GAME THEORY AND VIDEO GAME, A NEW APPROACH OF GAME THEORY TO ANALYZE AND CONCEIVE GAME SYSTEMS

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ABSTRACT

Numerous authors have tried to use game theory as a tool for computer game design and study. These works recall generally some basic concepts of the Neuman Mogenstern game analysis (payoff matrix, finite zero-sum game, dominant and mixed strategy…) and show how to model and interpret some simple gameplay situation. In this paper we consider solo games (one player against the computer) and we take the point of view of the game analyst or the game designer. Game theory is either used to understand the gameplay or to build a gameplay. In this case game theory hypothesis are never met: the computer has all the knowledge and is in charge to disclose the game rules to the player according to a driven progression scheme. But we show that, as the game designer is the creator of the universe of the game, he can choose the rules, the payoff matrices and the laws of the video game universe, that game theory can be a power full tool.

INTRODUCTION

The field of our work is game design of video game, more precisely the game system creation and the gameplay generation. The game design task didn't appear with video game. It remains the same from the board or paper games creation: conceive the game system, including game rules and eventual ludo-narrative structure, and the way the player experiments it: the gameplay.

Video game creation and research

In academicals researches the game design is studying on a high level by Espen Aarseth and the DIAC team of the ITU of Copenhagen. Their *ludologic* approach of game, in opposition of *narrativist* or *visualist* approaches, lets the game system and the simulation characteristics of video game being the centre of the playing experience and the main focus for the game designer. Characteristics of objects, relations between them and with the player are the main material to craft to build a game. Salen and Zimmerman (Salen 2004) made a large description of the game design process and highlight it with several approaches as cybernetic, game theory or information system. Those points of view on game provide concrete elements to start research on game design tools. At the CNAM, Liliana Vegas (Vegas 2003) develops new tool to analyze and conceive game system by using Petri network. Her model allows the construction of viable complex game structure.

The limit of the previous works on game design process is mainly the exclusion of the resulting tools to create gameplay (not game system) with all the difficulty and balancing stakes. As an instrument to analyze the conflict between agents, in our case the player and the game system, game theory seems to be the right operational tool to try filling this lake.

Basics of game theory

Game Theory is a tool permitting the studying of situations where agents take decisions and where the interaction of strategies is expressed into gain. We consider the mathematical games theory was invented by John Von Neuman and Oskar Morgenstern in 1944. Using game theory alone needs particular conditions, so games analysts often use it in conjunction with other branches of mathematics: statistics, probabilities and linear programming.

Typical game

Games have two mains representations: Trees (or *extensive form*) and Matrices. In our following works we chose to represent game by the matrix form. Tree form is not adapted for readability of large choices of strategies in a continuous time like most of video games playing are made of (Salen 2004).

A game is given by: A finite set of players, a finite tree or numbers of possibilities in the payoff matrix, rules of play (examples: player 1 chose a strategy then player 2 chose), and for each final node reach by the players in the tree or for each crossing of strategies in the matrix, each player receives a particular payoff.

The most classical game of the game theory is between two players A and B. The rule are determined by two payoff matrices AB and BA of real numbers having the same dimensions (k,n). This game is a sequence of independent rounds. At a given step player A choose a number i between 1 and k and, independently, player B a number j between 1 and n. They reveal simultaneously their choices. At the end of this step A wins AB(i,j) and B BA(i,j). The game ends either after a given number of rounds or, if the player has initially a finite amount of money, when one of the two players is ruined.

The preceding game is said to be a two players' game with perfect and incomplete information: the players knows all the rules but do not know all the decision of the other players.

Some others principles are important in video game design field. The *zero-sum game*, it means that AB(i,j)=-BA(i,j): the reward of player A is the loss of player B. The *dominant strategy*, strategy i of player A is dominant among the others if i gives better payoffs whatever is the choice of B. The *Nash equilibrium*, an equilibrium strategy is such that none of the players may changes unilaterally his strategy without taking the risk of loosing gain. Many finite games do not have a deterministic Nash equilibrium. All finite zero sum game has a probabilistic Nash equilibrium called a mixed strategy. A mixed strategy is a probability vector which defines the frequencies to play each individual strategy.

