

# Strategic Subchannel Resource Allocation for Cooperative OFDMA Wireless Mesh Networks

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**Abstract**—Wireless Mesh Networks (WMNs) are emerging as a key solution to provide broadband and mobile wireless connectivity in a flexible and cost effective way. In suburban areas, a common deployment model relies on OFDMA communications between mesh routers (MRs), with one MR installed at each user premises. In this paper, we investigate a possible user cooperation path to implement strategic resource allocation in OFDMA WMNs, under the assumption that users want to control their interconnection. In this case, a novel strategic situation appears: how much a MR can demand, how much it can obtain and how this shall depend on the interference with its neighbors. Strategic interference management and resource allocation mechanisms are needed to avoid performance degradation during congestion cases between MRs. In this paper, we model the problem as a bankruptcy game taking into account the interference between MRs. We identify possible solutions from cooperative game theory, namely the Shapley value and the Nucleolus, and show that they outperform two state-of-the-art schemes, namely Centralized-Dynamic Frequency Planning, C-DFP, and Frequency-ALOHA, F-ALOHA. In particular, the Nucleolus solution offers best performance overall in terms of throughput and fairness.

## I. INTRODUCTION

Wireless Mesh Networks (WMNs) are emerging as a key solution to provide broadband and mobile wireless connectivity in a flexible and effective way. A common deployment model is based on OFDMA communications between mesh routers (MRs), with a user subscription for the installation of one MR at user premises; the local access can then be guaranteed using classical WiFi and Ethernet networks. In this paper, we investigate a user cooperation path for strategic resource allocation in OFDMA WMNs, under the assumption that users want to control their interconnection. In this case, a novel strategic situation appears: how much a MR can demand, how much it can obtain and how this shall depend on the interference with its neighbors? These questions pose an interesting research challenge.

Interference can occur among neighboring MRs, especially in those suburban or emergency environments with a dense deployment of WMN equipment, when the coverage areas of MRs overlap. In such situations, it is likely that the shared spectrum is not enough to meet all demands, so that demand congestion can persistently occur; hence coordination or cooperation mechanisms are needed between independent users' routers to manage reciprocal interferences and resource allocation and avoid performance degradation during congestion cases. We can refer to such networking cases as collaborative wireless mesh networks. In collaborative WMNs, nodes' interference levels and demands should be taken into account when allocating resources to them. We propose to model these situations using cooperative game theory, so that resource allocation solutions are strategically justified.

Under the rationality hypothesis, users are willing to agree in a binding agreement fixing the game-theoretic resource allocation rule, motivated by the achievable gain in throughput and resiliency; indeed, our results show that such approaches can grant some improvements in throughput and fairness. More precisely, we model the resource allocation problem as a bankruptcy game taking into account the interference between MRs. We identify possible solutions from cooperative game theory, namely the Shapley value and the Nucleolus, and show through extensive simulations of realistic scenarios that they outperform two state-of-the-art OFDMA allocation schemes, namely Centralized-Dynamic Frequency Planning, C-DFP, and Frequency-ALOHA, F-ALOHA. In particular, the Nucleolus solution offers best performance overall in terms of throughput and fairness. The paper is organized as follows. Section II presents an overview of related works. In Section III, we analytically introduce the context of our work and formulate the problem as a bankruptcy game. Section IV describes our approach, followed by a presentation of simulation results in Section V. Finally, Section VI concludes the paper.

## II. RELATED WORK

Cooperative resource allocation in wireless networks has been considered in recent research works. The general objective is the computation of efficient allocations, while accounting for wireless node interference. In the following, we discuss a selection of relevant approaches. A simple solution to OFDMA resource allocation consists in allowing random access to the spectrum in a first-in-first-served fashion, as proposed in [1], where a variation of ALOHA for the time-frequency domain is presented. On the other hand, authors in [2] propose Centralized - Dynamic Frequency Planning (C-DFP) mechanism, implementable when the operator has full control of the WMN equipment. They present a suboptimal fair resource allocation scheme in WMNs that maximizes the throughput and guarantees a Quality of Service (QoS) level. In [3], authors stress the potential of effective interference detection for channel assignment, in virtual cut-through switching-based networks. Using information on link and possible interference, they solve the problem as an edge-coloring problem, where only chosen routes are considered for channel assignment. As decomposition of a master problem, in [4] the authors propose a distributed subcarrier allocation scheme based on the Lagrange dual approach and the Lambert-W function, consisting of maximizing the sum rate while satisfying minimum rate demand. Generally, centralized approaches do not take into account independency requirements for network nodes, which may appear as counter-productive in the situation considered in our work. In [5] authors show how node cooperation can improve system performance and user satisfaction in WMNs; they propose two cooperative

