeno models, unmeasurable premise variables, stability, robustness, interval observer design.

1. INTRODUCTION

Since more than 25 years, fuzzy model-based approaches with Takagi-Sugeno (T-S) fuzzy systems Takagi and Sugeno (1993) have been considerd as an effective tool to represent different types of nonlinear systems thanks to their ability to reduce mathematics complexity by decomposing the operation space into different zones to create a set of linear local models. The validity of each system is quantified by nonlinear weighting functions. The T-S fuzzy models with measurable and unmeasurable premise variables have been used in a variety of applications in the literature. Most of the works deal with the premise variables which are not always accessible and depend on a subset of the system state Li et al. (2016); Ghorbel et al. (2014). In these cases, estimation of the unmeasurable vectors becomes necessary to rescontruct the premise variables Ichalal et al. (2010). T-S fuzzy models are also investigated to describe nonlinear switched systems as in Zouari et al. (2014); Garbouj et al. (2019). Nonlinear switched systems are a particular class of hybrid systems. They exhibit simultaneously continuous and discrete dynamics and recently arise in a variety of engineering applications, such as DC-DC converters Loxton et al. (2009), robotic systems Petroff (2007), etc. The work in Zouari et al. (2014); **?** adopt a restrictive requirement of the premise variables: *they are supposed to be measurable*.

The state estimation problems of such switching representation is very challenging and become more involved if we consider that the output is subject to noises in which the conventional observers may not be an efficient method. Additionally, for the purpose of control such as stabilization and tracking, precise information of the state vector in transient periods is not necessary. However, practically there is a great demand for estimation of the state of a system with guarantees at all time and the notion of interval observer has been one of useful approaches to meeting this practical demand. That is why interval observer has attracted an ever growing attention in the past few years. Although design of interval observers requires bounds of the uncertainties/disturbances and bounds of the initial conditions to be known a priori, this requirement is accepted in many applications. Many classes of systems have been studied in both continuous and discrete time, e.g. linear systems Mazenc et al. (2014); Mazenc and Bernard (2011); Efimov et al. (2013a), bilinear systems Dinh and Ito (2016), nonlinear systems Raïssi et al. (2012); Mazenc et al. (2013); Dinh and Ito (2017), time-delay systems Efimov et al. (2013b).

Following right above-mentioned works, interesting results of interval observers have been devoted to switched systems, especially in the case of linear switched systems Ethabet et al. (2018); Guo and Zhu (2017); Marouani et al. (2018). However, in practice it is well-known that most of the systems need to be described by nonlinear switched behaviors, and thus interval estimation design becomes more complex and existing methods are not able to cope with such behaviors. Furthermore, the extension of interval observers to nonlinear switched systems modeled by T-S fuzzy systems has not been fully considered in the literature and this motivates our work. The main contribution of this paper is to design an interval observer for nonlinear switched systems subject to measurement noises and additive disturbances and represented by T-S fuzzy models with unmeasurable premise variables. The stability analysis is given in terms of Linear Matrix Inequalities (LMIs) and the restrictive nonnegativity assumptions on the observation error is relaxed through a change of coordinates.

The paper is organized as follows: preliminaries and problem formulation are given in Section II. Section III is devoted to the main results of designing a T-S interval observer. A numerical example is presented in Section IV to illustrate the efficiency of the proposed method. Finally, Section V concludes the paper.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 Preliminaries

The set of real numbers is denoted by $\mathbb{R}$. The set of nonnegative real numbers is denoted by $\mathbb{R}_{\geq 0}$, i.e., $\mathbb{R}_{\geq 0} := [0,+\infty)$. Inequalities must be understood *component-wise*, i.e., for $x_a = [x_{a,1},...,x_{a,n}]^\top \in \mathbb{R}^n$ and $x_b = [x_{b,1},...,x_{b,n}]^\top \in \mathbb{R}^n$, $x_a \leq x_b$ if and only if, for all $i \in \{1,...,n\}$, $x_{a,i} \leq x_{b,i}$. The symbol $M \succ 0$ (resp. $M \prec 0$) means that the symmetric matrix M is positive (resp. negative) definite. E_p is a $(p\times 1)$ vector whose elements are equal to 1. For a measurable and locally essentially bounded input $u : \mathbb{R}_+ \to \mathbb{R}$, the symbol $\|u\|_{[t_0,t_1]}$ denotes its $\mathscr{L}_\infty$ norm. If $t_1 = +\infty$, then we will simply write $\|u\|$. A matrix $A \in \mathbb{R}^{n\times n}$ is called Metzler if all its off-diagonal elements are nonnegative. A matrix $A \in \mathbb{R}^{n\times n}$ is said to be nonnegative if every entry of A is nonnegative. Given a matrix $A \in \mathbb{R}^{m\times n}$, we define $A^+ = \max\{0,A\}$, $A^- = A^+ - A$ and denote the absolute value of a matrix by $|A| = A^+ + A^-$ (similarly for vectors). For a square matrix T_i, we define $diag([T_1 \dots T_N])$ as a diagonal matrix composed by $T_1 \dots T_N$.

Lemma 1. (Mazenc and Bernard (2011)) The system described by:

$$\dot{x}(t) = Ax(t) + u(t),\ x(0) = x_0 \tag{1}$$

is said to be nonnegative if A is a Metzler matrix and $u(t) \geq 0$. For any initial condition $x_0 \geq 0$, the solution of (1) satisfies $x(t) \geq 0, \forall t \geq 0$.