Video game creation and Game Theory

References made to game theory in previous works, studies and handbooks on video game design, are focussed on the high level of the game structure. This approach is justified by the fact that the notion of strategy is supposed to cover the whole game or match.

In others words, a given walkthrough between all decisions nodes from start to end of the game defines one "strategy" (Salen2004).

Second, the matrix is supposed to be given to players (human and computer)… but the goal of the game designer is to build the matrix, to choose the possible strategies and the payoff. So the value in the matrix should be unknown variables.

Third, in solo game, the goal is not to found an equilibrium strategy, but a winning strategy for the human player which is not too easy to find.

At last but not least, in the model the human player is supposed to know the matrix, which is not the case in most of video games. The game theory analysis assumes that the game decision tree is known by all the players (in a solo game the human player and the computer) and can be analyzed. If we consider adventure or action games, this point of view leads to consider fast pace decisions as "turning right", "fire" or "jump" as subgame tasks of the overall strategy, which is supposed to be understood by the player. So, at a micro level, the player (in a video game meaning) is only accomplishing boring tasks to reach the final goal (Koster 2005). If we consider classical emergent games, like Chess and Go, the decision trees are so vast that it can not be considered as a whole in a game theory analysis. This contradiction seems to be a *must have* in game.

The equilibrium research and the known matrix principle contradict one of the fundamental key of most of video games is to reveal the rules progressively (Natkin 2004) (Juul 2004). The designer will not to build a single and simple game matrix which will be quickly understood by the human player. His goal is, either to implicitly built a huge matrix (in emergent games) or to built a sequence of matrices corresponding to new and increasing strategies that the player must discover (in progression games).

We consider in the next section a real video game example and show how game theory can be used for practical design purposed. To overcome the preceding points, we study the game at a micro level of decisions and, like swim against the stream, find an acceptable strategy dimension, a right subgame strategy level needed for analysis and design.

To experiment our point of view we have considered an off the shell typical video game, Ninja Gaiden (Tecmo 2004) We tried to build a model of this micro level in terms of the game theory: building a payoff matrix of a given situation revealed some game system characteristics. Our focus was not to analyze the player's decision but the game system itself and finally our method seems to be a tool to analyze and design game.

CASE STUDY: NINJA GAIDEN

The game

Developed by Team Ninja and published by Tecmo, Ninja Gaiden is a solo *Beat'em all* with a fast pace of action and a highly difficult gameplay. This real time 3D game was released exclusively on Xbox in 2004. It had a good evaluation from the game press and became a good commercial success.

Player performs the game as a super powered ninja overcoming armies of enemies in a great variety of backgrounds with non realistic constraints. Main gameplays aspects are fighting (close combat, range weapons and spells) and platforming (from the platform game genre).

Contents of this game match with our goal, for the player is always in conflict with many enemies at the same time. Game designers defined groups' behaviours that can be identified and allow "black box" analysis of the game system strategies.

The chosen situation corresponds to the start of the 4th mission, near the quarter of the game. At this point, the learning curve has ended its main growth: The player doesn't have to learn at a fast frequency new moves or controls. He is in a tactical exploitation of his knowledge. The second gameplay bloc of the walkthrough in this level is a classical situation in Ninja Gaiden: three enemies ban the player from a door and a confrontation is necessary to the player's progression. The topology is also typical: big flat walls surround the enemies and help the player to use air moves, a main feature.

We use two kinds of data for our experiment: a video sequence of the played situation and the game itself. Video sequences is just a record of decisions sequence, one of the possible solutions of the game situation. We use it only to measure the impact of some actions and events on gauges (for example: enemies' hits on avatar's life gauges). Our main tool is the game situation itself, played a great number of times, with for each session a clear question to solve.