resource allocation approaches: one based on a centralized approach and the other based on distributed control, while taking into account subcarrier allocation, power allocation, partner selection, service differentiation, and packet scheduling. Similarly, authors in [6] propose a fair subcarrier and power allocation scheme to maximize the Nash bargaining fairness: WMN nodes hierarchically allocate groups of subcarriers to the clients, so that each mesh client allocates transmit power among its subcarriers to its outgoing links. Adopting user cooperation assumptions and requirements close to [5] and [6], in this paper, we model the OFDMA allocation problem in WMNs as a cooperative game. We allow MRs to negotiate resources in multiple MR groups, where groups are locally detected as a function of interferer MR neighbors. Hence we target a solution in which the resource allocation is periodically pre-computed based on changing demands and interference maps. In particular, we consider dense environment situations in which the overall demand is quite often higher than the available bandwidth on the shared media, which mathematically corresponds to a bankruptcy game situation [7]. As detailed in the following, we investigate two solution concepts: the well-known Shapley value [8] (already adopted in a variety of situations in networking such as inter-domain routing [9] and network security [11]); and the less-known Nucleolus [10] (used in strategic transmission computation [12]), which shows additional interesting properties for bankruptcy situations.

### III. CONTEXT AND PROBLEM FORMULATION

We consider a WMN network meshed using OFDMA WiMAX. Resources are expressed in the time-frequency domain, and are organized in subchannels. More precisely, we consider a total of 60 subchannels, corresponding to WiMAX standard operating with OFDMA in the PUSC (Partial Usage of Sub-Channels) mode for a system bandwidth of 20 MHz. A certain number of clients is attached to each MR; client demands represent the required bandwidth, then translated in a number of required subchannels per MR. As already mentioned, for dense environments, we expect that the overall demand often exceeds the available resources. Therefore, our objective is to find, for such congestion situations, a strategic resource allocation that satisfies throughput expectations while controlling the inter-node interference. In the following, we present the corresponding optimization problem, then we highlight possible alternative solutions, and finally describe the properties of bankruptcy games along with possible solutions.

#### A. Notations

Let  $\mathcal{R}$  be the set of MRs,  $d_i$  the demand of  $R_i \in \mathcal{R}$ , and  $x_i$  the number of allocated resources to  $R_i$ . Also, let  $\mathcal{I}_i$  be the interference set of  $R_i$ , which corresponds to the set of nodes composed of  $R_i$  and the nodes causing interference to  $R_i$ .

#### B. Related centralized optimization problem

For the sake of clarity, we model here the resource allocation problem as a centralized mono decision-maker optimization problem, i.e., as the C-DFP approaches mentioned in Sec-

tion II. The problem can be formulated as:

$$\begin{aligned} &\text{objective} && f(d_i, x_i) \\ &\text{subject to} && 0 \leq x_i \leq d_i, \forall R_i \in \mathcal{R} \\ &&& \sum_{j|R_j \in \mathcal{I}_i} x_j \leq E, \forall \mathcal{I}_i \\ &&& x_i \in \mathcal{Z}^+, \forall R_i \in \mathcal{R} \end{aligned}$$

where  $E$  is the number of subchannels in an OFDMA frame. The objective typically depends on the demand and the allocated resources; in our case it is the minimization of the maximum gap between demand and allocation,  $\min \max_i \left( \frac{d_i - x_i}{d_i} \right)$ . The constraints are integrity constraints, on the allocated tiles to individual nodes and to nodes belonging to same interference sets. Later, we compare our approaches to this C-DFP solution highlighting the interest in strategic approaches and stressing the tradeoffs between them.

#### C. Possible distributed approaches

For each interference set, we have therefore a situation in which a group of WMN nodes can: (i) randomly access the spectrum hoping that collision will not occur (e.g., as in F-ALOHA [1]); (ii) self-organize to define an online joint scheduling; (iii) divide the available spectrum proportionally.

