# Comparison of different branch-and-bound methods for a quadratic separable multi-knapsack problem

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Integer Quadratic Programming problems with linear constraints are usually NP-hard problems. The quadratic separable multi-knapsack problem $(SMQKP)$ is a special case of integer quadratic programming; this problem consists in maximizing a positive concave separable quadratic integer function subject to $m$ linear capacity constraints. It can be expressed as follows:

$$(SMQKP)\left\{\begin{array}{ll} \max\ f(x) = \sum_{i=1}^{n} c_i x_i - d_i x_i^2 & \\ s.t.(\Omega_1) & \left|\begin{array}{l} \sum_{i=1}^{n} a_{ji} x_i \leq b_j\ \forall j = 1,..,m \\ 0 \leq x_i \leq u_i, i = 1...n \\ x, u \in N^n \end{array}\right. \end{array}\right.$$

where,
$D = ((d_i))_{1\leq i\leq n}$ is a positive semi-definite diagonal $(n \times n)$ matrix, $A = ((a_{ji}))_{1\leq j\leq m, 1\leq i\leq n}$ is a $(m \times n)$ matrix, $c \in R^n$, $b \in R^m$ and $u_i \leq (\frac{c_i}{2d_i})$, $\forall i = 1,..,n$.

Quadratic Integer Programming is often considered as a problem non easy to solve because of the nonlinearity of the objective function and the integrity of the variables. This explains perhaps the rareness of recent experimentations on this topic. In this communication we compare different reformulations of $(SMQKP)$. We compare the bounds relative to the different formulations by extending the results presented in [3]. Then we use these bounds in a branch & bound procedure in order to compare their efficiency. Our aim is to give some efficient proceedings for solving integer quadratic programs such as $(SMQKP)$.

More precisely, we will compare the following formulations of $(SMQKP)$ :

- We first solve $(SMQKP)$ by using an integer programming software (Cplex 9.0). This software requires the objective function to be concave. The bound used at each node is the continuous relaxation of the initial problem: it consists in solving a continuous ($[0, u_i]$), concave, quadratic program subject to $m$ linear capacity constraints;

- We re-formulate $(SMQKP)$ as an equivalent 0-1 linear program [2] denoted by $(MKP01)$. In this formulation, the integer variables are replaced into 0-1 variables and the objective function is linearized by using its characteristics of separability. We then use the same software to exactly solve the problem. The bound used at each node is computed by solving a continuous ([0,1]) linear program subject to $m$ linear capacity constraints;

- The previous bound is equal to the one obtained by applying continuous and surrogate relaxation to $(MKP01)$. We then improve it by computing the surrogate relaxation in which we do not continuously relax the variables. This bound consists in solving a 0-1 linear program subject to one (surrogate) linear capacity constraint;

- The two previous branch-and-bound procedures solve $(MKP01)$ which is equivalent to $(SMQKP)$. We test others branch-and-bound schemes in which the initial problem is not $(MKP01)$ but directly $(SMQKP)$ with the two same bounds as above. In the scheme, $(SMQKP)$ is reformulated at each node for computing the surrogate (continuous [1] or 0-1) bound.

We report computational experiments that allow us to compare the different techniques we present.

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