Building the matrix

Our goal is to build enough matrices to represent all the possible interactions between the human player and the game system player in the confrontation. We needed three elements: *human player strategies entries*, *game system player strategies entries* and a *way* to calculate payoff (The term "player" is here used in a game theory meaning).

Human player's strategies entries

According to the low level decision orientation we use the more pragmatic principle to create human player's entries: the immediate action as a result of paddle control. The player press a button, inclines a stick, holds or releases a trigger to obtain an action of the avatar.

The human player's entries must be defined by the two criteria, activated control and associate effect, for a given control leads to diffeent effects depending on the context. For example button (B) can be a bow attack, a shuriken attack or a no effect action. These variations influence the payoff calculation.

Human player entries in the matrix : Examples effects from the start of the game situation

Control (Xbox pad)	Effect
no input	standing
Left Stick	Move
A button	Jump
A button + Left Stick	Jump and move
Hold Right trigger	Block
Hold Right trigger + Left Stick	Roll
X button	Sword attack
B button	Range attack with bow
B +Y buttons	Spell (fire shield)
Y button	Powerful attack
Hold Y button	Load charge
Other buttons	Open menu, center camera…

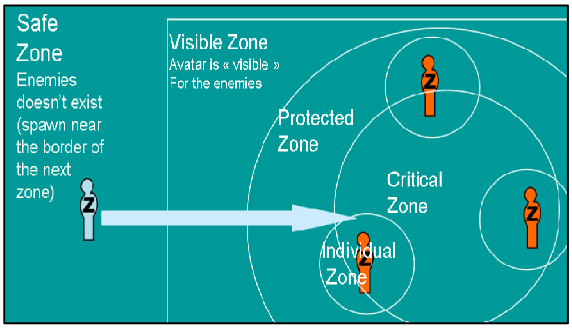
We can implement it in a matrix in the following way

Game system player strategies entries

A solo game system goal is not to win but to create a conflict which can be overcome by the human player. The main elements to consider, building our model, are the ones which induce changes of the human's strategies during the game. All obstacles on the journey to a victorious resolution can be included in the construction of the game system player strategies entries.

In our specific game situation, it's an enemies' trio. Their actions can be considered as game strategy in several ways. We chose to base the entries not on the individual states of the enemies but on the formation configuration. This choice can be validated as it has a simple and direct effect on the payoff (see next section).

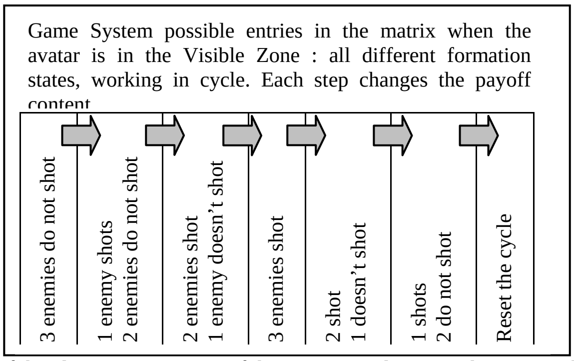
We have also to consider the evolution of the formation state from a *game system* point of view. To understand it, we have to look closely how the formation behaviour was conceived by the game designers. Enemies' formation has several ranges of detection of the player position. The enemies are assigned to protect a place in the topology. The avatar presence in each zone materialized on the figure activates specific group behaviour.



When the player stays in the Visible Zone, the behaviour of the formation loops in a cycle of states defined by the range weapons used by each enemy. If the player comes in the Protected Zone, the enemies cycle change to a "one tries to attack the avatar with close combat weapon, the others use range weapons". If the player comes in Critical Zone, only one enemy uses range weapon, the others fight hand to hand. To each zone and each state of a cycle corresponds a different payoff for a same human player's action.

We chose to use as entries for one matrix all the possible states of the enemies' formation for one human player's situation. This set of possible states remains the same as long as the relative position of the human's player stays the same.

