Optimized load planning for motorail transportation

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1. Introduction

Motorail service providers offer long distance transportation of cars and passengers on separate wagons. While passengers are carried in so-called sleeping cars, their vehicles are transported on motorail wagons. Due to long traveling distances, most journeys are carried out overnight, which allows a rested arrival at the destination for passengers in general and families in particular. Motorail transportation gained international popularity at the beginning of the 1950s. Today, motorail transportation services are globally offered by national as well as private rail transport companies. The densest network is provided in Europe, where motorail services are available in 13 countries. Further large networks exist in Australia, Chile and the United States. Due to increased capacity requirements by vehicles over the past 50 years – as a consequence of increasing vehicle lengths and widths, growing requests for optional equipment or the current popularity of large and heavy sports utility vehicles (SUV) – the present capacities of car transportation wagons become a critical issue.

We consider the problem of loading vehicles onto motorail wagons – in the following called motorail transportation problem (MTP) – which decides on which wagon, loading deck and position each particular vehicle should be loaded. Given a set of vehicles the objective is either to find a feasible allocation of all vehicles to transportation wagons or to maximize the number of vehicles loaded on a particular loading deck such that all other vehicles can be loaded as well. Integer programming formulations for the MTP as well as a variant of the MTP for order acceptance decision support, called motorail capacity problem (MCP), have been introduced by Lutter and Werners [1]. Practical applications of these models arise at motorail terminals (MTP) as well as during the booking process (MCP). In both cases fast solution times are essential.

At motorail terminals, vehicles consecutively arrive at the terminal. The goal is to select subsets of these vehicles to be loaded together, while assuring that all other vehicles can be loaded as well. Optimization based decision support aims at speeding up the entire loading process while guaranteeing the feasibility of the proposed loading plan at all times. Real-world decision support mainly relies on the computational performance of the optimization model. As the previous formulation of the MTP is hard to solve to optimality, this paper introduces two novel model formulations.

The MCP enables motorail companies to manage the order acceptance process when the exact vehicle weight is only known as an interval before acceptance of the order. Again, the computational performance plays a major role in real-world applications. Because the solution time of the original formulation of the MCP is slowed down by further binary and continuous variables, we propose a MTP-based formulation of the MCP. The novel formulation has the advantage that no additional variables are needed and it shows clearly superior computational performance. Both models have been successfully implemented at Deutsche Bahn Fernverkehr AG, a subcompany of Deutsche Bahn the largest...
German railway company, in order to optimize the order acceptance process and to improve the loading process at motorail terminals.

The contributions of this paper are as follows: Two novel integer linear programming formulations of the MTP and the MCP are introduced and evaluated. The first reformulation is derived from the four-index formulation of the MTP (see [1]) by reformulating height constraints in a combined height and capacity constraint which allows the use of three-fold indexed variables. It is shown that the three-index formulation provides stronger LP relaxations than the original four-index formulation. Second, on the basis of the three-index formulation an extended formulation is introduced, which provides the best LP relaxation but uses more variables than the other formulations. A customized branch-and-price approach is developed to solve the extended formulation. Third, it is shown that the reformulations of the MTP can be used to solve the motorail capacity problem as well. In contrast to previous work on these problems, we prove that both problems can be represented by the same set of decision variables. Extensive computational experiments with real-world data indicate that the novel formulations significantly outperform previous formulations of both problems and enable the real-world use in a decision support system.

The paper is organized as follows. In Section 2 related research is discussed. Two integer linear programming formulations for the MTP as well as the corresponding reformulations of the MCP are stated in Section 3. Computational results for all model formulations are provided in Section 4, while Section 5 concludes with some final remarks and perspectives for further research.

2. Literature review

Optimization models for motorail transportation have been introduced by Lutter and Werners [1,2]. The authors present an integer linear optimization model to assign vehicles to transportation wagons, which is called MTP. The model is extended to deal with order acceptance decisions when vehicle weights are only known within an interval. An extensive computational study for the order acceptance model which includes all MTP constraints is conducted using the commercial solver FICO Xpress. In total, 35 out of 82 tested instances were solved to optimality within a time limit of 1500 s. In the test set of large instances containing seven wagons and 59 up to 96 vehicles, only six out of 36 instances were solved to optimality. Many unsolved instances reveal considerable optimality gaps. These results show that solving the problem is far from being trivial. For real-world use large optimality gaps or run times are impractical and need to be reduced.

