

Transitivity Tables

We introduce transitivity tables to derive new temporal knowledge from the resulted qualitative temporal relations between time intervals and points that may be precise or imprecise.

- **Qualitative Temporal Relations between Time Intervals**

The Allen's transitivity table lets us obtain from $R1(I, J)$ and $R2(J, K)$ that $R3(I, K)$ holds, where $I = [I^-, I^+]$, $J = [J^-, J^+]$ and $K = [K^-, K^+]$ are precise time intervals and $R1, R2$ and $R3$ are Allen's relations. As already mentioned, we can deduce from "During(I, J)" and "Meet(J, K)" that "Before(I, K)" holds.

We generalize such deductions using imprecise time intervals. Let $I = [I^-, I^+]$, $J = [J^-, J^+]$ and $K = [K^-, K^+]$ be imprecise time intervals; where $I^- = \{I^{-(1)} \dots I^{-(B)}\}$, $I^+ = \{I^{+(1)} \dots I^{+(E)}\}$, $J^- = \{J^{-(1)} \dots J^{-(B)}\}$, $J^+ = \{J^{+(1)} \dots J^{+(E)}\}$, $K^- = \{K^{-(1)} \dots K^{-(B)}\}$ and $K^+ = \{K^{+(1)} \dots K^{+(E)}\}$. Based on Table 2, we can deduce from "During(I, J)" and "Meet(J, K)" that "Before(I, K)" holds. Indeed by "During(I, J)", we have "Precedes($J^{-(B)}, I^{-(1)}$) \wedge Precedes($I^{+(E)}, J^{+(1)}$)", and by "Meet(J, K)", we have "Same($J^{+(1)}, K^{-(1)}$) \wedge Same($J^{+(E)}, K^{-(B)}$)". From "Precedes($I^{+(E)}, J^{+(1)}$)" and "Same($J^{+(1)}, K^{-(1)}$)", we conclude that "Before(I, K)" holds. Our transitivity table coincides with the Allen's one.

- **Qualitative Temporal Relations between a Time Interval and a Time Point**

We introduce two transitivity tables to reason about the qualitative temporal relations between a time interval and a time point. Let I and J be time intervals and P and L be time points. They may be all precise or all imprecise. Table 1 shows the transitivity table that allows to reason $R3(P, L)$ from $R1(P, I)$ and $R2(I, L)$. $R1$ is a Point-Interval relation; $R2$ is an Interval-Point relation and $R3$ is a Point-Point relation. There are some composition relations that are "undecidable". For instance, the relations "After(P, I)" and "Before(I, L)" yields three possible relations: "Before(P, L)", "Equals(P, L)" and "After(P, L)". One of them is right.

Table 1. Transitivity table that allows reasoning $R3(P, L)$ from $R1(P, I)$ and $R2(I, L)$.

| R1(P, I) / R2(I, L) | Before(P, I) | After(P, I) | Starts(P, I) | During(P, I) | Ends(P, I) |
|----------------------------|---------------------------------|---------------------------------|---------------------|---------------------------------|-------------------|
| Before(I, L) | Before(P, L) | One of the time point relations | Before(P, L) | Before(P, L) | Before(P, L) |
| After(I, L) | One of the time point relations | After(P, L) | After(P, L) | After(P, L) | After(P, L) |
| Started-by(I, L) | Before(P, L) | After(P, L) | Equals(P, L) | After(P, L) | After(P, L) |
| Contains(I, L) | Before(P, L) | After(P, L) | Before(P, L) | One of the time point relations | After(P, L) |
| Ended-by(I, L) | Before(P, L) | After(P, L) | Before(P, L) | Before(P, L) | Equals(P, L) |

Table 2 allows to reason $R3(I, J)$ from $R1(I, P)$ and $R2(P, J)$. $R1$ is an Interval-Point relation; $R2$ is a Point-Interval relation and $R3$ is an Interval-Interval relation.

Table 2. Transitivity table that allows reasoning $R_3(P, L)$ from $R_1(P, I)$ and $R_2(I, L)$.

| $R_1(I, P)$ $R_2(P, J)$ | Before(I, P) | After(I, P) | Started-by(I, P) | Contains(I, P) | Ended-by(I, P) |
|----------------------------|---|--|--|---|--|
| Before(P, J) | Before(I, J) | One of the time interval relations | Contains(I, J) \vee Ended-by(I, J) \vee Met-by(I, J) \vee Before(I, J) \vee Overlaps(I, J) | Contains(I, J) \vee Ended-by(I, J) \vee Overlapped-by(I, J) \vee Meets(I, J) \vee Before(I, J) | Before(I, J) |
| After(P, J) | One of the time interval relations | After(I, J) | After(I, J) | After(I, J) \vee Met-by(I, J) \vee During(I, J) \vee Started-by(I, J) \vee Overlapped-by(I, J) | During(I, J) \vee Started-by(I, J) \vee Overlapped-by(I, J) \vee Met-by(I, J) \vee After(I, J) |
| Starts(P, J) | Before(I, J) | After(I, J) \vee Met-by(I, J) \vee Overlapped-by(I, J) \vee Ended-by(I, J) \vee Contains(I, J) | Started-by(I, J) \vee Equals(I, J) \vee Starts(I, J) | Contains(I, J) \vee Ended-by(I, J) \vee Overlaps(I, J) | Meets(I, J) |
| During(P, J) | During(I, J) \vee Starts(I, J) \vee Overlaps(I, J) \vee Meets(I, J) \vee Before(I, J) | After(I, J) \vee Met-by(I, J) \vee Overlapped-by(I, J) \vee Ended-by(I, J) \vee Contains(I, J) | Contains(I, J) \vee Ends(I, J) \vee Overlapped-by(I, J) | Overlaps(I, J) \vee Overlapped-by(I, J) \vee Starts(I, J) \vee Started-by(I, J) \vee During(I, J) \vee Contains(I, J) \vee Ends(I, J) \vee Ended-by(I, J) \vee Equals(I, J) | Overlapped-by(I, J) \vee Starts(I, J) \vee During(I, J) |
| Ends(P, J) | During(I, J) \vee Starts(I, J) \vee Overlaps(I, J) \vee Meets(I, J) \vee Before(I, J) | After(I, J) | Met-by(I, J) | Contains(I, J) \vee Started-by(I, J) \vee Overlapped-by(I, J) | Equals(I, J) \vee Ended-by(I, J) \vee During(I, J) |

- **Qualitative Temporal Relations between Time Points**

We introduce a transitivity table to reason about the resulting qualitative temporal relations between time points. It allows to reason $R_3(P, M)$ from $R_1(P, L)$ and $R_2(L, M)$, where P, L and M are precise or imprecise time points. R_1 , R_2 and R_3 are Point-Point relations.

Table 9. Transitivity table that allows reasoning $R_3(P, M)$ from $R_1(P, L)$ and $R_2(L, M)$.

| $R_1(P, L)$ | Before(P, L) | Equals(P, L) | After(P, L) |
|-------------------------------|---------------------------------|---------------------|---------------------------------|
| $R_2(L, M)$ | | | |
| Before(L, M) | Before(P, M) | Before(P, M) | One of the time point relations |
| Equals(L, M) | Before(P, M) | Equals(P, M) | After(P, M) |
| After(L, M) | One of the time point relations | After(P, M) | After(P, M) |