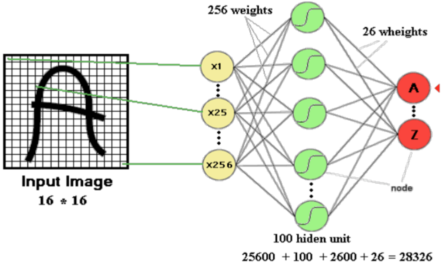
Entreposage et fouille de données (STA211)
Neural Networks and Deep Learning

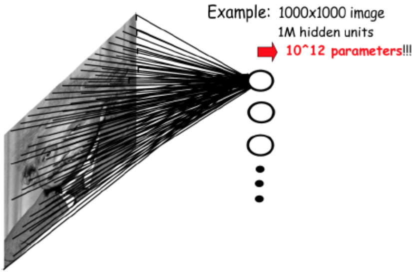
Nicolas Thome
Conservatoire Nationnal des Arts et Métiers (CNAM)
Laboratoire CEDRIC - équipe Vertigo

Outline

Fully Connected Networks: Limitations

- Scalability issue with Fully Connected Networks (MLP)

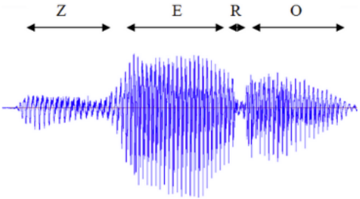


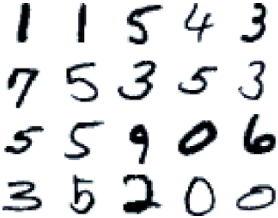


⇒ # Parameter explosion even for a single hidden layer !

Limitations of Fully Connected Networks

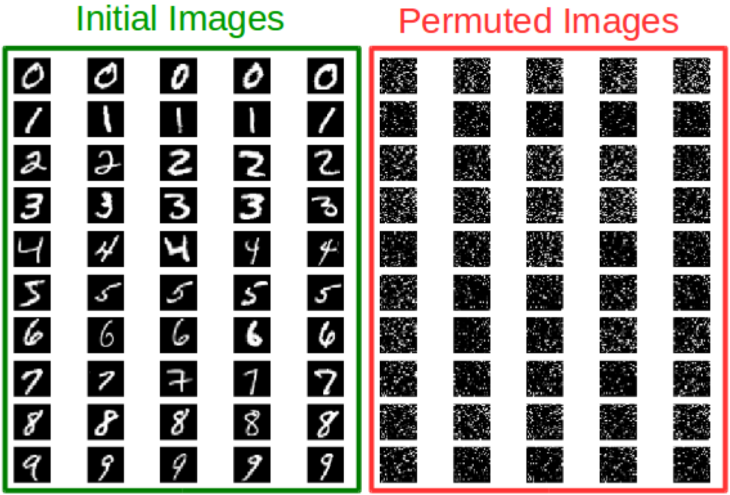
- **Signal data: importance of local structure**
  - 1D signals: local temporal structure
  - 2D signal data: local spatial structure





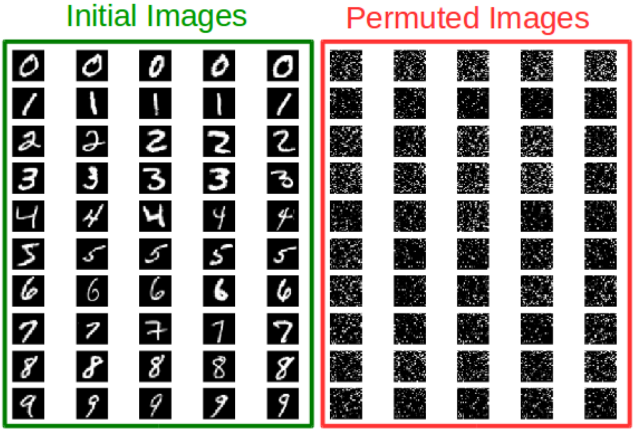
Limitations of Fully Connected Networks

- BUT: vectorial representation of inputs: dimensions arbitrary!



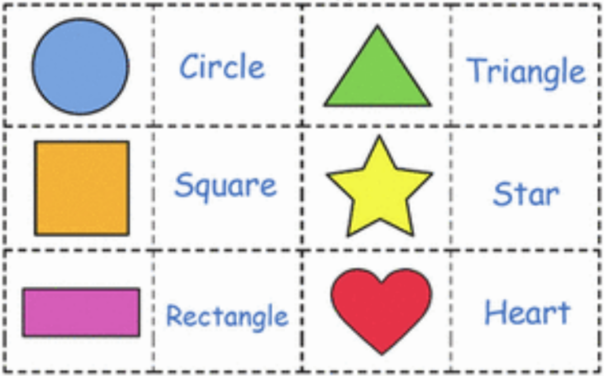
Limitations of Fully Connected Networks

- MNIST ex: same performances with initial and permuted images!
  - However, local information obviously useful