Lemma 2. (Chebotarev et al. (2015)) Let $x \in \mathbb{R}^n$ be a vector such that $\underline{x} \leq x \leq \overline{x}$.
(1) if $A \in \mathbb{R}^{m\times n}$ is a constant matrix, then

$$A^+\underline{x} - A^-\overline{x} \leq Ax \leq A^+\overline{x} - A^-\underline{x} \tag{2}$$

(2) if $A \in \mathbb{R}^{m\times n}$ is a matrix satisfying $\underline{A} \leq A \leq \overline{A}$, for some $\underline{A},\overline{A} \in \mathbb{R}^{m\times n}$, then

$$\begin{aligned} &\underline{A}^+\underline{x}^+ - \overline{A}^+\underline{x}^- - \underline{A}^-\overline{x}^+ + \overline{A}^-\overline{x}^- \leq Ax \\ &\leq \overline{A}^+\overline{x}^+ - \underline{A}^+\overline{x}^- - \overline{A}^-\underline{x}^+ + \underline{A}^-\underline{x}^- \end{aligned} \tag{3}$$

Lemma 3. (Boyd et al. (1994)) Consider x and y with appropriate dimensions and Ω a positive definite matrix. the following property is verified:

$$x^Ty + y^Tx \leq x^T\Omega x + y^T\Omega^{-1}y \tag{4}$$

Consequently, let $\lambda \succ 0$ be a scalar and $M \in \mathbb{R}^n$ be a symmetric positive definite matrix, then:

$$2x^Ty \leq \frac{1}{\lambda}x^TMx + \lambda y^TM^{-1}y \quad x,y \in \mathbb{R}^n \tag{5}$$

Lemma 4. (Liberzon and Morse (1999)) Let

$$\dot{x} = A^\sigma x,\ \forall\sigma \in \{1,2,\dots,N\} \tag{6}$$

The switched system (6) is asymptotically stable if there exists a matrix $M = M^T > 0$ such that

$$\dot{V}(x) = x^T(A^{\sigma^T}M + MA^\sigma)x < 0, \forall\sigma \in \{1,2,\dots,N\} \tag{7}$$

where $V(x) = x^TMx$ is the common Lyapunov function.

2.2 T-S model formulation for nonlinear switched system

Consider a continuous time nonlinear switched system described as follows :

$$\begin{aligned} &\Sigma_{\sigma(t)} : \begin{cases} \dot{x}(t) = f_{\sigma(t)}(x(t),u(t),d(t)) \\ y(t) = g_{\sigma(t)}(x(t),v(t)) \end{cases} \\ &\sigma(t) : [0,+\infty[\to \{1,2,\dots,N\} \end{aligned} \tag{8}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the input and $y \in \mathbb{R}^p$ is the output. $d \in \mathbb{R}^\ell$ and $v \in \mathbb{R}^p$ are respectively the bounded additive disturbances and measurement noises. σ is the switching law such that $\sigma(t) \in \{1,...,N\}$ is the index of the active mode. For example, if one has $\sigma(t) = i,\ i \in \{1,2,\cdots,N\}$, the system is said to be in the mode i at the instant t. f_σ and g_σ are nonlinear functions. The additive disturbances d and the measurement noises v are assumed to be unknown but bounded.

Each nonlinear mode $\sigma(t)$ can be approximated by T-S fuzzy models which consist of a set of linear sub-models interpolated through weighting functions. Each linear model represents local dynamics given by the following structure:

$$\begin{cases} \begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{r}\mu_i^\sigma(\xi(t))(A_i^{\sigma(t)}x(t) + B_i^{\sigma(t)}u(t) + F_i^{\sigma(t)}d(t)) \\ y(t) = C^{\sigma(t)}x(t) + v(t) \end{cases} \\ \sigma(t) : [0,+\infty[\to \{1,2,\dots,N\} \end{cases} \tag{9}$$

where $A_i^{\sigma(t)} \in \mathbb{R}^{n\times n}, B_i^{\sigma(t)} \in \mathbb{R}^{n\times m}, F_i^{\sigma(t)} \in \mathbb{R}^{n\times \ell}$ *and* $C^{\sigma(t)} \in \mathbb{R}^{p\times n}$ are known constant matrices. $\mu_i^\sigma(\xi(t))$ are the weighting functions depending on the so-called decision variable $\xi(t)$ that can be internal or external to the system. When these variables are internal, they can be measurable such as $\{u(t),y(t)\}$ or unmeasurable as the state $x(t)$ of the system. The weighting functions satisfy the following convex sum property:

$$\begin{cases} 0 \leq \mu_i^\sigma(\xi(t)) \leq 1, \forall i \in \{1,\dots r\} \\ \sum\limits_{i=1}^{r}\mu_i^\sigma(\xi(t)) = 1 \end{cases} ,\forall\sigma \in \{1,...,N\} \tag{10}$$

Assumption 1. For all $i \in \{1,\dots,r\}$ and $\sigma(t)$ defined in (9), $\frac{r+1}{r}A_i^{\sigma(t)} + \frac{1}{r}\sum\limits_{\substack{j=1\\ j\neq i}}^{r}A_j^{\sigma(t)}$ are Metzler matrices and $\frac{r+1}{r}B_i^{\sigma(t)} + \frac{1}{r}\sum\limits_{\substack{j=1\\ j\neq i}}^{r}B_j^{\sigma(t)}$ are nonnegative matrices.

Assumption 1 is not restrictive because (i) a T-S model representation of a nonlinear one is not unique, so there may have T-S representations which satisfy Assumption 1 and (ii) it covers positive T-S fuzzy systems Benzaouia et al. (2014) which require A_i Metzler and B_i nonnegative for all $i \in \{1,\dots,r\}$.

Assumption 2.

$$\underline{d} \leq d(t) \leq \overline{d},\ |v(t)| \leq \overline{V}E_p,\ \forall t \geq 0 \tag{11}$$

where $\underline{d},\overline{d} \in \mathbb{R}^\ell$ and the scalar $\overline{V}$ are a priori known.

Assumption 2 is basic in the literature of interval observers where the uncertainties are assumed bounded with known bounds.

