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MARIUSZ MYCEK
Institute of Telecommunications, Warsaw University of Technology, Poland
MICHAŁ PIÓRO
Institute of Telecommunications, Warsaw University of Technology, Poland
Department of Electrical and Information Systems, Lund University, Sweden
STEFANO SECCI
Institut Telecom, Telecom ParisTech, LTCI CNRS, France; Politecnico di Milano, Italy
JEAN-LOUIS ROUGIER
Institut Telecom, Telecom ParisTech, LTCI CNRS, France
ACHILLE PATTAVINA
Politecnico di Milano, Italy

A SHAPLEY VALUE-BASED INCENTIVE SCHEME FOR COOPERATIVE MULTI-PROVIDER TRAFFIC MANAGEMENT

Abstract

This paper refines a distributed optimization mechanism for reliable cooperative optimization of flow reservation levels in multi-domain networks (introduced in [1]), by considering the fundamental issue of fair income distribution. The proposed idea of income distribution mechanism has been adopted from the theory of cooperative games.[1]

Key words

inter-domain routing, multi-carrier, Shapley value

1. Introduction

The goal of this paper is to improve the distributed scheme for cooperative optimization of inter-domain traffic flow presented in [1] and [3]. These papers define an iterative distributed optimization process, run either in the control plane or in the management plane of the network, in which domains, cooperating in a coalition, calculate the optimal pattern of inter-domain traffic flow. The referred papers are mostly devoted to the description of an optimization process whose objective is to maximize the sum of incomes of individual domains under the assumption that the income of a domain depends linearly on the amount of inter-domain traffic the domain injects into the network. As a globally optimal solution might prefer that a domain should rather transit than inject traffic, such an implicit distribution rule could lead to unfair distribution of the total income. In fact, as a domain has no guarantees to gain any additional profit (in reality, it may even loose) there is no incentive to enter the coalition. This paper aims in closing that gap – it extends the model of distributed cooperative optimization with a mechanism of provably fair distribution of the coalition's income adopted from the theory of cooperative games.

The paper is organized as follows. Section 2. summarizes the distributed optimization model presented in [3] and [1] and introduces a necessary notation. Sections 3. and 4. present a model for application income distribution mechanisms adopted from the theory of cooperative games, namely the notion of the Shapley value. Section 5. presents a method and the algorithm for computation of the Shapley values from the results of the distributed optimization process. Section 6. presents numerical results illustrating the effect of the proposed distribution mechanism on the incomes of particular domains. Eventually, Section 7. gives concluding remarks together with a sketch of the plan for further investigations.

2. Distributed routing optimization framework

In [1] a generic multi-domain routing problem (consisting in optimization of bandwidth reservation levels on inter-domain links for traffic flows identified by traffic classes and traffic destinations) is formulated, and its possible decompositions are discussed. In [3] it is shown how to decompose the problem with respect to individual domains using sub-gradient optimization based on Lagrangean relaxation and it is demonstrated how to resolve an inter-domain routing optimization problem using a distributed process based on sub-gradient optimization combined with recovering of near-optimal bandwidth reservation levels. The original approach was further refined by [4] (where it is demonstrated how to take advantage of different aggregation models of intra-domain topology in order to reduce the size and increase the computational efficiency of the proposed method) and, at last, in [5] where an effective architecture and reasonable stopping criteria for the distributed optimization process have been introduced. Hereafter there are reminded only the necessary elements of the original formal notation – for further details please refer the original papers [1], [3], [4] and [5].

The considered model of the network consists of a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with the set of nodes $\mathcal{V}$ and the set of directed links $\mathcal{E}$ ($\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$). $\mathcal{M}$ is the set of network domains. Each node $v \in \mathcal{V}$ belongs to exactly one domain denoted by $\mathcal{A}(v)$. Set $\mathcal{D}$ represents traffic demands between pairs of nodes. The originating and terminating node of $d \in \mathcal{D}$ is denoted by $s(d)$ and $t(d)$, respectively, and h_d is the traffic volume of d, expressed in the same units of bandwidth as capacity of links. $\mathcal{D}(s,t) = \{d \in \mathcal{D} : s(d) = s \wedge t(d) = t\}$ denotes the set of all demands from node $s \in \mathcal{V}$ to node $t \in \mathcal{V}$.