Fully Connected Networks: Limitations

- Fully connected networks: no assumption on data structure
  - Structure can be learned but need lots of annotated data
  - Prior knowlege on data structure ⇒ useful
- Example: MLP training for shape recognition from color images

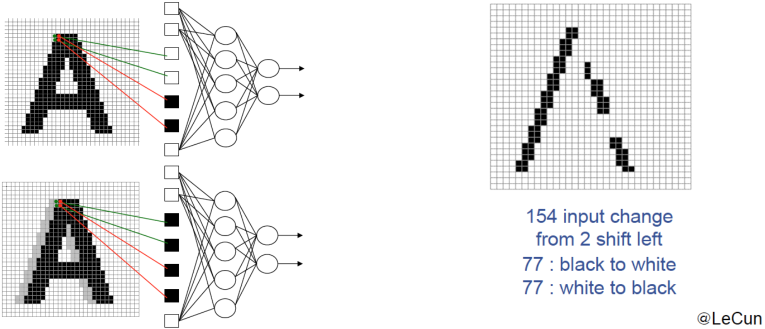


Input image encoding:

- Color (RBG) ?
- Grayscale $L = \frac{R+B+G}{3}$?

Fully Connected Networks: Limitations

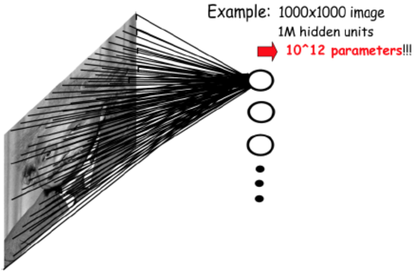
- Invariance and robustness to deformation (stability)
- What we expect:
  - Small deformation in input space ⇒ similar representations
  - Large transfo in input space ⇒ very dissimilar representations
- Example (image): impact of a 2 pixel shift



Fully Connected Networks: Limitations

Conclusion of MLP on raw data

- Brute force connection of images as input of MLP NOT a good idea
  - No Invariance/Robustness of the representation because topology of the input data completely ignored
    $\Rightarrow$ *e.g.* indifferent to permutations of input pixel
  - Nb of weights grows largely with the size of the input image

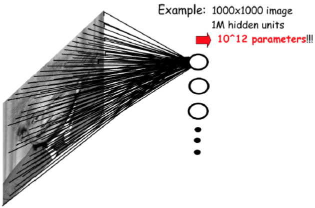


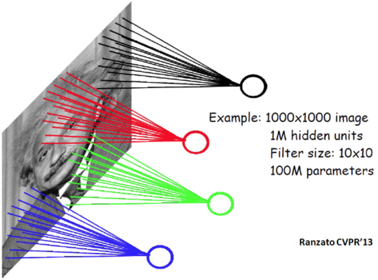
$\Rightarrow$ How keep spatial topology?
$\Rightarrow$ How to limit the number of parameters?

Taking advantage of structure: Convolution

How to limit the number of parameters?

1. Sparse connectivity: hidden unit only connected to a local patch
   - Weights connected to the patch: **filter** or **kernel**
   - Inspired by biological systems: cell only sensitive to a small sub-region of the input space (receptive field). Many cells tiled to cover the entire visual field

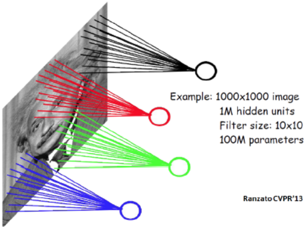


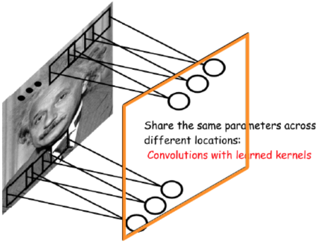


Taking advantage of structure: Convolution

How to limit the number of parameters?

2. Shared Weights
    - Hidden nodes at different locations share the same weights
        - Substantially reduces the number of parameters to learn
    - Keep spatial information in a 2D feature map (hidden layer map)





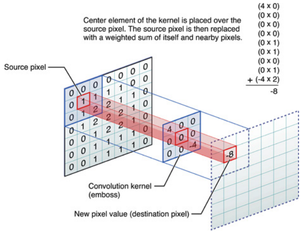
⇒ Computing responses at hidden nodes equivalent to convolving input image with a linear filter (learned)
⇒ A learned filter as a feature detector