Assumption 3. The input $u \in \mathbb{R}^m$ is upper and lower bounded, i.e., $u_{\min} \leq u(t) \leq u_{\max}$ for all $t \geq 0$ where $u_{\min}, u_{\max} \in \mathbb{R}^m$. Moreover given $\underline{\varepsilon}, \overline{\varepsilon} \in \mathbb{R}^n$, for all $i \in \{1,\ldots,r\}$, $\sigma \in \{1,\ldots,N\}$, there exist $\overline{u} \in \mathbb{R}^m$ and $\underline{u} \in \mathbb{R}^m$ such that for all $\varepsilon \in \mathbb{R}^n$, $\underline{\varepsilon} \leq \varepsilon \leq \overline{\varepsilon}$,

$$\mu_i^\sigma(\underline{\varepsilon})\underline{u} \leq \mu_i^\sigma(\varepsilon)u(t) \leq \mu_i^\sigma(\overline{\varepsilon})\overline{u} \tag{12}$$

Assumption 3 is not restrictive. The requirement that the input u is bounded is accepted in many applications. Besides because the function μ_i^σ is known and always bounded between 0 and 1 as in (10), then one can easily select $\overline{u} \in \mathbb{R}^m$ and $\underline{u} \in \mathbb{R}^m$ to satisfy Assumption 3, e.g., $\overline{u} = \frac{u_{\max}}{\min_{i\in\{1,\ldots,r\},\sigma\in\{1,\ldots,N\}} \mu_i^\sigma(\overline{\varepsilon})}$ and $\underline{u} = -|u_{\min}|$.

Let us consider the matrices $\overline{A}_i^{\sigma(t)}$ and $\overline{B}_i^{\sigma(t)}$ defined as follows:

$$\begin{aligned} A_0^{\sigma(t)} &= \frac{-1}{r}\sum_{i=1}^{r} A_i^{\sigma(t)}, \overline{A}_i^{\sigma(t)} = A_i^{\sigma(t)} - A_0^{\sigma(t)} \\ B_0^{\sigma(t)} &= \frac{-1}{r}\sum_{i=1}^{r} B_i^{\sigma(t)}, \overline{B}_i^{\sigma(t)} = B_i^{\sigma(t)} - B_0^{\sigma(t)} \end{aligned} \tag{13}$$

The matrices $A_0^{\sigma(t)}$ and $B_0^{\sigma(t)}$ characterize the nominal local model of each mode. The matrices $\overline{A}_i^{\sigma(t)}$ and $\overline{B}_i^{\sigma(t)}$ describe the variation around the nominal model. Substituting (13) into the system (9), an equivalent system is obtained:

$$\begin{cases} \begin{cases} \dot{x}(t) = A_0^{\sigma(t)}x(t) + B_0^{\sigma(t)}u(t) + \sum_{i=1}^{r} \mu_i^\sigma(\xi(t))F_i^{\sigma(t)}d(t) \\ \quad + \sum_{i=1}^{r} \mu_i^\sigma(\xi(t))(\overline{A}_i^{\sigma(t)}x(t) + \overline{B}_i^{\sigma(t)}u(t)) \\ y(t) = C^{\sigma(t)}x(t) + v(t) \end{cases} \\ \sigma(t) : [0, +\infty[\to \{1,2,\ldots,N\} \end{cases} \tag{14}$$

In this work, we suppose that the weighting functions depend on the system state which is unmeasurable (i.e. $\xi(t) = x(t)$). The main contribution of this paper is to design a T-S interval state estimation of the system described in (14). Starting from the initial state x_0 which verifies $\underline{x}_0 \leq x_0 \leq \overline{x}_0$ and taking into account the uncertainties, the designed interval observer satisfy $\underline{x}(t) \leq x(t) \leq \overline{x}(t), \forall t \geq 0$. In the sequel, for the sake of simplicity, the time variable t will be omitted.

3. T-S INTERVAL STATE ESTIMATION FOR NONLINEAR SWITCHED SYSTEM

Based on the structure of the nonlinear switched system represented by the T-S fuzzy models given in (14), the following T-S framer composed of $\underline{x}$ and $\overline{x}$ is proposed in the Luenberger form:

$$\begin{cases} \dot{\overline{x}} = (A_0^\sigma - L^\sigma C^\sigma)\overline{x} + B_0^\sigma u + L^\sigma y + |L^\sigma|\overline{V}E_p + \sum_{i=1}^{r} \overline{\varphi}_{F_i^\sigma}^{+} \\ \quad + \sum_{i=1}^{r} \left(\overline{A}_i^\sigma \overline{x}^+ + \mu_i^\sigma(\overline{x})\overline{B}_i^\sigma \overline{u}\right) \\ \dot{\underline{x}} = (A_0^\sigma - L^\sigma C^\sigma)\underline{x} + B_0^\sigma u + L^\sigma y - |L^\sigma|\overline{V}E_p - \sum_{i=1}^{r} \underline{\varphi}_{F_i^\sigma}^{-} \\ \quad - \sum_{i=1}^{r} \left(\overline{A}_i^\sigma \underline{x}^- - \mu_i^\sigma(\underline{x})\overline{B}_i^\sigma \underline{u}\right) \end{cases} \tag{15}$$

where $\overline{\varphi}_{F_i^\sigma} = F_i^{\sigma+}\overline{d} - F_i^{\sigma-}\underline{d}$, $\underline{\varphi}_{F_i^\sigma} = F_i^{\sigma+}\underline{d} - F_i^{\sigma-}\overline{d}$.