[1]*This work has been carried out within the EU IST Euro-NF INCAS (INter Carrier Alliance Strategies) S.JRA.1.7 activity, and within the scope of a COST Action 293, GRAAL (Graphs and algorithms in communication networks).*

In the sequel, z_d will denote the variable specifying the percentage of volume h_d actually handled in the network, i.e., $z_d h_d$ is the carried traffic of demand d.

Let x_{et} denote a variable specifying the amount of aggregated bandwidth (called *flow* in the sequel) reserved on intra-domain e link for the traffic destined for (a remote) node $t \in \mathcal{V}$. Then, for each inter-domain link e we introduce two flow variables: x^+_{et} and x^-_{et}. Variable x^+_{et} (respectively, x^-_{et}) denotes the amount of bandwidth reserved for traffic carried on e and destined for t that is reserved by domain $\mathcal{A}(a(e))$ (respectively, $\mathcal{A}(b(e))$) at which link e originates (respectively, terminates).

3. Rationales

According to a review of Internet agreements that can be found in [7], the framework proposed in [1] can be considered as a sort of "extended peering agreement" from which providers obtain mutual benefit without side payments. However, we claim that for such frameworks the agreement shall rely on side payments since the multi-provider optimization can arise disparities. In order to reserve bandwidth for external connections for which no direct earning is obtained, a provider may need a form of economical incentive. It is indeed possible that, by reserving bandwidth for external connections, a provider grants earnings to its "peers" bigger than the earnings related to its own services. Instead of "extended peering agreement" it seems more appropriate to refer to an economical "alliance of providers" that wish to cooperate for multi-provider connection-oriented (ASON/GMPLS) services, sharing the related incomes (besides the costs of an integrated technical architecture such as proposed in [2]). It is thus needed to define a fair scheme for multi-provider income distribution that rewards a provider in a way that is not solely based on the generated traffic (Content provider behavior, see [10]) or absorbed traffic (Eyeball provider behavior, see [10]) but also accordingly to its *alliance transit contribution*, i.e., that takes into account how much a provider supports the services of the other providers allocating its network's resources. With a far-sighted standpoint, in [9] it is proven that, if part of the profits due to inter-provider services were shared, the Internet providers would behave less selfishly, yielding better global routing with lower routing cost than under the current practice. Using the Shapley value concept from *cooperative games*, in [9] it is argued that profits and costs may be fairly imputed considering the importance of each AS in the interconnected "coalition" composed of ASs routing "common" inter-AS flows [10]. In this way, it is proven that ASs have incentive to better route yielding to a common inter-domain routing cost lower than with the current practice, besides than interconnection cost savings.

4. A Shapley Value Perspective for Income Distribution

The Shapley Value concept is a game theoretic solution for value imputation problems that offers interesting properties recalled below [6]. For this reason, it has been applied to very diverse fields [11]. In game theory, interacting agents are modeled as players that take decisions rationally following the utility functions of all the players. In cooperative games, since some players may contribute more than others for the collaboration, the value imputation problem consists in how to distribute a global value (or revenue) among the players. How important is each player to the coalition, and what payoff can be reasonably expected, are questions to which cooperative coalitional game theory answers with many theoretical concepts - not worth being all reviewed here (for a review consider [8]). Among these concepts, the Shapley value considers the strategic weight (importance) of each player in the alliance to share the alliance value.

The Shapley value is thus equal to zero for null players, which do not offer any marginal contribution to a coalition in any case, and equal to the single-player payoff for dummy players, which are indifferent in staying in the coalition or not. In our multi-provider framework, dummy players are those that reserve resources for external inter-provider connections but do not obtain the same from the other providers, while null player are those that not even reserve resources.

The Shapley value can be used to assign the payoff (income) of a player as function of his marginal contribution to the coalition. Given that the marginal contribution that a player brings to a coalition (i.e. the alliance income related to its connection services), varies as function of the players that already form the coalition, it is essential considering the order in which the player enters the coalition (or would enter if a coalition evaluates the opportunity of joining the new player).