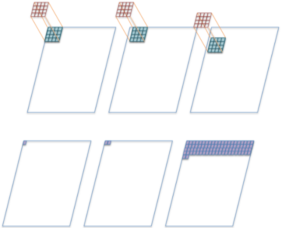
Convolution: Scalar Images

- 2D convolution with a Finite Impulse Response (FIR) h of size d (odd):

$$f'(i,j) = (f \star h)(i,j) = \sum_{n=-\frac{d-1}{2}}^{\frac{d-1}{2}} \sum_{m=-\frac{d-1}{2}}^{\frac{d-1}{2}} f(i-n, m-j)h(n,m)$$

- Simply centering filer h in pixel (x,y) $\Rightarrow$ weighted sum



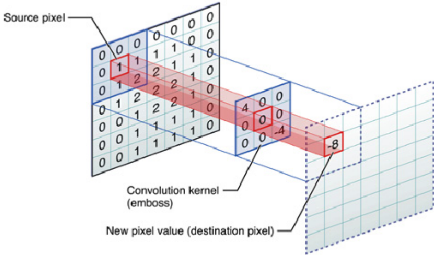


- Output for 1 filer (resp. K filters): 1 2D map (resp. K 2D maps)

2D Convolution *vs* Cross-Correlation

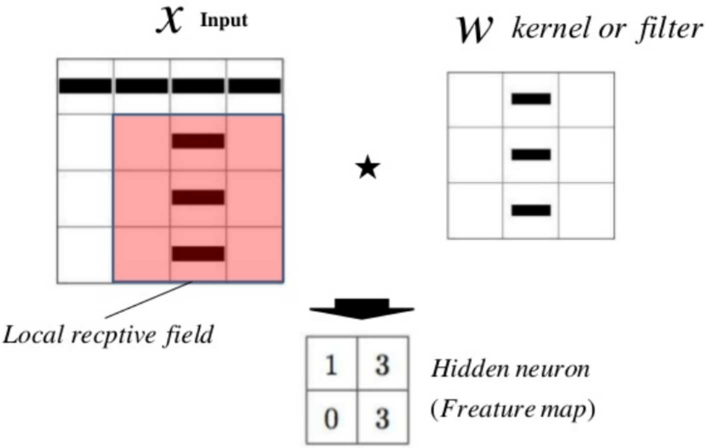
- 2D Convolution:
  $$f'(i,j) = (f \star h)(i,j) = \sum_n \sum_m f(i-n, m-j)h(n,m)$$
- Cross-Correlation:
  $$f'(i,j) = (f \otimes h)(i,j) = \sum_n \sum_m f(i+n, m+j)h(n,m)$$
  - Cross-Correlation ~ Convolution without symmetrizing mask!

$$h = \begin{pmatrix} -4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix} \Rightarrow g = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -4 \end{pmatrix}$$



2D Convolution / Cross-Correlation: Example

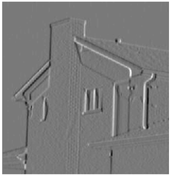
- Cross-Correlation: output maps ⇔ location in input image similar to mask



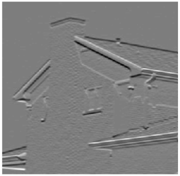
Convolution: Example for Gradient Computation

- Gradient: $\overrightarrow{G}(x, y) = \begin{pmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{pmatrix}^T = \begin{pmatrix} Ix & Iy \end{pmatrix}^T$
- Convolution approximation: $I_x \approx I \star M_x$, $I_y \approx I \star M_y$

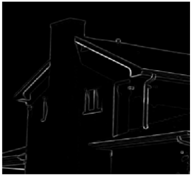
$$M_x = \frac{1}{4} \cdot \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad M_y = \frac{1}{4} \cdot \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$



$I_x \approx I \star M_x$



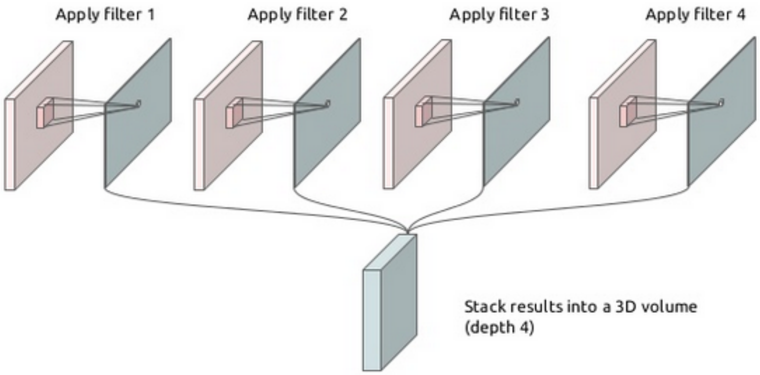
$I_y \approx I \star M_y$



$I_e = Ix^2 + Iy^2$

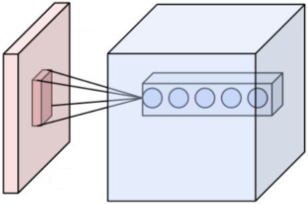
Convolution Layer

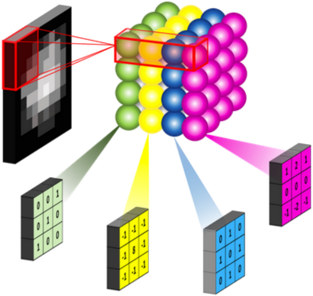
- 2D convolution: each filter $\Rightarrow$ 2D map (image)
- Convolution Layer: staking maps from multiple Filters
  $\Rightarrow$ **Tensor: multi-dimensional array**