Clearly, (i) excludes any form of coordination and would favor opportunistic wealth-averse behaviors (e.g., setting a minimum waiting time upon collision in F-ALOHA) that other nodes can not control. Approaches like (ii) risk to generate enormous signaling for large interference sets (likely in dense environments). Under (iii), inefficiency can arise whether many demands are less than the proportional share, and a weighted proportional share would favor cheating demands (higher claims than what is really needed). The path forward is therefore towards cooperative approaches that dissuade malicious behaviors in setting demands, under an adequate binding agreement fixing common rules on shared information and allocation scheme. Before detailing our algorithmic approach, let us introduce the bankruptcy game that can model interactions among WMN nodes belonging to the same interference set.

#### D. Bankruptcy game modeling

With a dense deployment of WMN nodes, one should expect situations in which the overall resource claim (i.e., sum of the demands) surpasses the number of available subchannels in the shared spectrum. Assuming that WMN nodes, belonging to the same interference set, share information about respective demands, the interaction can be modeled as a cooperative coalitional game. The choice of the game characteristic function, representing the profit attributed to each coalition of players in a canonical coalitional game, is an important tie-break. We stay under the assumption that a coalition  $S$  of nodes, within the same given interference set  $\mathcal{I}_i$ , group apart so as to decide among them how to share the spectrum. In the most pragmatic case, they will be able to share what the other nodes have left after getting what they claimed. In order to avoid secessions, the utility function of the game should be superadditive, that is, the best coalition should be the grand coalition grouping all nodes in the same interference set. Such a utility or characteristic function corresponds, in fact, to what is known as ‘bankruptcy game’ precisely defined hereafter.

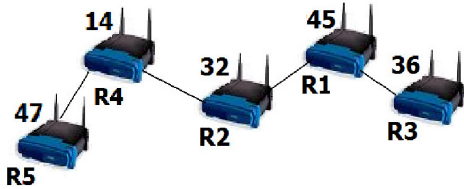


Fig. 1. An example of 5-node Wireless Mesh Network

**Definition III.1.** A bankruptcy game [7] is defined as  $G(\mathcal{N}, v)$  where  $\mathcal{N}$  represents the claimants of the bankruptcy situation and  $v$  is the characteristic function that associates to each coalition its worth defined as the part of the estate not claimed by its complement:

$$v(S) = \max(0, E - \sum_{i \in \mathcal{N} \setminus S} d_i), \forall S \subseteq \mathcal{N} \setminus \{\emptyset\} \quad (1)$$

where  $E \geq 0$  is an estate that has to be divided among the members of  $\mathcal{N}$  (the claimants) and  $d \in \mathbb{R}_+^{|\mathcal{N}|}$  is the claim vector such that  $E < \sum_{i \in \mathcal{N}} d_i$ .

Equation (1) has been proven to be supermodular which means that the marginal utility of increasing a player's strategy rises with the increase in other player strategies.

#### E. Possible imputation schemes

Solutions to cooperative games are essentially qualified with respect to the satisfaction of rationality constraints, desirable properties and existence conditions. A common solution for cooperative games in networking is the Shapley value, because it shows desirable properties in terms of null player, symmetry, individual fairness, and additivity [8]. It is computed by averaging the marginal contributions of each router in the network in each strategic situation. Nevertheless, the Shapley value is not consistent [7], in the following sense.

**Definition III.2.** An allocation  $x = (x_1, x_2, \dots, x_N)$  is consistent if  $\forall i \neq j$  the division of  $x_i + x_j$ , prescribed for claims  $d_i$  and  $d_j$ , is  $(x_i; x_j)$ .

This means that no player or group of players can gain more by unilaterally deviating from a consistent solution since it will always obtain the same profit. For cooperative WMNs, this discourages clustering-like solutions inside an interference set. Another appealing solution concept, the Nucleolus, is the unique consistent solution in bankruptcy games. It is the imputation that minimizes the worst inequity. It is computed by minimizing the largest excess  $e(x, S)$ , expressed as:

$$e(x, S) = v(S) - \sum_{j \in S} x_j, \forall S \subset \mathcal{N} \quad (2)$$

The excess  $e(x, S)$  measures the amount by which the coalition  $S$  falls short of its potential  $v(S)$  in the allocation  $x$ ; the Nucleolus corresponds to the lexicographic minimum imputation of all possible excess vectors.

## IV. AN ALGORITHMIC GAME APPROACH

The game-theoretic approach we propose is composed of two main phases: an Interference Set Detection phase and a Bankruptcy Game Iteration phase. Formally, it represents a binding agreement between cooperating MRs.

