

# Optimizing an hybrid energy system

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## 1 Introduction

The development of renewable energy brought new complex combinatorial optimization problems as the one studied here: the conception of an autonomous hybrid energy system. Such systems involve several sources of energy, see for instance [1], [2] and [3]. In this paper we study the optimization of the design of a hybrid system including photovoltaic arrays, wind turbines, batteries storage and an auxiliary fuel generator. An excess in the production of electricity obtained from wind or sun allows loading batteries which could be used when the demand will exceed the wind and solar production; in case of total shortage of these three sources of energy, the generator is activated. In addition, the stochastic behavior of both solar and wind energy and of demand needs to be taken into account for a robust solution: here, we only consider the variation of the demands.

The aim is to determine the optimal number of photovoltaic panels, wind turbines and batteries in order to meet demand in any case while minimizing the cost function. The study is made considering an annual demand which is known day by day and hour by hour and which is denoted by  $D_t$  with  $\bar{D}_t - \varepsilon_t \leq D_t \leq \bar{D}_t + \varepsilon_t$  where the data are  $\bar{D}_t$  and  $\varepsilon_t$ .

We assume here that there is only one type of wind turbine and photovoltaic array which are defined respectively by their unit time output  $E_t^w$  and  $E_t^p$  where  $t$  is the time period, and by their cost  $C^w$  and  $C^p$ . A battery is defined by its storage capacity  $CAP$  which has a lower bound  $CAP_{min}$  a minimum performance (discharge efficiency)  $\gamma$ , maximum hourly input  $E^{in}$  and output  $E^{out}$ , and its cost  $C^b$ . Notice that a battery is either in a load state or in a discharge state during each time unit.  $C^{aux}$  is the cost of one energy unit (kW-h) produced by the auxiliary fuel generator.

## 2 The model

We modeled the problem as a mixed integer linear program. The variables are the following:  $x^w$ ,  $x^p$  and  $x^b$  are respectively the (integral) number of wind turbines, photovoltaic array and batteries to install.

$e_t^{in}$  is the total battery input and  $e_t^{out}$  is the total battery output, between  $t$  and  $t+1$ .

$e_t^{bat}$  is the total load of the batteries at time  $t$ .

$e_t^{aux}$  is the output of the auxiliary fuel generator, between  $t$  and  $t+1$ .

The program is:

$$\begin{aligned}
\min C &= C^p x^p + C^w x^w + C^b x^b + C^{aux} \sum_t e_t^{aux} \\
E_t^p x^p + E_t^w x^w - e_t^{in} + \gamma e_t^{out} + e_t^{aux} &\geq D_t, \forall t, \\
0 &\leq e_t^{in} \leq E^{in}, \\
0 &\leq e_t^{out} \leq E^{out}, \\
e_t^{bat} &= e_{t-1}^{bat} - e_t^{out} + e_t^{in}, \\
x^b CAP_{min} &\leq e_t^{bat} \leq x^b CAP,
\end{aligned}$$

### 3 Robustness

As in [4], we assume that there is an uncertainty on the demands only for a limited number  $p$  of time periods: then we consider the worst case, i.e. a maximum demand for these  $p$  time periods. Then, we show that finally the cost function can be written as:

$$\min_x C^p x^p + C^w x^w + C^b x^b + \max_{\sum_{i=1}^M z_i \leq p, z_i \in \{0,1\}} \min C^{aux} \sum_{t=1}^M e_t^{aux}$$

under the above constraints, where  $z_i=1$  if we allow  $D_i$  to vary and  $z_i=0$  otherwise.

This allow then to consider a dual sub-problem where the demands and the variables  $x^w$ ,  $x^p$  and  $x^b$  are fixed. Then we can transform the "max min" of the cost function in a linear maximization .

### 4 Results

We give the results obtained for real data which can be found in [5].

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