Convolution Layer

- Tensor: stacking several filters outputs
  - Depth ⇔ # filters
  - Each spatial position: output for the different filters
- Ex: 2D conv with gray-scale images, input tensor depth=1
- **Convolution on color images / hierarchies:**
  - **Convolution on tensors!**
  - **Input Tensor ⇒ output Tensor**

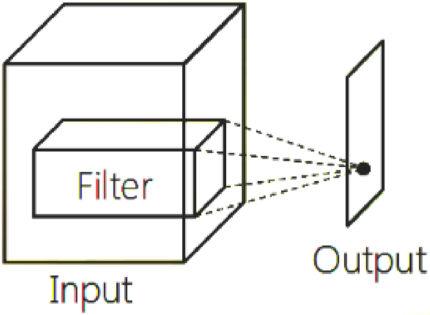


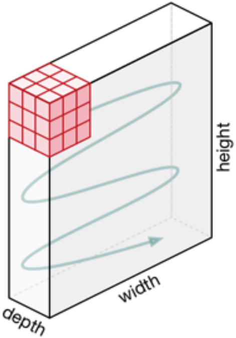


Convolution Layer for Tensors

$$f'(i,j) = (f \star h)(i,j) = \sum_{k=1}^{K} \sum_{n=-\frac{d-1}{2}}^{\frac{d-1}{2}} \sum_{m=-\frac{d-1}{2}}^{\frac{d-1}{2}} f(i-n, m-j, k) h(n,m,k) \; + \; b$$