TABLE I  
INTERFERENCE RELATIONSHIPS

WMN node	Interferers
$R_1$	$\{R_2, R_3\}$
$R_2$	$\{R_1, R_4\}$
$R_3$	$\{R_1\}$
$R_4$	$\{R_2, R_5\}$
$R_5$	$\{R_4\}$

TABLE II  
INTERFERENCE SETS

Steps	WMN node sets
1	$\{R_1, R_2, R_3\}$
2	$\{R_2, R_4, R_5\}$
3	$\{R_1, R_2, R_4\}$
4	$\{R_1, R_3\}$
5	$\{R_4, R_5\}$

TABLE III  
COALITIONAL PAYOFFS

Coalition	$v(S)$
$\emptyset$	0
$R_1$	0
$R_2$	0
$R_3$	0
$R_1 \cup R_2$	24
$R_1 \cup R_3$	28
$R_2 \cup R_3$	15
$R_1 \cup R_2 \cup R_3$	60

TABLE IV  
SHAPLEY VALUE COMPUTATION

Permutation	$R_1$	$R_2$	$R_3$
$R_1, R_2, R_3$	0	24	36
$R_1, R_3, R_2$	0	32	28
$R_2, R_1, R_3$	24	0	36
$R_2, R_3, R_1$	45	0	15
$R_3, R_1, R_2$	28	32	0
$R_3, R_2, R_1$	45	15	0
Average	24	17	19

#### A. Interference Set Detection

Upon each significant change in demands or in network topology, each node determines the set of interferer nodes included inside its coverage area. MRs are able to share their interference set with other nodes in the network. Next, the list of interference sets are sorted, firstly with respect to their cardinality, and secondly with respect to the overall demands, both in a decreasing fashion.

#### B. Bankruptcy Game Iteration

In the second phase, resources are eventually allocated, proceeding with solving a bankruptcy game for each interference set, following the order of the list from the first phase. The rational behind such an agreement is that we first solve the most critical bankruptcy situations. Strategically, in this way we do not penalize nodes that interfere less compared to nodes that interfere more, as well as nodes that claim a little compared to nodes that claim a lot. Since a node can belong to many interference sets, if it has already participated to a game in a previous game iteration, it is excluded from the next game iteration in which it appears. Each game iteration therefore includes only the nodes for which an allocation has not been computed yet. It corresponds to a game where:

- $\mathcal{N}$  includes only the unallocated nodes in the set;
- the estate  $E$  is decreased by the amount already allocated to the set's nodes.

#### C. An illustrative example

We consider a WMN composed of five routers as shown in Fig. 1; the number near each router represents the number of required subchannels of attached clients, and the lines between routers represent an interference relationship, as reported in Table I. The interference set list is presented in Table II; the first step includes the players of a bankruptcy game  $G(\mathcal{N}, v)$  where  $\mathcal{N} = \{R_1, R_2, R_3\}$ , and the coalitional payoffs are given in Table III;  $v(\mathcal{N}) = E = 60$  since no node has participated to any previous game. Table IV reports the

TABLE V  
NUCLEOLUS COMPUTATION

Step 1:				
Coalition	$e(x, S)$	(30, 10, 20)	(25, 16, 19)	(26, 16, 18)
$R_1$	$-x_1$	-30	-25	-26
$R_2$	$-x_2$	-10	-16	-16
$R_3$	$-x_3$	-20	-19	-18
$R_1 \cup R_2$	$24-x_1-x_2$	-16	-20	-18
$R_1 \cup R_3$	$28-x_1-x_3$	-22	-16	-16
$R_2 \cup R_3$	$15-x_2-x_3$	-15	-20	-19

Step 3:			
Coalition	$e(x, S)$	(10, 34)	(7, 37)
$R_4$	$-x_5$	-10	-7
$R_5$	$30-x_5$	-34	-7