For instance, if the player'avatar is in the Visible Zone, the formation loops according to the following cycle, each step changes the payoff content:



If the player comes nearer of the enemies and enter in the Protected Zone and a different cycle is activated:

Using these entries and crossing them with the human player controls and effects, it is possible to define the rows and the column of the game matrix.

Defining the payoff values

To define the payoff values we need a unit. There are four mains resources in Ninja Gaiden: hits points (and all heal potion and other objects linked), magic resources (mana and all objects linked to the restoration of mana), special range weapons (ammunitions) and "yellow Aura". No one of these resources is regenerated automatically in the game: The human player spends, uses, buys and wins them. Yellow aura have the function of money and is particularly interesting, all other resources can be calculated in term of yellow aura. So, all the consequences of the player's actions can be measured in Yellow aura units. For example:

When an enemy is killed, he drops a 100 Yellow Aura bonus on ground. If the player hits him with is normal sword attack, four hits are necessary to kill him. So for each hit with normal attack, the gain is +25 Yellow Aura.

If the player is hit in a fight, his life gauge losses a given amount of points. Life has a cost linked to the price of heal potions. So each type of hit can be calculated in Yellow Aura.

The following table defines significant values of payoff according to a given action of the human player:

Type of gain/loss: Value in yellow aura in the matrix
Use one range ammunition (here Arrows): -10
Use a spell: - 2000
All attack out of range, spell include: 0
Hit with a default range weapon: + 2,5
Hit with normal sword attack: + 25 at less*
Hit with an Arrow: + 50
Hit with a spell : here A fire shield: + 50 at less*
Hit with a loaded attack: +50 at less*
Hit with a powerful attack: +50 at less*
Hit with a counter attack: +50
Take a Heal bonus: + 150
No damage possible on the avatar: 0
Range hit from these type of enemy: - 20 X the number of attacking enemies
Hand to hand hit from these type of enemy: - 60 X the number of attacking enemies
An enemy cut of the throat: - 300
* multiply by the number of enemies in range

Another element must be taken into account to fill the matrix: the accuracy ratio of the enemies depends on the action of the player. For example if the player jumps constantly and when an enemy fires, he has one chance on eight to be hit. So, in the matrix, at this crossing of entries, we can implement a cost expectation of *(-20 yellow aura by 1 chance on 8 to be hit =) -2,5* for the human player.

One notion is not measured in our method: the advance of the human player to a victorious resolution of the whole game.

The down page figure represents a matrix build with the method. For sake of simplicity the buttons opening menus are not represented.

Matrix 1 : Avatar on ground in Visible Zone			Game system player			
	Human player Control	Effect on avatar	3 do not shoot	1 shots 2 do not shot	2 shot 1 doesn't shot	3 shot
Human player	no input	standing	0	- 20	- 40	-60
	L Stick	Move to the next zone, n the same zone, in the previous zone	0	-15	-30	-45
	A	Jump	0	-2,5	-5	-7,5
	A + L Stick	Jump and move	0	-2,5	-5	-7,5
	Hold R trigger	Block	0	0	0	0
	Hold R trigger + L Stick	Roll	0	-0,1	-0,2	-0,4
	X	Sword attack	0	-15	-30	-45
	B	Range attack with bow	+50	+45	+40	+35
	B+Y	Spell (fire shield)	-2000	-2000	-2000	-2000
	Y	Powerful attack	0	- 20	- 40	-60
	Hold Y	Load charge(	0	- 20	- 40	-60

From a matrix to another

To covert the all possible payoffs in the chosen game situation we must conceive several matrix. The main variable for evaluating the changes is the player situation in space. It affects: the Controls possibilities and Effects in human player entries, the content of the enemies cycle and the payoffs values. There are three positions relative to the enemies' formation: in the Visible Zone, in the Protected Zone, in the Critical Zone. There are also three positions relative to the topology: on ground, in the air, on the wall. Each time the player's avatar change one of the values, the matrix change.