- **Convolution: linear, bias b ⇒ affine**

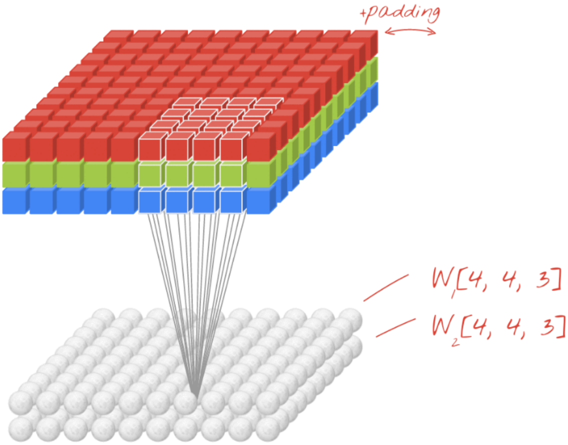




- **Filtering on depth:** correlation between feature maps

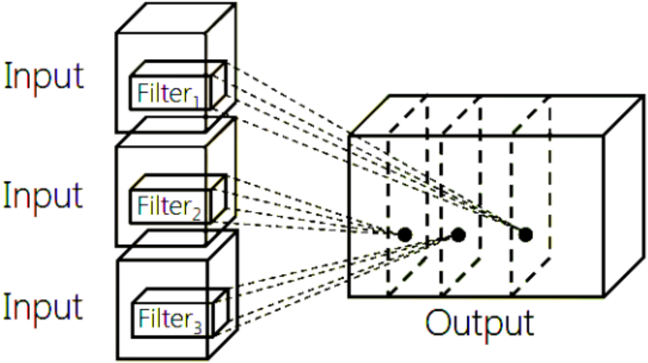
Convolution Layer for Tensors

Ex: input color image



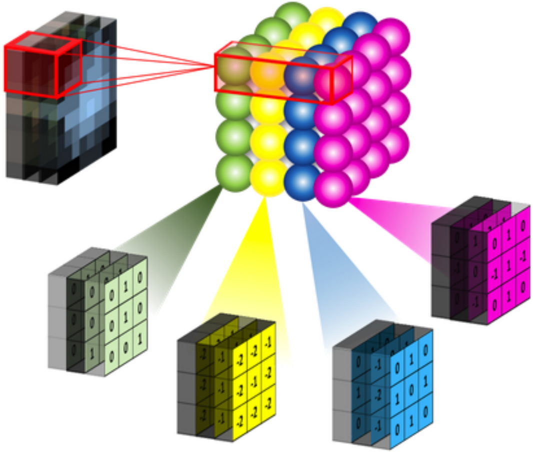
Convolution Layer for Tensors

Natural extension for multiple filters



Convolution Layer for Tensors

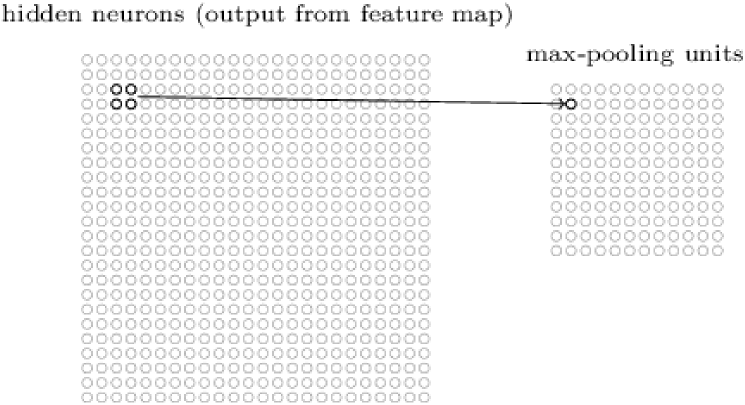
Ex: input color image



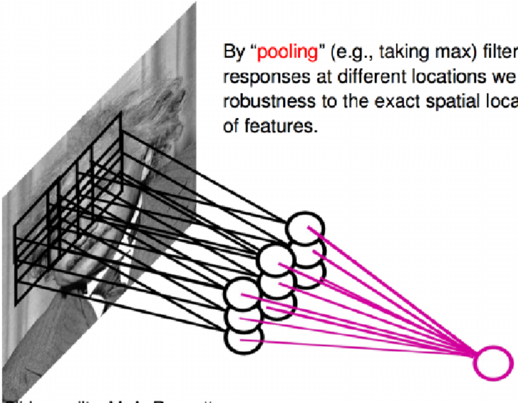
Pooling Layers

How to gain (local) shift invariance ?

- Spatial agregation for each layer
- If stride $s > 1$, sptial resolution decreases (subsampling) $\Rightarrow$ gaining invariace to local translations



Pooling Layers



By "pooling" (e.g., taking max) filter responses at different locations we gain robustness to the exact spatial location of features.

Slide credits: M. A. Ranzatto

Pooling Layers: Examples

Max-pooling:

$$h_j^n(x, y) = max_{\bar{x} \in N(x), \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})$$

Average-pooling:

$$h_j^n(x, y) = 1/K \sum_{\bar{x} \in N(x), \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})$$

L2-pooling:

$$h_j^n(x, y) = \sqrt{\sum_{\bar{x} \in N(x), \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})^2}$$

L2-pooling over features:

$$h_j^n(x, y) = \sqrt{\sum_{k \in N(j)} h_k^{n-1}(x, y)^2}$$

Slide credits: M. A. Ranzatto

Max Pooling & Translation Invariance

- Translation invariance wrt vector $\overrightarrow{T} = (t_x, t_y)^t$ if:
  - $\overrightarrow{T} \not\Rightarrow$ new largest element at pooling region edge
  - $\overrightarrow{T} \not\Rightarrow$ remove max from pooling region
- Ex: 5×5 conv map, 3×3 max pooling centered at 15: $max = 15$,
- **Invariance OK: $\forall$ translation $(t_x, t_y) \in \pm 1$ px**
  $\Rightarrow max = 15$

$$C = \begin{bmatrix} 11 & -5 & 1 & -2 & 0 \\ 1 & 3 & 0 & 0 & 5 \\ 8 & 4 & 15 & -10 & 4 \\ 8 & 6 & 5 & 3 & 7 \\ 3 & 0 & -2 & 9 & 3 \end{bmatrix}$$

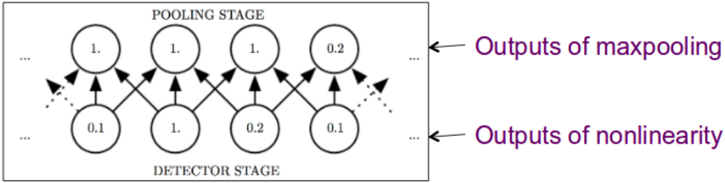
Max Pooling & Translation Invariance

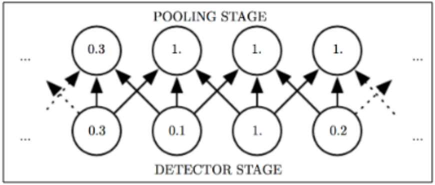
- Translation invariance wrt vector $\vec{T} = (t_x, t_y)^t$ if:
  - $\vec{T} \nRightarrow$ new largest element at pooling region edge
  - $\vec{T}$ remove max from pooling region
- Ex: 5×5 conv map, 3×3 max pooling centered at 15: $max = 15$,
- **Invariance KO: right translation $t_x = +1$ px**
  $\Rightarrow max = 7$

$$C = \begin{bmatrix} 11 & -5 & 1 & -2 & 0 \\ 1 & 3 & 0 & 0 & 5 \\ 8 & 15 & 4 & -10 & 4 \\ 8 & 6 & 5 & 3 & 7 \\ 3 & 0 & -2 & 9 & 3 \end{bmatrix}$$