TABLE VI  
COALITIONAL PAYOFFS

Coalition	Payoff
$\emptyset$	0
$R_4$	0
$R_5$	30
$R_4 \cup R_5$	44

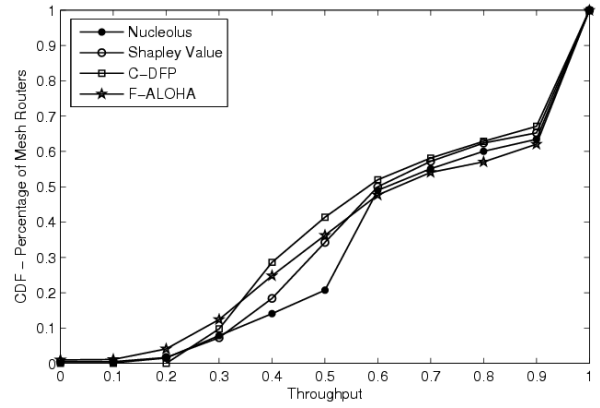
TABLE VII  
SHAPLEY VALUE COMPUTATION

Permutations	$R_4$	$R_5$
$R_4, R_5$	0	44
$R_5, R_4$	14	30
Average	7	37

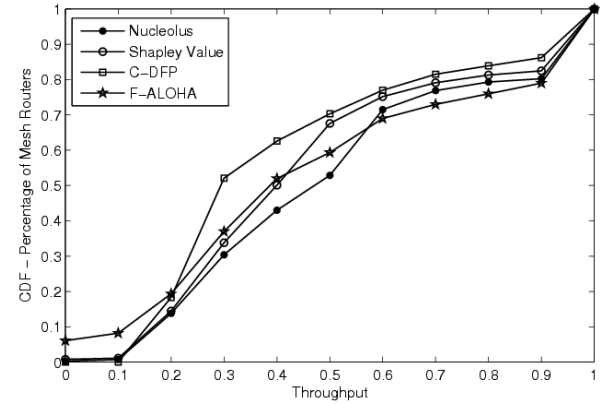
Shapley values (rounded) as well as the detail on each node's marginal contributions. For the Nucleolus, one starts at an arbitrary point such that  $x_1 + x_2 + x_3 = 60$ , e.g., (30, 10, 20), as in the step-1 part of Table V. Then, one minimizes the largest excess, corresponding to coalition  $R_2$  in our case; but, this coalition can claim that every other coalition is doing better than it is. So, one tries to improve this coalition by making  $x_2$  larger or, equivalently,  $x_1 + x_3$  smaller since  $x_3 = 60 - x_1 - x_2$  (feasibility property); but, decreasing the excess of  $R_2$ , the excess of  $R_1 \cup R_3$  increases at the same rate and these excesses then meet at  $-16$ , when  $x_2 = 16$ . Clearly, no allocation  $x$  can make the excess smaller than  $-16$  since at least one of the coalitions  $R_2$  or  $R_1 \cup R_3$  can have at least an excess of  $-16$ . Hence,  $x_2 = 16$  is the first component of the Nucleolus. Proceeding in the same manner, one finally obtains the Nucleolus allocation (26, 16, 18). We move now to the second step, in this case the total estate to share among MRs is not 60 subchannels since  $R_2$  has already participated to a game and obtained its resources; thus the new game is formed of two players,  $R_4$  and  $R_5$ , and the total payoff  $v(\mathcal{N})$  is then equal to  $E - x_2 = 60 - 16 = 44$  subchannels ( $x_2 = 16$  in the obtained Nucleolus solution), as reported in Table VI. The Shapley value computation for this second game is illustrated in Table VII. Moreover, for the Nucleolus, we obtain the step-3 part of Table V. The algorithm stops at this point since all nodes have received their resources. We notice that the Nucleolus smoothes the maximum and the minimum allocation, preventing from extremely low and high allocations for nodes that interfere a lot and a little, respectively.

## V. PERFORMANCE EVALUATION

In this section, we evaluate the proposed game-theoretic approaches (i.e., Shapley value and Nucleolus). C-DFP and F-ALOHA schemes, presented in Section II, are used as benchmarks: the first represents the centralized solution, and



(a) 50 nodes



(b) 100 nodes

Fig. 2. Throughput Cumulative Distribution Function for the two cases.

the second the non-collaborative solution. We simulated realistic scenarios with two network sizes (50 and 100 nodes) representing respectively low, and large densities. MRs are randomly distributed in a 5km $\times$ 5km area. Mesh clients are uniformly distributed within the MR radius of 275m, and each one of them uniformly generates its traffic demand that can be directly translated to a number of subchannels. We consider a typical OFDMA frame (downlink WiMAX frame) consisting of 60 subchannels. Let us focus on the comparison among different strategies based on the offered throughput and the allocation fairness.

### A. Throughput analysis

Fig. 2 reports the mean normalized throughput (i.e., mean ratio of the number of allocated subchannels to the total demand; in the following referred to as throughput) for the two considered datasets. We can here appreciate how much the strategic constraints in game theory approach, and in particular the individual and collective rationality, contribute in avoiding low throughputs. In particular, we can assess that:

- At low throughputs, F-ALOHA and C-DFP offer very low performance, especially in dense environments; e.g., in the 100-node case, in F-ALOHA around 6% of the MRs obtain null throughput, and about 23% in C-DFP obtain a throughput less than 30%, while these numbers are roughly halved with game-theoretic approaches.