Mathematically, we use the formulation of a coalitional game. We start with a function $\mu : \mathcal{P}(\mathcal{M}) \to \mathbb{R}$, that goes from subsets of players (partition set of $\mathcal{M}$) to reals, called the "worth function", with the properties:
i) $\mu(\emptyset) = 0$;
ii) $\mu(S \cup T) \geq \mu(S) + \mu(T)$, $\forall S, T \subseteq \mathcal{M} \mid S \cap T = \emptyset$.
The computation of μ will be explicited in the next section. The interpretation of the function μ is as follows: if S is a coalition of players which agree to cooperate, then $\mu(S)$ describes the total expected gain from this cooperation, independent of what the actors outside of S do. ii) is the "super-additivity" condition, hypothesis of classical cooperative game theory, which expresses the fact that collaboration can only help, and never hurts. A shapley value imputation ω_i can thus be calculated for each player $i \in \mathcal{M}$ as function of μ:

$$\omega_i(\mu) = \sum_{S \subseteq \mathcal{M} \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} (\mu(S \cup \{i\}) - \mu(S)) \quad (1)$$

where the sum extends over all subsets S of $\mathcal{M}$ not containing player i. The formula can be justified if one imagines the coalition being formed one player at a time, with each player demanding its contribution $\mu(S \setminus \{i\}) - \mu(S)$ as a fair compensation, and then averaging over the possible different permutations in which the coalition can be formed.

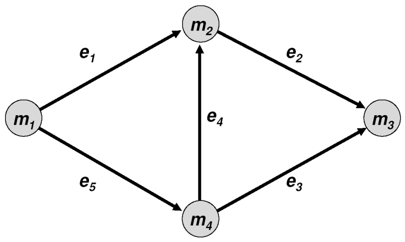

Fig 1. *Connectivity graph of an exemplary multi-domain network*

The Shapley Value satisfies desirable properties of individual fairness, efficiency, symmetry, additivity and null player modeling (for a detailed characterization see [11]). In fact, the vector of Shapley values is the only payoff vector - defined on the class of all superadditive games - that satisfies these five properties. Namely, under a Shapley value distribution, in our framework every provider gets at least as much as it would have got without any collaboration, and two strategically equivalent providers obtain the same value. Moreover, the Shapley value distribution supports anonymity. That is, the labeling of the players doesn't play a role in the assignment of their payoffs, i.e., if i and j are two players, and μ^1 is the worth function that acts just like μ^2 except that the roles of i and j have been exchanged, then $\omega_i(\mu^1) = \omega_j(\mu^2)$. Finally, the Shapley value is the single imputation rule that supports marginality, i.e., which uses only the marginal contributions of a player as argument [11].

5. Coalitional game characterization

The computation of Shapley values for every domain $m \in \mathcal{M}$ of the coalition requires a procedure for evaluation of worth function $\mu(\mathcal{S})$ of arbitrary sub-coalition $\mathcal{S} \subseteq \mathcal{M}$. Lets consider a simple example of a multidomain network (connectivity graph presented in Figure 1) where node $m \in \mathcal{M} = \{m_1, m_2, m_3, m_4\}$ represents a single domain of the original network and, edge $e \in \mathcal{E} = \{e_1, e_2, e_3, e_4, e_5\}$ represents an aggregate of all the directed links between the pair of domains. Assume that there is a single demand d such that $s(d) = m_1$ and $t(d) = m_3$ of nominal volume $h_d = 1$. Let the optimal routing solution (of Problem (2) in [1]) define that $z_d = 1$ and particular reservation levels are: $x_{e_1 m_3} = 0.5, x_{e_2 m_3} = 0.75, x_{e_3 m_3} = 0.25, x_{e_4 m_3} = 0.25, x_{e_5 m_3} = 0.5$. A worth function $\mu(\mathcal{S})$ for the sub-coalition $\mathcal{S} \subseteq \mathcal{M}$ is defined as the value of the objective function (cf. (2a) in [1]) that could be achieved when only nodes $m \in \mathcal{S}$ and links $e \in \mathcal{E}_\mathcal{S}$ ($\mathcal{E}_\mathcal{S} = \{e \in \mathcal{E} : a(e) \in \mathcal{S}, b(e) \in \mathcal{S}\}$) would be active.

Therefore, $\mu(\mathcal{S})$ is computed upon the optimal routing solution for the grand coalition, so independently from which sub-coalition is active, the corresponding reservation levels are fixed.