According to eq. (2) of Lemma 2, we have for any $d \in \mathbb{R}^\ell$:

$$F_i^{\sigma+}\underline{d} - F_i^{\sigma-}\overline{d} \leq F_i^\sigma d \leq F_i^{\sigma+}\overline{d} - F_i^{\sigma-}\underline{d} \tag{16}$$

From eq. (3) and note that $0 \leq \mu_i^\sigma(x) \leq 1$ for all $i \in \{1,\ldots,r\}$, $\sigma \in \{1,\ldots,N\}$, we deduce that

$$-\underline{x}^- \leq \mu_i^\sigma(x)x \leq \overline{x}^+ \tag{17}$$

$$-\underline{\varphi}_{F_i^\sigma}^- \leq \mu_i^\sigma(x)F_i^\sigma d \leq \overline{\varphi}_{F_i^\sigma}^+ \tag{18}$$

From Assumption 3, we have for all $i \in \{1,\ldots,r\}$, $\sigma \in \{1,\ldots,N\}$,

$$\mu_i^\sigma(\underline{x})\underline{u} \leq \mu_i^\sigma(x)u(t) \leq \mu_i^\sigma(\overline{x})\overline{u} \tag{19}$$

Let's define $\overline{e} = \overline{x} - x$ and $\underline{e} = x - \underline{x}$, the corresponding observation error dynamics are given by:

$$\begin{cases} \dot{\overline{e}} = \dot{\overline{x}} - \dot{x} = (A_0^\sigma - L^\sigma C^\sigma)\overline{e} + \overline{\Psi}^\sigma \\ \dot{\underline{e}} = \dot{x} - \dot{\underline{x}} = (A_0^\sigma - L^\sigma C^\sigma)\underline{e} + \underline{\Psi}^\sigma \end{cases} \tag{20}$$

where

$$\begin{aligned} \overline{\Psi}^\sigma &= L^\sigma v + |L^\sigma|\overline{V}E_p + \sum_{i=1}^{r} \overline{\varphi}_{F_i^\sigma}^+ - \sum_{i=1}^{r} \mu_i^\sigma(x)F_i^\sigma d \\ &\quad + \sum_{i=1}^{r} (\overline{A}_i^\sigma \overline{\delta}_i^\sigma + \overline{B}_i^\sigma \overline{\Delta}_i^\sigma), \\ \underline{\Psi}^\sigma &= |L^\sigma|\overline{V}E_p - L^\sigma v + \sum_{i=1}^{r} \mu_i^\sigma(x)F_i^\sigma d + \sum_{i=1}^{r} \underline{\varphi}_{F_i^\sigma}^- \\ &\quad + \sum_{i=1}^{r} (\overline{A}_i^\sigma \underline{\delta}_i^\sigma + \overline{B}_i^\sigma \underline{\Delta}_i^\sigma), \end{aligned} \tag{21}$$

with the vectors:

$$\begin{aligned} \overline{\delta}_i^\sigma &= \overline{x}^+ - \mu_i^\sigma(x)x \geq 0 \\ \underline{\delta}_i^\sigma &= \mu_i^\sigma(x)x + \underline{x}^- \geq 0 \end{aligned} \tag{22}$$

$$\begin{aligned} \overline{\Delta}_i^\sigma &= \mu_i^\sigma(\overline{x})\overline{u} - \mu_i^\sigma(x)u \geq 0 \\ \underline{\Delta}_i^\sigma &= \mu_i^\sigma(x)u - \mu_i^\sigma(\underline{x})\underline{u} \geq 0 \end{aligned} \tag{23}$$

Assumption 4. The following conditions hold throughout this paper for all $i \in \{1, r\}$ and for all $\sigma \in \{1,...,N\}$:

- $\left|\overline{\delta}_i^\sigma\right| \leq \alpha_i^\sigma |\overline{x} - x|$, $\left|\underline{\delta}_i^\sigma\right| \leq \alpha_i^\sigma |x - \underline{x}|$,
- $\left|\overline{\Delta}_i^\sigma\right| \leq \gamma_i^\sigma |\overline{x} - x|$, $|\underline{\Delta}_i^\sigma| \leq \gamma_i^\sigma |x - \underline{x}|$,

Assumption 4 ensures Lipschitz conditions on the weighting functions $\mu_i : \mathbb{R}^n \to \mathbb{R}_+$. For all $i \in \{1,\ldots,r\}$, $\sigma \in \{1,...,N\}$, the computation of the matrices $\alpha_i^\sigma \in \mathbb{R}^{n\times n}$, $\gamma_i^\sigma \in \mathbb{R}^{m\times n}$ is introduced in (Ichalal et al., 2010, Section 10) and they are real nonnegative matrices.

Theorem 1. Assume that the Assumptions 1-4 are satisfied and there exist L^σ such that the matrices $A_0^\sigma - L^\sigma C^\sigma$ are Metzler for all $\sigma \in \{1,2,\ldots,N\}$. The initial state x_0 verifies $\underline{x}_0 \leq x_0 \leq \overline{x}_0$. Given the matrices $\alpha_i^\sigma, \gamma_i^\sigma$, $i \in \{1,\ldots,r\}$, $\sigma \in \{1,...,N\}$ and the scalars $\lambda^\sigma > 0$. Then if there exist a positive definite matrix $M \in \mathbb{R}^{n\times n}$, matrices $K^\sigma \in \mathbb{R}^{n\times p}$, positive definite matrices $\Omega_{1i}^\sigma \in \mathbb{R}^{n\times n}$ and $\Omega_{2i}^\sigma \in \mathbb{R}^{m\times n}, i \in \{1,\ldots,r\}$ such that the following LMI holds:

$$\begin{bmatrix} \mathscr{A}^\sigma & \Xi_A^\sigma & \Xi_B^\sigma & \Xi_\alpha^\sigma \Omega_A^\sigma & \Xi_\gamma^\sigma \Omega_B^\sigma \\ \Xi_A^{\sigma^T} & -\Omega_A^\sigma & 0 & 0 & 0 \\ \Xi_B^{\sigma^T} & 0 & -\Omega_B^\sigma & 0 & 0 \\ \Omega_A^{\sigma^T}\Xi_\alpha^{\sigma^T} & 0 & 0 & -\Omega_A^\sigma & 0 \\ \Omega_B^{\sigma^T}\Xi_\gamma^{\sigma^T} & 0 & 0 & 0 & -\Omega_B^\sigma \end{bmatrix} \prec 0 \tag{24}$$