Max Pooling & Translation Invariance

- Max pooling: partial translation invariance (under some conditions)
  - **At least local stability:** every value in bottom changed, only half values in top changed ⇒ Distance after pooling decreases





From [Goodfellow et al., 2016]

Convolutional Neural Networks (ConvNets)

- An elementary block: Convolution + Non linearity + pooling
- Stack several blocks: Convolutional Neural Networks (ConvNets)

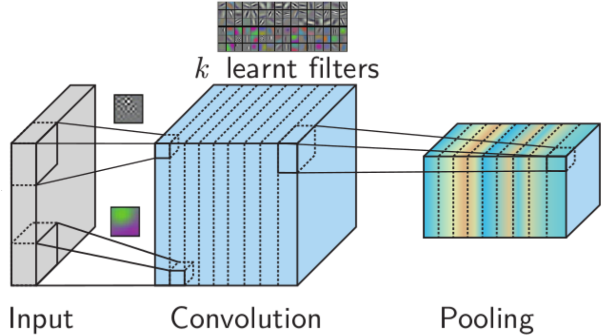


Figure: Important building blocks in CNN

Convolutional Neural Networks (ConvNets)

- Generally, Feature maps stacked together at one point $\Rightarrow$ fully connected layers

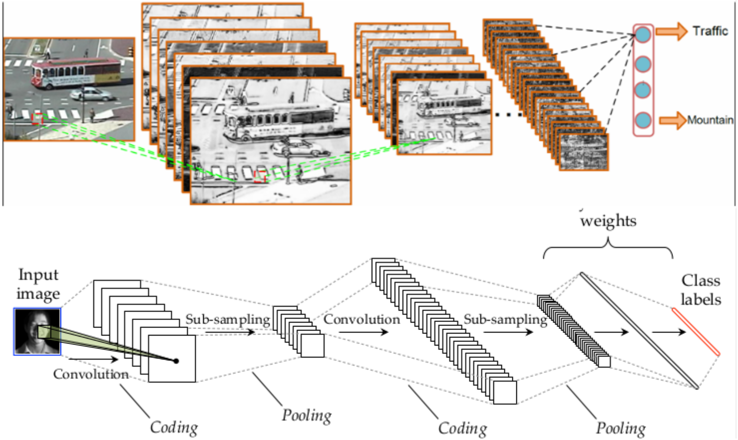
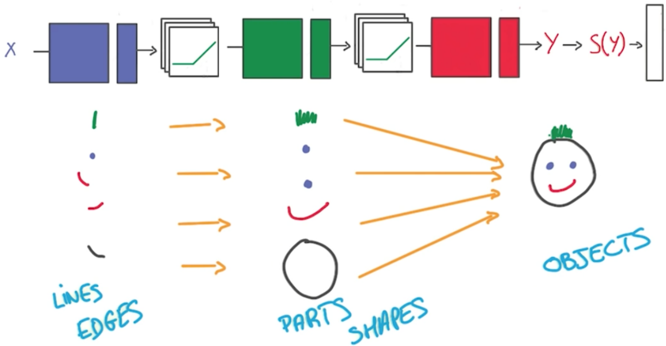


Figure: Important building blocks in CNN

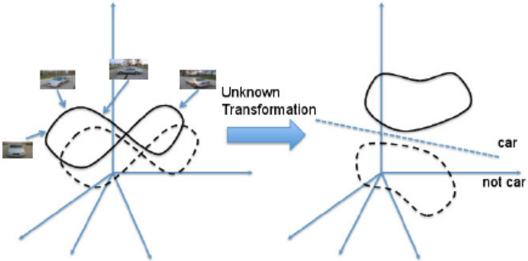
ConvNets: Conclusion

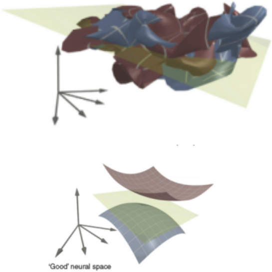
- Crucial step for tacking advantage of structure ⇒ local processing
- Useful for many data types and applications:
  - Low-level signal, *e.g.* image, audio (speech, music)
  - More semantic data, *e.g.* modern text embedding (`word2vec`) or RNN
- Block [Convolution + Non linearity + pooling] intuitive for modeling hierarchical information extraction