One can observe that the value of the worth function $\mu(\mathcal{S})$ is equal to zero for every sub-coalition $\mathcal{S} \subseteq \mathcal{M}$ such that does not contain both the source and the destination domain of demand d (i.e., domains m_1 and m_3) together with at least one from two transit domains (either m_2 or m_4). Hence the only profitable sub-coalitions are $\mathcal{S}_1 = \{m_1, m_2, m_3, m_4\}, \mathcal{S}_2 = \{m_1, m_2, m_3\}$ and $\mathcal{S}_3 = \{m_1, m_4, m_3\}$ with respective worth functions $\mu(\mathcal{S}_1) = 1$, $\mu(\mathcal{S}_2) = .5$ and $\mu(\mathcal{S}_3) = .25$.

permutations	m_1	m_2	m_3	m_4
$m_1m_2m_3m_4$	0	0	.5	.5
$m_1m_2m_4m_3$	0	0	1	0
$m_1m_4m_2m_3$	0	0	1	0
$m_1m_4m_3m_2$	0	.75	.25	0
$m_1m_3m_4m_2$	0	.75	0	.25
$m_1m_3m_2m_4$	0	.5	0	.5
$m_2m_3m_4m_1$	1	0	0	0
$m_2m_3m_1m_4$	.5	0	0	.5
$m_2m_1m_3m_4$	0	0	.5	.5
$m_2m_1m_4m_3$	0	0	1	0
$m_2m_4m_1m_3$	0	0	1	0
$m_2m_4m_3m_1$	1	0	0	0
$m_3m_4m_1m_2$	.25	.75	0	0
$m_3m_4m_2m_1$	1	0	0	0
$m_3m_2m_4m_1$	1	0	0	0
$m_3m_2m_1m_4$	.5	0	0	.5
$m_3m_1m_2m_4$	0	.5	0	.5
$m_3m_1m_4m_2$	0	.75	0	.25
$m_4m_1m_2m_3$	0	0	1	0
$m_4m_1m_3m_2$	0	.75	.25	0
$m_4m_3m_1m_2$	.25	.75	0	0
$m_4m_3m_2m_1$	1	0	0	0
$m_4m_2m_3m_1$	1	0	0	0
$m_4m_2m_1m_3$	0	0	1	0
Shapley values	.3125	.229167	.3125	.14833

T a b l e 1. Intermediate and the final Shapley values

The Shapley values are then computed using (1). The intermediate and the final results of this process are presented in Table 1. The first column of the table contains all the possible permutations of domains of the coalition $\mathcal{M}$, the next four columns contain marginal contributions of domains m_1, m_2, m_3 and m_4 respectively. The last row of the table contains the final Shapley values for every domain $m \in \mathcal{M}$ of the coalition.

Worth function for the optimal routing solution

The Shapley value computation is complex. This is due to additional intrinsic complexity related to the structure of the optimal routing solution: a flow directed to a particular destination domain t is usually aggregated and has many source domains s, and its sub-flow paths are a-priori unknown.

Let $d_t(v), v \in \mathcal{M}, t \in \mathcal{M}$ denote the total traffic volume generated within v and directed to t. Let $\phi_t(\mathcal{S}, v), \mathcal{S} \subseteq \mathcal{M}, v \in \mathcal{S}, t \in \mathcal{S}$ denote the traffic volume that domain v has to direct to domain t when sub-coalition $\mathcal{S}$ is active. Let $\mathcal{E}_\mathcal{S}$ denote set of links active for sub-coalition $\mathcal{S}$ – i.e., $e \in \mathcal{E}$ such that $a(e) \in \mathcal{S}$ and $b(e) \in \mathcal{S}$. At last, let $\varphi_t(\mathcal{S}, e), \mathcal{S} \subseteq \mathcal{M}, e \in \mathcal{E}_\mathcal{S}, t \in \mathcal{S}$ (we refer to it as volume of link e) denote the volume of traffic to domain t carried on link e when sub-coalition