where

$$\mathscr{A}^\sigma = A_0^{\sigma^T} M + M A_0^\sigma - C^{\sigma^T} K^{\sigma^T} - K^\sigma C^\sigma + \frac{4}{\lambda^\sigma}$$
$$\Xi_A^\sigma = \left[\, M\overline{A}_1^\sigma \ \dots \ M\overline{A}_r^\sigma \,\right], \Xi_B^\sigma = \left[\, M\overline{B}_1^\sigma \ \dots \ M\overline{B}_r^\sigma \,\right]$$
$$\Xi_\alpha^\sigma = [\, \alpha_1^\sigma \ \dots \ \alpha_r^\sigma \,], \Xi_\gamma^\sigma = [\, \gamma_1^\sigma \ \dots \ \gamma_r^\sigma \,]$$
$$\Omega_A^\sigma = diag([\, \Omega_{11}^\sigma \ \dots \ \Omega_{1r}^\sigma \,]), \Omega_B^\sigma = diag([\, \Omega_{21}^\sigma \ \dots \ \Omega_{2r}^\sigma \,]) \tag{25}$$

with $K^\sigma = ML^\sigma$, then the framer (15) is an interval observer for the system (14).

Proof.

Since $A_0^\sigma - L^\sigma C^\sigma$ are assumed Metzler. Additionally using Assumption 1 yields $\overline{A}_i^\sigma$ Metzler and $\overline{B}_i^\sigma \geq 0$. As $|v| \leq \overline{V} E_p, \forall t \geq 0$ and thank to (18), if $\overline{x}_0$ and $\underline{x}_0$ are chosen such that:

$$\begin{cases} \overline{e}(0) = \overline{x}_0 - x_0 \geq 0 \\ \underline{e}(0) = x_0 - \underline{x}_0 \geq 0 \end{cases}$$

then according to Lemma 1, the dynamics of the estimation errors $\overline{e}$ and $\underline{e}$ stay positive $\forall t \geq 0$ and thus $\underline{x}(t) \leq x(t) \leq \overline{x}(t), \forall t \geq 0$.
Let us show that the errors $\overline{e}$ and $\underline{e}$ stay bounded $\forall t \geq 0$. The stability of (20) is proved based on Lemma 4. Consider the common Lyapunov function for the upper estimation error $V(\overline{e}) = \overline{e}^T M \overline{e}$, $M \succ 0$, then:

$$\begin{aligned} \dot{V}(\overline{e}) &= \dot{\overline{e}}^T M \overline{e} + \overline{e}^T M \dot{\overline{e}} \\ &= \overline{e}^T ((A_0^\sigma - L^\sigma C^\sigma)^T M + M(A_0^\sigma - L^\sigma C^\sigma))\overline{e} \\ &+ 2\overline{e}^T M L^\sigma v + 2\overline{e}^T M |L^\sigma| \overline{V} E_p \\ &+ 2\overline{e}^T M \sum_{i=1}^r \overline{\varphi}^+_{F_i^\sigma} - 2\overline{e}^T M \sum_{i=1}^r \mu_i^\sigma(x) F_i^\sigma d \\ &+ 2\overline{e}^T M \sum_{i=1}^r \overline{A}_i^\sigma \overline{\delta}_i^\sigma + 2\overline{e}^T M \sum_{i=1}^r \overline{B}_i^\sigma \overline{\Delta}_i^\sigma \end{aligned} \tag{26}$$

From Lemma 3, the following inequalities are obtained for all $\delta \in \{1,...,N\}$:

$$2\overline{e}^T M L^\sigma v \leq \frac{1}{\lambda^\sigma} \overline{e}^T M \overline{e} + \lambda^\sigma v^T L^{\sigma^T} M L^\sigma v$$

$$2\overline{e}^T M |L^\sigma| \overline{V} E_p \leq \frac{1}{\lambda^\sigma} \overline{e}^T M \overline{e} + \lambda^\sigma E_p^T \overline{V} |L^\sigma|^T M |L^\sigma| \overline{V} E_p$$

$$2\overline{e}^T M \sum_{i=1}^r \overline{\varphi}^+_{F_i^\sigma} \leq \frac{1}{\lambda^\sigma} \overline{e}^T M \overline{e} + \lambda^\sigma \left(\sum_{i=1}^r \overline{\varphi}^+_{F_i^\sigma} \right)^T M \sum_{i=1}^r \overline{\varphi}^+_{F_i^\sigma}$$

$$\begin{aligned} 2\overline{e}^T M \sum_{i=1}^r \mu_i^\sigma(x)(-F_i^\sigma) d &\leq \frac{1}{\lambda^\sigma} \overline{e}^T M \overline{e} \\ &+ \lambda^\sigma \left[\sum_{i=1}^r \mu_i^\sigma(x) d^T F_i^{\sigma^T} M \sum_{i=1}^r \mu_i^\sigma(x) F_i^\sigma d \right] \end{aligned}$$

On the other hand using Lemma 3 and Lipschitz conditions proposed in Assumption 4, it follows that for all $i \in \{1,\dots,r\}$:

$$\begin{aligned} \overline{\delta}_i^{\sigma^T} \overline{A}_i^{\sigma^T} M \overline{e} + \overline{e}^T M \overline{A}_i^\sigma \overline{\delta}_i^\sigma &\leq \overline{\delta}_i^{\sigma^T} \Omega_{1i} \overline{\delta}_i^\sigma + \overline{e}^T M \overline{A}_i^\sigma \Omega_{1i}^{-1} \overline{A}_i^{\sigma^T} M \overline{e} \\ &\leq \overline{e}^T \alpha_i^{\sigma^T} \Omega_{1i} \alpha_i^\sigma \overline{e} + \overline{e}^T M \overline{A}_i^\sigma \Omega_{1i}^{-1} \overline{A}_i^{\sigma^T} M \overline{e} \end{aligned}$$

and

$$\begin{aligned} \overline{\Delta}_i^{\sigma^T} \overline{B}_i^{\sigma^T} M \overline{e} + \overline{e}^T M \overline{B}_i^\sigma \overline{\Delta}_i^\sigma &\leq \overline{\Delta}_i^{\sigma^T} \Omega_{2i} \overline{\Delta}_i^\sigma + \overline{e}^T M \overline{B}_i^\sigma \Omega_{2i}^{-1} \overline{B}_i^{\sigma^T} M \overline{e} \\ &\leq \overline{e}^T \gamma_i^{\sigma^T} \Omega_{2i} \gamma_i^\sigma \overline{e} + \overline{e}^T M \overline{B}_i^\sigma \Omega_{2i}^{-1} \overline{B}_i^{\sigma^T} M \overline{e} \end{aligned}$$