ConvNets and Manifold Untangling

Manifold Untangling

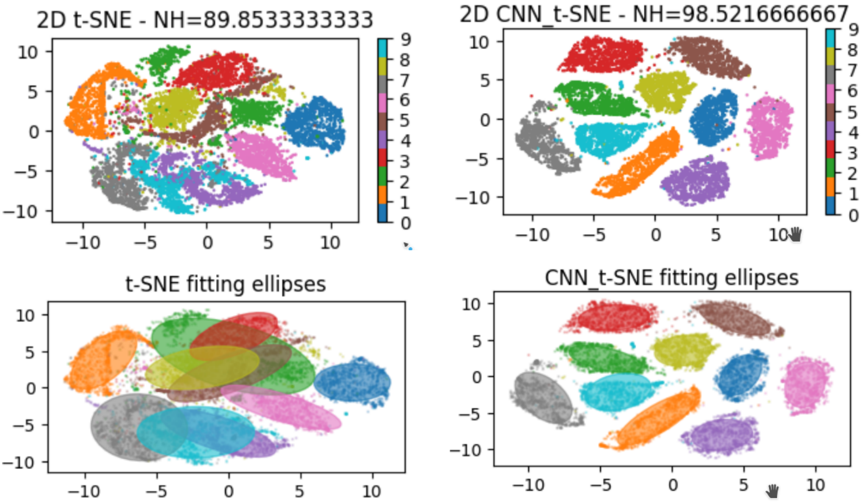




Credit: DiCarlo

ConvNets and Manifold Untangling

Convnet: 2 conv and 1 FC layer: latent space *vs* input space visu



Input space

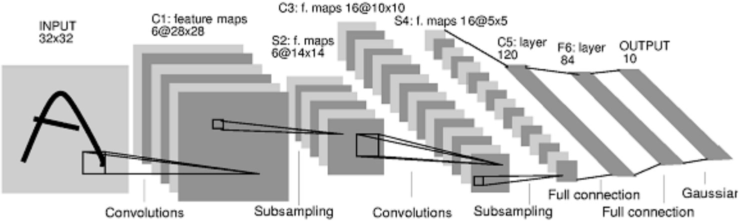
Latent space

Outline

Deep Learning: Trends and methods in the last four decades

80's: 1^{st} Convolutionnal Neural Networks

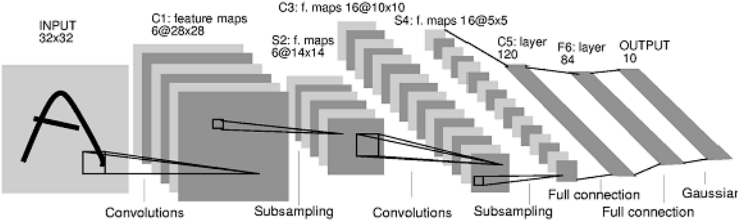
- LeNet 5 Model [LeCun et al., 1989], trained using back-prop



- Input: 32x32 pixel image. Largest character is 20x20
- 2 successive blocks [Convolution + Sigmoid + Pooling (+sigmoid)]
  Cx: Convolutional layer, Sx: Subsampling layer
- C5: convolution layer ~ fully connected
- 2 Fully connected layers Fx

80's: LeNet 5 Model

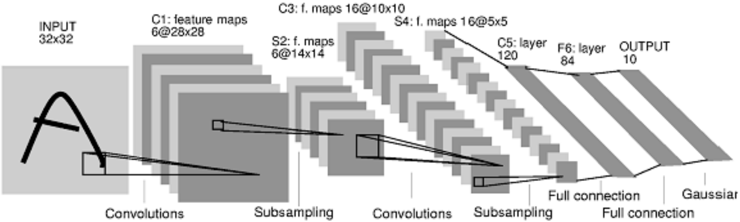
C1 Layer



- Convolutional layer with 6 5x5 filters $\Rightarrow$ 6 feature maps of size 28x28 (no padding).
- # Parameters: 5^2 per filer + bias $\Rightarrow (5 * 5 + 1) * 6 = 156$
  - If it was fully connected: (32*32+1)*(28*28)*6 parameters $5 \sim 10^6$!

80's: LeNet 5 Model

S2 Layer

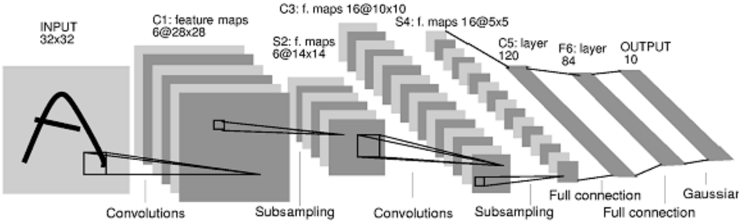


- Subsampling layer = pooling layer
- Pooling area : 2x2 in C1
- Pooling stride: 2 ⇒ 6 features maps of size 14x14
- Pooling type : sum, multiplied by a trainable param + bias
  ⇒ 2 parameters per channel
- Total # Parameters: $2 * 6 = 12$