$\mathcal{S}$ is active. Distribution of volume of domain v to its outgoing links is trivial if this domain volume exceeds the sum of reservation levels over active outgoing links – i.e., $\phi_t(\mathcal{S}, v) \geq \sum_{e \in \delta^+(v) \cap \mathcal{E}_\mathcal{S}} x_{et}, \mathcal{S} \subseteq \mathcal{M}, v \in \mathcal{S}, t \in \mathcal{S}$ – as in such a case, every active outgoing links gets volume equal to its reservation ($\varphi_t(\mathcal{S}, e) = x_{et}$). In the opposite case, where there is a surplus of reservation to use and due to the optimal routing solution got from the distributed optimization (cf. Section 2.) does not specify paths for particular demands (or sub-flows), it seems reasonable and simple to assume a fair weighted distribution of volume of domain v to its outgoing links – i.e., $\varphi_t(\mathcal{S}, e) = x_{et}/(\sum_{f \in \delta^+(v) \cap \mathcal{E}_\mathcal{S}} x_{ft})\phi_t(\mathcal{S}, v) \quad \mathcal{S} \subseteq \mathcal{M}, t \in \mathcal{S}, e \in \mathcal{E}_\mathcal{S}, v \in \mathcal{S}$.

Algorithm 5..1: worthFunctionComponent($\mathcal{S}, t$)

procedure domainvolume($\mathcal{S}, m$)
  if **not** $domainReady(m)$
    then $\begin{cases} \phi_t(\mathcal{S}, m) \leftarrow 0 \\ \textbf{for each } e \in \delta^-(m) \cap \mathcal{E}_\mathcal{S} \\ \quad \textbf{do } \phi_t(\mathcal{S}, m) \leftarrow \phi_t(\mathcal{S}, m) + \text{linkvolume}(\mathcal{S}, e, t) \\ domainReady(m) \leftarrow true \end{cases}$
  return $(\phi_t(\mathcal{S}, m))$

procedure linkvolume($\mathcal{S}, e, t$)
  if $a(e) \in \mathcal{S}$
    then return $(\phi_t(\mathcal{S}, v)x_{et}/(\sum_{f \in \delta^+(a(e)) \cap \mathcal{E}_\mathcal{S}} x_{ft}))$
    else return (0)

main
  for each $m \in \mathcal{M}$
    do $domainReady(m) \leftarrow$ **false**
  return (domainvolume($\mathcal{S}, t$))

Let $\mu(S, t)$ denote a component of worth function of sub-coalition $\mathcal{S} \subseteq \mathcal{M}$ for traffic to destination domain t. To compute value of that component one may use Algorithm 5..1. The algorithm assumes that a flow of traffic to domain t, which is induced by values of reservations taken form the optimal routing solution, forms an acyclic graph (in fact, the optimal routing solution does not always induce acyclic flows still, they could be easily made acyclic by simple preprocessing). Algorithm 5..1 takes advantage of the observation that $\mu(S, t) = \phi_t(\mathcal{S}, t)$ – i.e., that value of worth function component, for particular sub-coalition $\mathcal{S}$ and destination domain t, is equal to the volume of domain t.

Finally, the worth function of the sub-coalition $\mathcal{S} \subseteq \mathcal{M}$ can be computed as

$$\mu(S) = \sum_{t \in S} \mu(S, t) \qquad (2)$$

Then, (1) computes the Shapley value imputation for all domains of the coalition.

6. Numerical experiments

In our experiments we tested the Shapley value based distribution algorithm for a single multi-domain network consisting of seven domains (the domain connectivity graph of that network is presented in Figure 2, where a single line represents a pair of oppositely directed unidirectional links of equal capacity). The considered traffic matrix is random.

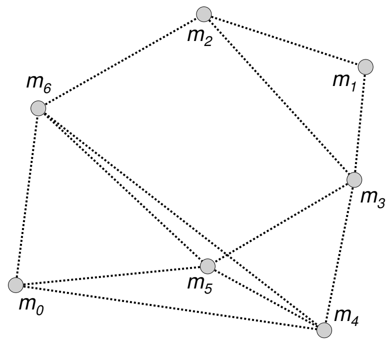

Fig 2. The domain connectivity graph

Lets consider flow to the arbitrarily chosen destination domain – e.g., flow to domain m_3. That flow is depicted in Figure 3, where number beside a link denote the amount of bandwidth reserved on that link for traffic to domain m_3. Considering these reservation levels, one can easily compute the amount of traffic to domain m_3 that each domain injects into the network, terminates or transits (results for such computations are presented in Table 2). Table 3 shows how the two considered distributions divide the income related to flow to domain m_3 between particular domains (the original distribution refers to the implicit distribution rule assumed in [1] and [3], where a domain was awarded only for traffic that it injects into the network – there were no income components related to transiting nor terminating traffic). There are four columns for each distribution – i denotes income component related to traffic a domain injects into the network, t income component related to traffic terminated within a domain and tr income component related to traffic transited by a domain. Finally, column $\sum$ denotes the total income that is attributed to a domain by particular distribution.