Taking into account the above-mentioned inequalities, the derivative of the common Lyapunov function (26) can be bounded as follows:

$$\dot{V}(\overline{e}) \leq \overline{e}^T \mathscr{A}^\sigma \overline{e} + \overline{e}^T \mathscr{B}^\sigma \overline{e} + \mathscr{C}^\sigma \tag{27}$$

where $\mathscr{A}^\sigma$ defined in (24) and

$$\begin{aligned} \mathscr{B}^\sigma &= \sum_{i=1}^r M \overline{A}_i^\sigma \Omega_{1i}^{-1} \overline{A}_i^{\sigma^T} M + M \overline{B}_i^\sigma \Omega_{2i}^{-1} \overline{B}_i^{\sigma^T} M + \alpha_i^{\sigma^T} \Omega_{1i} \alpha_i^\sigma \\ &+ \gamma_i^{\sigma^T} \Omega_{2i} \gamma_i^\sigma \\ \mathscr{C}^\sigma &= v^T \left[\lambda^\sigma L^{\sigma^T} M L^\sigma \right] v + E_p^T \left[\lambda^\sigma \overline{V} |L^\sigma|^T M |L^\sigma| \overline{V} \right] E_p \\ &+ \left(\sum_{i=1}^r \overline{\varphi}^+_{F_i^\sigma} \right)^T [\lambda^\sigma M] \sum_{i=1}^r \overline{\varphi}^+_{F_i^\sigma} \\ &+ \sum_{i=1}^r \mu_i^\sigma(x) d^T F_i^{\sigma^T} [\lambda^\sigma M] \sum_{i=1}^r \mu_i^\sigma(x) F_i^\sigma d \end{aligned}$$

Because the weighting functions μ_i^σ for all $i \in \{1,\dots,r\}$, $\sigma \in \{1,\dots,N\}$, the additive disturbances d and the measurement noises v are bounded in norm, it follows that $\mathscr{C}^\sigma$ is bounded. Thus, (27) is guaranteed if $\mathscr{A}^\sigma + \mathscr{B}^\sigma \prec 0$, which can be rewritten by employing definitions given in (25) as follows:

$$\begin{aligned} &\mathscr{A}^\sigma + \Xi_A^\sigma \Omega_A^{\sigma^{-1}} \Xi_A^{\sigma^T} + \Xi_B^\sigma \Omega_B^{\sigma^{-1}} \Xi_B^{\sigma^T} + \Xi_\alpha^\sigma \Omega_A^\sigma \Xi_\alpha^{\sigma^T} \\ &+ \Xi_\gamma^\sigma \Omega_B^\sigma \Xi_\gamma^{\sigma^T} \prec 0 \end{aligned} \tag{28}$$

Thank to Schur complement and the fact that $K^\sigma = ML^\sigma$, (28) is equivalent to LMI (24). Therefore, the upper estimation error $\overline{e}$ is input-to-state stable (ISS). Similarly, we can show that the lower estimation error $\underline{e}$ is also ISS $\forall t \geq 0$. ■

Note that the existence of the observer gains L^σ in Theorem 1 is conservative because of the requirement stating that the matrices $A_0^\sigma - L^\sigma C^\sigma$ should be Metzler. In fact, it is not usually possible to ensure this constraint. Naturally, one can think about finding a nonsingular transformation $z = Px$ such that the matrices $P(A_0^\sigma - L^\sigma C^\sigma)P^{-1}$ are Metzler. Subsequently, a framer can be constructed in these new coordinates. However, the existence of a common transformation P for all $\sigma \in \{1,\dots,N\}$ is not obvious, even impossible. Thus, a new methodology is proposed. It is based on the design, in the original base, of two conventional observers. The structure is inspired by the one proposed in Dinh et al. (2014) for non-switched systems.

Assumption 5. There exist changes of coordinates P^σ such that the matrices

$$P^\sigma (A_0^\sigma - L^\sigma C^\sigma) P^{\sigma^{-1}}$$

are nonnegative for all $\sigma \in \{1,\dots,N\}$.

Remark 1. The second approach based on changes of coordinates is general since it is always possible to transform any real square matrix into a nonnegative form. The existence of such a transformation is not a strong assumption. For instance, it has been shown in Mazenc and Bernard (2011) that based on the Jordan canonical form, it is always feasible to transform any square constant matrix into a nonnegative form with a constant or a time-varying transformation.

The two conventional observers are described by:

$$\begin{aligned} \dot{\hat{x}}^+ &= (A_0^\sigma - L^\sigma C^\sigma)\hat{x}^+ + B_0^\sigma u + L^\sigma y + P^{\sigma^{-1}} |P^\sigma L^\sigma| \overline{V} E_p \\ &+ P^{\sigma^{-1}} \left(P^{\sigma^+} \sum_{i=1}^r \overline{\varphi}^+_{F_i^\sigma} + P^{\sigma^-} \sum_{i=1}^r \underline{\varphi}^-_{F_i^\sigma} \right) \\ &+ P^{\sigma^{-1}} \sum_{i=1}^r \Big(P^{\sigma^+} \overline{A}_i^\sigma \overline{x}^+ + P^{\sigma^-} \overline{A}_i^\sigma \underline{x}^- \\ &+ P^{\sigma^+} \overline{B}_i^\sigma \mu_i(\overline{x}) \overline{u} - P^{\sigma^-} \overline{B}_i^\sigma \mu_i(\underline{x}) \underline{u} \Big) \end{aligned} \tag{29}$$