To the best of our knowledge, no further literature on motorail optimization exist. Related problems arise in the field of auto-carrier transportation and container loading. The auto-carrier transportation problem (ACT) deals with simultaneous loading and routing of auto-carriers. The problem is motivated by the real-world problem of delivering new cars to car dealers on demand. Auto-carrier loading comprises the selection and allocation of vehicles to parking positions on car transporters while routing decisions affect the itinerary planning of auto-carriers. The goal is to minimize the sum of transportation costs as well as penalty costs which occur when cars are not delivered by due date. The problem was introduced by Agbegha et al. [3] and further developed by Tadei et al. [4]. Both publications only consider a simplified version of the loading constraints. In a recent publication, Dell’Amico et al. [5] present a variant of the problem that incorporates the loading constraints in a more detailed fashion than the previous authors. Due to its computational complexity, the problem is commonly tackled heuristically. Tadei et al. [4] propose a decomposition of routing and load planning decisions and solve the sub-problems as integer programs with a standard commercial solver. Dell’Amico et al. [5] develop an iterated local search algorithm to solve the problem. In contrast to auto-carrier loading, motorail transportation requires different loading constraints due to significant technical differences in the used transportation technology and differing legal requirements.

Another related problem deals with the loading of containers onto freight trains. Container loading problems are characterized by a detailed treatment of physical loading requirements. In analogy to the motorail transportation problem, timetabling and routing decisions are not part of the optimization problem. Many papers on container loading aim at providing real-world decision support for container terminals [6–9]. While some containers are stored at container terminals before the beginning of the loading process, others arrive just-in-time. Thus, loading plans need to be revised during the loading process. Additionally, run-times are of high importance when focusing on real-world decision support. The same holds true for optimization at motorail terminals. Despite these commonalities, motorail transportation is distinctively different from container loading regarding physical loading constraints and the handling of transportation goods at motorail terminals. In the following, related literature with emphasis on real-world applications at container terminals is reviewed.

Corry and Kozan [6,7] first introduced an integer programming formulation for container loading at a realistic level of detail. The authors propose a dynamic load planning model to optimize the load planning of trains at intermodal terminals. Different types of containers are modeled with a finite set of different load patterns for every wagon. Local search as well as simulated annealing heuristics are proposed, since real-world instances of the original problem cannot be solved by exact methods in adequate time. Three different integer linear programming formulations for the container loading problem which vary regarding the consideration of length and weight constraints were considered by Bruns and Knust [8]. Their approach is related to what is considered in this paper in the sense that different model formulations of the same problem are provided and tested. In contrast to Corry and Kozan, they solve their problems with commercial and non-commercial solvers. In a subsequent paper [9], previous results are extended regarding the integration of data uncertainty. Novel formulations to hedge against different cases of uncertainties regarding container weight, container overhang as well as wagon failure are introduced. Again commercial solvers are used to solve their model formulations. More recently, a mathematical model for the train loading problem arising at automated terminals with an innovative transfer system for loading and unloading containers was introduced by Anghinolfi et al. [10]. To solve the model heuristically a Greedy Randomized Adaptive Search heuristic is developed which is able to generate good quality solutions in short time.

This study presents two new models to solve the MTP as well as the MCP and thus provides a significant extension to the developments made in the previous studies by Lutter and Werners [1,2]. While most approaches in the literature concerning auto-carrier loading and container load planning propose various heuristics to solve their problems, this paper focuses on exact solution methods.

3. Improved formulations of the motorail transportation problem

This section describes the problem of loading vehicles onto motorail wagons. The first subsection discusses relevant technical
details of motorail trains and vehicle loading. In a recent publication [11], a mathematical formulation of the MTP as well as an extension to support order acceptance decisions (MCP) is presented. We briefly describe the four-index formulation of the MTP. This is followed by two alternative representations of the MTP and a novel formulation of the motorail capacity problem which is derived from the MTP.