80's: LeNet 5 Model

C3 Layer: Convolutional

- C3: 16 filters ⇒ 16 feature maps of size 10x10 (no padding)



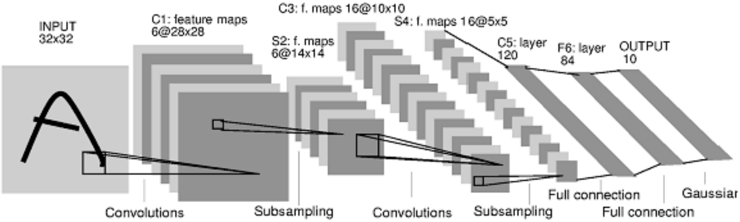
- 5x5 filters connected to a subset of S2 maps
  ⇒ 0-5 connected to 3, 6-14 to 4, 15 connected to 6

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	X				X	X	X			X	X	X	X		X	X
1	X	X				X	X	X			X	X	X	X		X
2	X	X	X				X	X	X			X		X	X	X
3		X	X	X			X	X	X	X			X		X	X
4			X	X	X			X	X	X	X		X	X		X
5				X	X	X			X	X	X	X		X	X	X

- # Parameters: 1516
  $(5 * 5 * 3 + 1) * 6 + (5 * 5 * 4 + 1) * 9 + (5 * 5 * 6 + 1) = 456 + 909 + 151$

80's: LeNet 5 Model

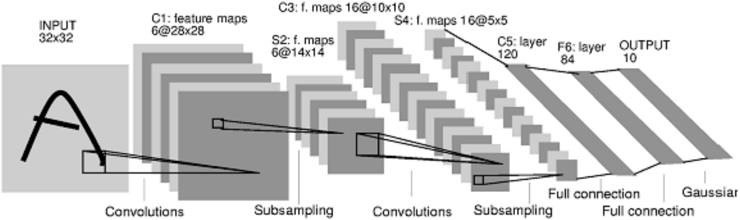
S4 Layer



- Subsampling layer = pooling layer
- Pooling area : 2x2 in C3
- Pooling stride: 2 $\Rightarrow$ 16 features maps of size 5x5
- Pooling type : sum, multiplied by a trainable param + bias
  $\Rightarrow$ 2 parameters per channel
- Total # Parameters: $2 * 6 = 12$

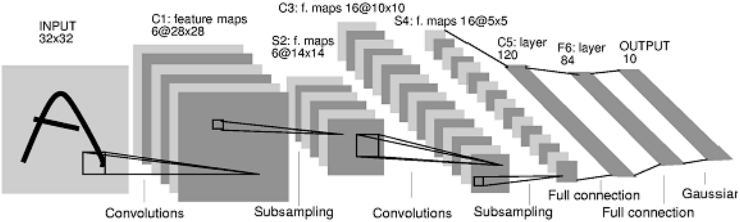
80's: LeNet 5 Model

C5 Layer: Convolutionnal layer



- 120 5x5x16 filters ⇒ whole depth of $S4$ ($\neq C3$)
- Each maps in $S4$ is 5x5 ⇒ single value for each $C5$ maps
- $C5$ 120 features map of size 1x1 (vector of size 120)
  ⇒ spatial information lost, $\sim$ to a fully connected layer
- Total # Parameters: $(5 * 5 * 16 + 1) * 120 = 48210$

80's: LeNet 5 Model



F6 Layer: Fully Connected layer

- 84 fully connected units.
- # Parameters: 84*(120+1)=10164

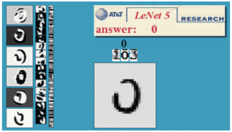
F7 Layer (output): Fully Connected layer

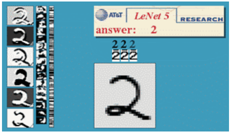
- 10 (# classes) fully connected units.
- # Parameters: 10*(84+1)=850

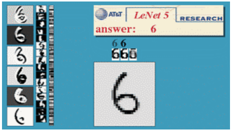
80's: LeNet 5 Model

- Evaluation on MNIST
- Total # parameters ~ 60000
  - 60,000 original datasets: test error: 0.95%
  - 540,000 artificial distortions + 60,000 original: Test error: 0.8%

- **Successful deployment for postal code reading in the US**







References I

Goodfellow, I., Bengio, Y., and Courville, A. (2016).
Deep Learning.
MIT Press.
http://www.deeplearningbook.org.

LeCun, Y., Boser, B., Denker, J. S., Henderson, D., Howard, R. E., Hubbard, W., and Jackel, L. D. (1989).
Backpropagation applied to handwritten zip code recognition.
Neural computation, 1(4):541–551.