and

$$\dot{\hat{x}}^- = (A_0^\sigma - L^\sigma C^\sigma)\hat{x}^- + B_0^\sigma u + L^\sigma y - P^{\sigma^{-1}}|P^\sigma L^\sigma|\overline{V}E_p$$
$$- P^{\sigma^{-1}}\left(P^{\sigma^+}\sum_{i=1}^r \underline{\varphi}^-_{F_i^\sigma} + P^{\sigma^-}\sum_{i=1}^r \overline{\varphi}^+_{F_i^\sigma}\right)$$
$$- P^{\sigma^{-1}}\sum_{i=1}^r\Big(P^{\sigma^+}\overline{A}_i^\sigma \underline{x}^- + P^{\sigma^-}\overline{A}_i^\sigma \overline{x}^+$$
$$- P^{\sigma^+}\overline{B}_i^\sigma \mu_i(\underline{x})\underline{u} + P^{\sigma^-}\overline{B}_i^\sigma \mu_i(\overline{x})\overline{u}\Big) \quad (30)$$

where $P^\sigma, \sigma \in \{1,2,\ldots,N\}$ are changes of coordinates presented in Assumption 5.

Theorem 2. Assumptions 1-5 hold. If the initial condition x_0 verifies $\underline{x}_0 \le x_0 \le \overline{x}_0$ then the equations,

$$\overline{x} = Q^{\sigma^+}P^\sigma\hat{x}^+ - Q^{\sigma^-}P^\sigma\hat{x}^-,\ \underline{x} = Q^{\sigma^+}P^\sigma\hat{x}^- - Q^{\sigma^-}P^\sigma\hat{x}^+ \quad (31)$$

associated with the suitably selected initial conditions:

$$\hat{x}_0^+ = Q^\sigma\left(P^{\sigma^+}\overline{x}_0 - P^{\sigma^-}\underline{x}_0\right),\ \hat{x}_0^- = Q^\sigma\left(P^{\sigma^+}\underline{x}_0 - P^{\sigma^-}\overline{x}_0\right), \quad (32)$$

are a framer for (14) with $Q^\sigma = P^{\sigma^{-1}}$.

In addition, given the matrices $\alpha_i^\sigma, \gamma_i^\sigma, i \in \{1,\ldots,r\}$ and the scalars $\lambda^\sigma > 0$. If there exist a positive definite $M \in \mathbb{R}^{n\times n}$, matrices $K^\sigma \in \mathbb{R}^{n\times p}$, positive definite matrices $\Omega_{1i}^\sigma \in \mathbb{R}^{n\times n}$ and $\Omega_{2i}^\sigma \in \mathbb{R}^{m\times n}$, $i \in \{1,\ldots,r\}$ such that LMI (24) holds, then the framer (31) is an interval observer for the system (14) and the observer gains can be computed as $L^\sigma = M^{-1}K^\sigma$.

Proof. The proof is similar to Theorem 1.

For framer property, based on Assumption 5, we prove that $\dot{\overline{E}}(t) = P^\sigma\dot{\hat{x}}^+(t) - P^\sigma\dot{x}(t)$ and $\dot{\underline{E}}(t) = P^\sigma\dot{x}(t) - P^\sigma\dot{\hat{x}}^-(t)$ are nonnegative dynamics. For the stability property, we consider the Lyapunov functions $(\hat{x}^+ - x)^T M(\hat{x}^+ - x)$ and $(x - \hat{x}^-)^T M(x - \hat{x}^-)$. ■

Remark 2. The nonnegativity property has motivated the need of a state transformation. The interest of the second structure proposed above is that even by using changes of coordinates $z(t) = P^\sigma x(t)$, the dynamics (29) and (30) are designed in the original coordinates "x" (i.e. $\dot{\hat{x}}^+(t) = (A_0^\sigma - L^\sigma C^\sigma)\hat{x}^+ + \ldots$) instead of in the basis "z" (i.e. $\dot{\hat{x}}^+(t) = P^\sigma(A_0^\sigma - L^\sigma C^\sigma)Q^\sigma\hat{x}^+(t) + \ldots$) as in Guo and Zhu (2017). This makes the stability analysis simpler and allows one to avoid jumping of the framer state and a hybrid behavior in the coordinates "z".

4. NUMERICAL EXAMPLE

In order to illustrate the effectiveness of the proposed methodology, we consider a switched system characterized by two nonlinear modes (i.e. $\sigma \in \{1,2\}$). Each mode is represented by T-S models with two local models (i.e. $r = 2$) as the form (9) where:

For Mode 1:

$$A_1^1 = \begin{bmatrix} -1.51 & -0.262 \\ 0 & -1 \end{bmatrix},\ A_2^1 = \begin{bmatrix} -0.86 & 1.47 \\ 0 & -3 \end{bmatrix}$$
$$B_1^1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ B_2^1 = B_1^1,\ F_1^1 = \begin{bmatrix} 0.5 \\ 0.6 \end{bmatrix},\ F_2^1 = F_1^1 \quad (33)$$
$$C^1 = [\,1\ \ 0\,]$$

For Mode 2:

$$A_1^2 = \begin{bmatrix} -0.55 & 2.1 \\ 0 & -1 \end{bmatrix},\ A_2^2 = \begin{bmatrix} -2.65 & 0.34 \\ 0 & -3 \end{bmatrix}$$
$$B_1^2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ B_2^2 = B_1^2,\ F_1^2 = \begin{bmatrix} 0.5 \\ 0.8 \end{bmatrix},\ F_2^2 = F_1^2 \quad (34)$$
$$C^2 = [\,1\ \ 0\,]$$