Observing Tables 2 and 3 one can easily conclude that the original distribution is unfair – as there are significant unpaid volumes of traffic terminated by domain m_3 and transited through domains m_2, m_4 and m_5. The second part of Table 3 shows that the proposed Shapley value distribution schemes offers significantly fairer results, as domains are awarded for every type of their contribution in the total income (the 'x's mean that the Shapley value attributed to transit domain cannot be easily divided into components related to injecting and transiting of traffic). Namely, the Shapley scheme assigns to m_3 the biggest share while the original scheme would assign a null income: without m_3 12833 units of traffic (c.f. the total ingress traffic at m_3 in Figure 3) could not be provided, so the corresponding revenue is distributed fairly also to m_3 recognizing to it an income share of 6010. Or, m_0 and m_6 not reserving bandwidth for any external connection, receive roughly one third of the original share.

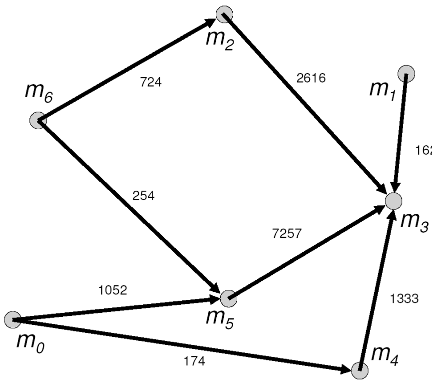

Fig 3. Flow to domain m_3

domain	injects	terminates	transits
m_0	1226	0	0
m_1	1627	0	0
m_2	1892	0	724
m_3	0	12833	0
m_4	1159	0	174
m_5	5951	0	1306
m_6	978	0	0

T a b l e 2. Flow m_3 components

	Original distribution				Shapley distribution			
	i	t	tr	$\sum$	i	t	tr	$\sum$
m_0	1225	0	0	1225	408	0	0	408
m_1	1619	0	0	1619	809	0	0	809
m_2	1902	0	0	1902	x	0	x	1188
m_3	0	0	0	0	0	6010	0	6010
m_4	1091	0	0	1091	x	0	x	602
m_5	5948	0	0	5948	x	0	x	3408
m_6	1045	0	0	1045	323	0	0	323

T a b l e 3. Flow m_3 related income distribution

7. Summary

In this paper we have presented a cooperative multi-provider routing optimization framework in which providers cooperate to the resource reservation for inter-provider connections. We discussed under which circumstances this might result economically feasible. In order to support the adoption of such a multi-provider routing optimization framework, we proposed a fair income distribution scheme relying on the Shapley Value concept from cooperative game theory, showing how the complex issue of computing the Shapley values using decomposition result parameters can be solved heuristically.

By comparison with the original implicit income distribution policy, we show the benefits of the adoption of the Shapley value distribution scheme. Those domains that attract large volumes of traffic can receive an income for such a contribution. Those providers that do not balance their injected traffic volume with bandwidth reserved for external connection transit, see their income share decreased. Those domains that do not offer transit at all are fairly penalized. Our approach is a further step (after a few others such as [10]) toward the definition of feasible cooperative routing frameworks and acceptable business models for the future Internet.

As a further work we aim to refine the optimization decomposition method so as to allow a pro-active integration of the Shapley values. The idea is to control the amount of traffic volume a provider is allowed to inject within the alliance. It might be desirable to allow rewarding a provider's transit contribution directly with intra-alliance traffic injection ability by bounding the inter-provider throughput.

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MARIUSZ MYCEK
mariusz@tele.pw.edu.pl
ul. Nowowiejska 15/19, 00-665 Warszawa

STEFANO SECCI
secci@telecom-paristech.fr
DA 512, 37/39, rue Dareau, 75014 Paris, France