Another variable changing the content of the matrix is the avatar state. For example if the player changes his range weapon to Shuriken instead of Bow, then the payoff for "Range attack" changes. If the player activates his spell "Fire shield," even his moves can provide damages. The payoffs of "Jump", "Move" and "Roll" to the enemies gain positives values if he's evaluating near enemies. Those modifications are minor compared to the first variable. We consider them as variations more than new matrices.

The number of matrices is limited to 9 and the player jumps from one to another at fast pace.

Game design rules determine the possible links between matrices. The player cannot go from one matrix to anyone of the others. In the previous figure you can see that if the player's current matrix is "Critical Zone / On the wall", his next possible matrices are "Protected Zone/on the wall", "Protected Zone/in the air" or Critical Zone/in the air". This principle reveals one of the characteristic of the player's relation to the game system: he can chose to cross an unwanted matrix on his way to reach a more interesting one. This is one of the objects of our next section.

ANALYSING THE GAMEPLAY USING THE GAME THEORY MATRICES

The good accumulation of payoffs problematic

Analysing the first matrix, we can see that the player has a dominant strategy, fire with his bow. The player can also "Block" as long as he wants without taking damages, the cycle of enemies' states doesn't change the payoff for this human entry. This strategy allows observing enemies' cycle. We can notice the advantage to act during the first state of the cycle, when no enemy shots. Combining these two first comments we can imagine a succession of strategies in the same matrix: Hold the Right Trigger, "Block", until the cycle is in the first state, then press B button, "Range Attack Weapon with Bow". It's the optimal combination for this specific matrix. But at this point in the game the player knows something important that push him not to use the bow: He cannot carry more than 12 arrows and can only fill his quiver by buying ammunitions in a shop or by discovering some specific bonus. In this particular level, at this specific moment, he doesn't know where or when he can find ammunitions or bonuses. Moreover, by the past he faced situations where arrows were the only way to eliminate specifics enemies and avoid the game-over. So the player pays attention of the way he uses arrows. Even if "Block and Bow" seems to be a good strategy by analysing this single matrix, in a more high level consideration this strategy may not be the best one.

As we can see, using one micro level matrix is not sufficient to have the complete vision of human player strategies and real payoff. We must consider this level as a subgame in a succession of matrices.

The down page Matrix corresponds to a player located in the Protected Zone on the ground.

We try now to identify game strategies using several matrices. In the Visible Zone (1st matrix), the player can "Block" until he can "Move" without taking hits, then enter in the protected zone (2nd matrix), and make a "Sword Attack". These three successive sub game strategies allow the player to have a positive payoff.

But, is it the only final payoff of the situation? Not really: we can consider the real payoff at the death of one enemy. Then a new game situation arises: the Avatar versus two enemies setting of matrices. But the player may choose bad payoff strategies to kill the first enemy with in mind to create an opportunity to destroy in an easier way the two remaining ones. So, must we consider the destruction of all the enemies as the good payoff? And we can ask same question for the save points, the end of the level or even the whole game. We reach an interesting propriety of the game system showed by this method: the game system provide the player a numerous levels of choices.

Those levels of interaction, in terms of game theory, are a good tool to measure the depth of gameplay. Definition of gameplay is linked to the players' choices as Sid Meier, Ernst Adams, Andrew Rollins (Rollins and Adams 2003) or Ed Byrne (Byrne 2005) defined them. Here we can obviously see that the payoff can be good in different ways. The player has a panel of possible layers of decision. He can find a good reason to act differently depending