The Assumption 1 is satisfied because the matrices $A_1^1 + \frac{1}{2}(A_1^1 + A_2^1)$, $A_2^1 + \frac{1}{2}(A_1^1 + A_2^1)$, $A_1^2 + \frac{1}{2}(A_1^2 + A_2^2)$, $A_2^2 + \frac{1}{2}(A_1^2 + A_2^2)$ are Metzler and the matrices $B_i^\sigma > 0$ for all $i,\sigma \in \{1,2\}$. The weighting functions are hyperbolic tangent functions and depend on the state of the switched system such as:

$$\begin{cases} \xi(t) = x_1(t) \\ \mu_1^\sigma(\xi(t)) = \frac{1}{2}(1 - \tanh(x_1(t))),\ \forall\sigma \in \{1,2\} \\ \mu_2^\sigma(\xi(t)) = 1 - \mu_1(x_1(t)),\ \forall\sigma \in \{1,2\} \end{cases} \quad (35)$$

For the simulation, the disturbances and the measurement noises are chosen such as:

$$\begin{aligned} d(t) &= 0.05\sin(0.8t),\ \overline{d} = -\underline{d} = 0.05 \\ v(t) &= 0.08\sin(0.8t),\ \overline{V} = 0.08 \end{aligned} \quad (36)$$

Thus, Assumption 2 is satisfied. For all $\sigma \in \{1,2\}$, the matrices $\alpha_1^\sigma, \alpha_2^\sigma, \gamma_1^\sigma$ and γ_2^σ in Assumption 4 are computed from the method given in Ichalal et al. (2010) and they are defined by:

$$\alpha_1^\sigma = \alpha_2^\sigma = \begin{bmatrix} 1.2 & 0 \\ 0 & 0 \end{bmatrix},\ \gamma_1^\sigma = \gamma_2^\sigma = [\,0.5\ \ 0\,] \quad (37)$$

To design the T-S interval observer given in (31) for the system (9), LMI (24) can be solved using the Yalmip toolbox (Lofberg (2004)). The scalar $\lambda^\sigma = 2.96, \forall\sigma \in \{1,2\}$ is considered. One feasible solution is given by:

$$L^1 = \begin{bmatrix} 24.5664 \\ 14.3821 \end{bmatrix},\ L^2 = \begin{bmatrix} 26.1506 \\ 15.4306 \end{bmatrix}$$
$$M = \begin{bmatrix} 0.0769 & -0.0160 \\ -0.0160 & 0.0313 \end{bmatrix}$$
$$\Omega_A^1 = diag([\,0.7830\ \ 1.3446\ \ 0.7437\ \ 1.9034\,])$$
$$\Omega_A^2 = diag([\,0.7113\ \ 1.3037\ \ 0.7890\ \ 1.3874\,])$$
$$\Omega_B^1 = \begin{bmatrix} 0.1650 & 0 \\ 0 & 1.1589 \end{bmatrix},\ \Omega_B^2 = \begin{bmatrix} 0.2175 & 0 \\ 0 & 1.0285 \end{bmatrix}$$

Clearly, the matrices $A_0^\sigma - L^\sigma C^\sigma$ are not Metzler for all $\sigma \in \{1,2\}$, then changes of coordinates are required. We propose changes of coordinates making P^σ such that the matrices $P^\sigma(A_0^\sigma - L^\sigma C^\sigma)Q^\sigma$ are Metzler for all $\sigma \in \{1,2\}$ as follows

$$P^1 = \begin{bmatrix} 0.6251 & -0.0161 \\ -0.6251 & 1.0161 \end{bmatrix},\ P^2 = \begin{bmatrix} 0.6365 & -0.0311 \\ -0.6365 & 1.0311 \end{bmatrix}$$
$$Q^1 = \begin{bmatrix} 1.6256 & 0.0258 \\ 1.0000 & 1.0000 \end{bmatrix},\ Q^2 = \begin{bmatrix} 1.6200 & 0.0488 \\ 1.0000 & 1.0000 \end{bmatrix}$$

Assumption 5 is then satisfied. The simulation results are depicted in Fig.1 with the initial condition $x_0 = [\,0\ \ 0\,]^T$ and $\overline{x}_0 = -\underline{x}_0 = [\,0.5\ \ 0.8\,]^T$. The switching law $\sigma(t)$ between the two modes of the considered system is plotted in Fig.2.

Since all assumptions of Theorem 2 are satisfied, the T-S interval observer (31) is applied. The simulation results show that the state stays in the estimated interval all the time, even when the measurement noises and the additive disturbances (36) are present. In addition, the upper and lower bounds remain stable despite the switching instants.

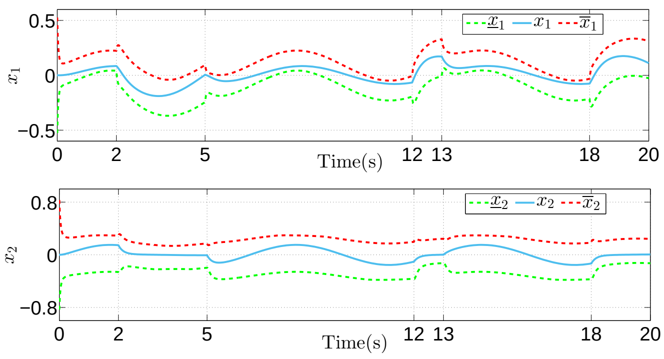

Fig. 1. Evolution of the states and their estimated bounds

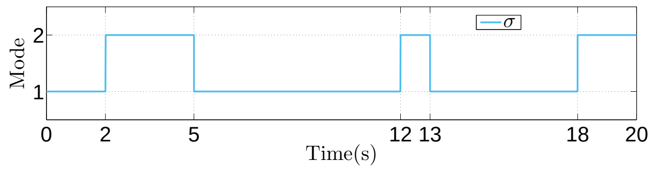

Fig. 2. Switching law σ

5. CONCLUSION

The problem of interval observers design for nonlinear switched systems described by T-S fuzzy models has been investigated. The problem is challenging because of the unmeasurable premise variables. The convergence of the error dynamics has been established using a common Lyapunov function. The nonnegativity property has been ensured through a change of coordinates. The effectiveness of the proposed method was illustrated through a numerical example. Future works will be focused on improving this present method when the switching signal is unknown and has to be estimated. Moreover multiple Lyapunov functions are also expected.

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