TABLE VIII  
MEAN FAIRNESS INDEXES

Nodes	Nucleolus	Shapley Value	C-DFP	F-ALOHA
50	0.863358	0.85666	0.83489	0.839741
100	0.756731	0.729936	0.700218	0.69025

- The median throughput is higher for the Nucleolus; e.g., in the 100-node case, 47% for the Nucleolus, 39% for the Shapley value, 37% for F-ALOHA and 29% for C-DFP.
- At high throughputs, F-ALOHA shows a small benefit over the Nucleolus, but in all cases the median throughput of the Nucleolus is still the highest among all approaches.
- Among the game-theoretic approaches, the Nucleolus persistently outperforms the Shapley value, with relevant differences at medium-low throughputs.

All in all, the Nucleolus seems the most appropriate approach with respect to the offered throughput, especially in high density environments. Moreover, the C-DFP approach appears as the most inadequate one, and the F-ALOHA offers low throughputs to a significant portion of the MRs.

### B. Fairness analysis

We evaluate the fairness of the solutions using two aspects.

(i) with respect to the Jain's fairness index [14], defined as:

$$FI = \left( \sum_{i=1}^N (x_i/d_i) \right)^2 / \left( N \sum_{i=1}^N (x_i/d_i)^2 \right) \quad (3)$$

reported in Table VIII. It is easy to notice that game-theoretic approaches give the highest fairness, thanks to the strategic constraints that avoid penalizing nodes presenting low interference degree and those with lower demands.

(ii) Fig. 3 further investigates how the node interference degree is taken into account, illustrating the mean normalized throughput as a function of the interference degree (that corresponds to the cardinality of its interference set) for the 100-node case. We can assess that:

- The Nucleolus outperforms the other methods especially at high interference degrees.
- The Shapley value shows a roughly 5% better throughput than F-ALOHA and C-DFP in high interference degrees.
- Globally, C-DFP appears as the less performant solution.

It seems appropriate to conclude that the interference degree is taken into account in a significantly different way with the Nucleolus, showing an interesting fairness performance certainly, especially desirable for dense environments.

## VI. CONCLUSION

Wireless mesh networks based on Orthogonal Frequency Division Multiple Access is a promising solution for high-speed data transmissions and wide-area coverage. In the case WMN customers desire a control of the MR coming with their subscription, strategic resource allocation mechanisms appear as desirable solutions. In this paper, we have investigated novel approaches based on the theory of cooperative games motivated by the fact that such approaches allow accounting for strategic interactions among independent WMN nodes, and by the intuition that they can offer better performance in dense environments. In particular, this paper presented a game-theoretic

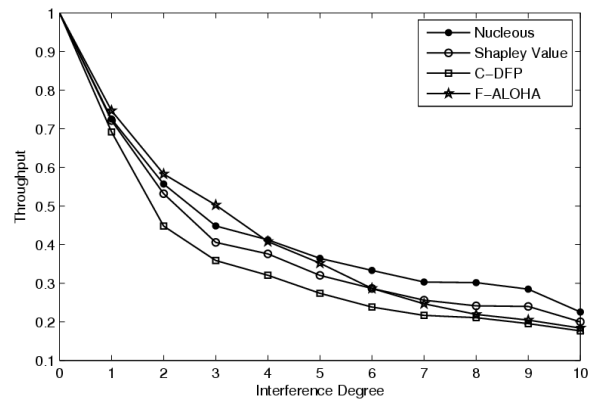


Fig. 3. Throughput distribution as a function of the interference degree (100 nodes).

approach for strategic resource allocation in OFDMA-based cooperative WMNs. Upon distributed detection of interference maps, our approach iterates bankruptcy games from the largest interference set with highest demand to the lower sets. We motivated the adoption of solutions from coalitional game theory, the Nucleolus and the Shapley value, highlighting how their properties can help meeting performance goals. Through extensive simulations using realistic datasets, we compared our game-theoretic approaches to state-of-the-art proposals. With respect to throughput and fairness, our approaches outperform the others. In particular, the Nucleolus approach is superior to all the others, achieving higher throughputs, therefore it represents a promising approach for resource allocation in future wireless mesh network deployments.

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