Matrix 2 : Avatar on ground in Protected Zone			Game system player					
			A – The close combat oriented enemy try to reach the player			B – The close combat oriented enemy make an attack		
	Human player Control	Effect on avatar	2 do not shot 1 prepares close combat	1 shots 1 doesn't shot 1 prepares close combat	2 shot 1 prepares close combat	2 do not shot 1 makes a close combat attack	1 shots 1 doesn't shot 1 makes close combat attack	2 shot 1 makes a close combat attack
Human player	no input	standing	0	-20	-40	-60	-80	-100
	L Stick	Move to the next zone, n the same zone, in the previous zone	0	-15	-30	-60	-75	-90
	A	Jump	0	-2,5	-2,5	-60	-62,5	-65
	A + L Stick	Jump and move	0	-2,5	-2,5	-60	-62,5	-65
	Hold R trigger	Block	-300	-300	-300	0	0	0
	Hold R trigger + L Stick	Roll	0	-0,1	-0,2	0	-0,1	-0,2
	X	Sword attack	+25*	+20*	+15*	+25*	+20*	+15*
	B	Range attack with bow	+50	+45	+40	-10	-15	-20
	B+Y	Spell (fire shield)	-1950*	-1950*	-1950*	-1950*	-1950*	-1950*
	Y	Powerful attack	+50*	+45*	+40*	+50*	+45*	+40*
	Hold Y	Load charge	+ 50*	-20	-40	-60	-80	-100

* The gain is to multiply by the number of enemies in range. In this situation, most of cases are versus one.

witch one of the level of matrices accumulation interests him. At one precise moment in the game, more the levels of good payoff are numerous, more the gameplay is deep.

Learning in game

Considering our previous section, the game theory approach is useful to manage the *learning curve* of the player. This curve formalizes the growth of the repertoire of the player. As Jesper Juul describes it, the solo games constantly offer new varieties of situations, increasing the tools the player must master. At the start of the game, the challenges are basics and ask, for instance, little performance. The more you go through the game, the more you have to find new tactics (beat new enemies with new behaviours) or to master some specific manipulation (the enemy is faster so you must be more accurate to hit him).

The way the learning curve appears in this game theory approach is simple: each time the player has experiment a combination of matrices, as our 9 ones for the game situation, he learns good strategies to progress. At the moment the designer consider the player masters this kind of situation, the matrix is changed: by the game system strategies entries or starting state of the avatar in the situation. Some examples follow.

The player meets several times the same trio of this specific type of enemy. But the first times were in an interior topology or by falling near them. Understand: the situation didn't start in the on ground/in the visible zone but on ground/in the protected zone or on ground/in the critical zone. With the situation of our case of study we can see that the game designer use this trio component to push the player to adopt different way to manage the conflict.

After our case of study situation, the player meets a new configuration: two of our typical enemies and a new one, firing bombs. This formation has a completely different matrix. The player cannot be safe by blocking at distance. The bomber enemy makes this strategy provide a negative payoff. At least, the player must move constantly. He has to learn a new way to beat enemies.

By making a walkthrough of all the matrices of the game we can show the evolutions of the winning strategies, a visibility of what the player learns.

CONCLUSION

We have shown that game theory can be an efficient tool to model and understand local properties of a gameplay. As we have experiment this method from a black box approach without the knowledge of game design documents or meeting with creative teams the experiment was rather fastidious to perform. But this work shows that the game theory tools can be a good model of existing practices in game design. The creations of game matrices when the gameplay is thought can be an efficient tool to formalized and discover hidden effects of payoff principle

We see several possibilities for the next steps:

The first one is analyzing others games (racing, open world…). Those new titles must include some space problematic to challenge the model. Ninja Gaiden has a strong aiming assistant so we didn't have to manage, for instance, the probabilities of hit and the player's mastering of manipulations. They must be confronting to the approach.

We can also use the method as a design tool and conceive a game system. We reveal some "must have": player entries (controls and effects), game system entries (obstacles), table to calculate the pay off (find a unique unit), moderator to calculate the payoff (as the ratio of hit in Ninja Gaiden) and the rules of transition from a matrix to another. This method provides us a way to manage the interesting choice factor: at all moment the player must have several scales of interesting payoff.

At least, one of the most interesting uses is linked to the learning curve of the player. By using the revealed principle of level of payoff, we can conceive guidelines for a player profiling A.I. conception. By measuring the choices made in the Immediate Action and the Perceived Situations scales, a game system can manage what the player know and, with moderation variables, create a dynamic model of the player.

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