3.1. The motorail transportation problem

The MTP determines an optimal assignment of vehicles to vehicle transportation wagons, loading decks, and positions considering physical and route specific requirements. The problem focuses on vehicle transportation as the occupants are carried without difficulty in passenger wagons. As each train has a given starting point and a single destination point, we concentrate on the load planning problem for one particular train and all its assigned vehicles. The order acceptance problem (MCP) is discussed in Section 3.5. Here, the most fundamental variant of the problem which arises at motorail terminals is described. In case of terminal optimization, pre-booked vehicles consecutively arrive at motorail terminals and are required to be loaded. Operational decision support can be achieved by solving the MTP within a rolling horizon framework to consecutively prove the feasibility and revise the loading plan accordingly. Hence, the fast generation of optimal solutions is required while the original formulation suffers from poor bounds leading to long computation times. Thus, two new formulations of the MTP generating better LP bounds are presented and evaluated.

Let \( V \) denote the number of vehicles. Then, the set of vehicles is given by \( V = \{1, \ldots, |V|\} \). A vehicle \( v \in V \) is defined by five characteristics, namely, weight \( W_v \), height \( H_v \), roof width category \( R_v \), position capacity requirement \( C_v \), and type \( T_v \). A motorail train consists of \( I \) identically shaped vehicle transportation wagons \( i \in I = \{1, \ldots, I\} \), each consisting of two loading decks \( e \in E = \{1, 2\} \). The upper loading deck is indicated by \( e = 1 \) while the lower loading deck is given by \( e = 2 \). Both loading decks provide \( P \) different positions \( p \in P = \{1, \ldots, P\} \) each. Each loading deck offers the same maximal weight capacity of \( \mathcal{W} \) and provides identical length limits. The length limit of the loading deck is implicitly included into the position capacity requirement parameter \( C_v \) and is incorporated by position capacity constraints. The position capacity demand \( C_v \) of a vehicle \( v \) is determined by its type coding \( T_v \). The parameters \( C_v \) are normalized such that one car corresponds to one position. Hence, cars \( (T_v = 1) \) require one position and thus \( C_v = 1 \). Motorcycles with a sidecar \( (T_v = 2) \) have a position requirement of \( \frac{1}{2} \) position and motorcycles without a sidecar \( (T_v = 3) \) need \( \frac{1}{2} \) position. In order to ease notation, cars are summarized in the set \( T_1 \subset V \), motorcycles with sidecars are given by \( T_2 \subset V \), while motorcycles are indicated by the set \( T_3 \subset V \). In general, each loading deck is of the same length and consists of five positions which are numbered in ascending order corresponding to the driving direction.

Fig. 1 visualizes both loading decks of a transportation wagon and shows the special structure of maximal heights: on the upper loading deck, the maximum height differs at each position and depends on the vehicle’s roof width. The parameter \( R_v \in \{0, 1\} \) is set to 0 for vehicles with normal roof width. Vehicles with wide roof width are identified with \( R_v = 1 \). Solid lines indicate maximal heights for vehicles with a roof width less than or equal to 135 cm, while dashed lines show maximal heights for vehicles with a roof width greater than 135 cm. On the lower loading deck, all positions feature the same, but smaller maximum height and no distinction is made between different vehicle roof widths. In the following, a given wagon \( i \) and loading deck \( e \) are indicated by the 2-tuple \((i, e)\) while a specific position \( p \) on \((i, e)\) is identified by the 3-tuple \((i, e, p)\). Each position \((i, e, p)\) is associated with specific maximum heights \( \Pi_{i, e, p} \), depending on vehicle width \( R_v \). This is necessary since vehicles with wide roof width \((>135 \text{ cm})\) are stronger affected by the circulation of air than vehicles with normal roof width \((\leq 135 \text{ cm})\). As a consequence, the allowed maximum height is reduced by 10 cm for vehicles with wide roof width. Due to identically shaped vehicle transportation wagons, the use of the wagon index \( i \) in the parameter \( \Pi_{i, e, p} \) becomes redundant.

3.2. Four-index formulation of the MTP

The four-index formulation of the motorail transportation problem (MTP-4Idx) was introduced by [11]. Given a set of vehicles \( V \) and a set of wagons \( I \), the goal is to find a subset of vehicles \( V' \subset V \) with maximal cardinality. The decision variable \( x_{i, e, p} \) indicates the assignment of vehicle \( v \) to position \((i, e, p)\) and equals 1 if and only if vehicle \( v \) is assigned to position \( p \) on loading deck \( e \) on wagon \( i \).

With these definitions, the integer linear formulation of the MTP can be written as follows:

\[
\text{max } \sum_{v \in V} \sum_{i \in I} \sum_{e \in E} \sum_{p \in P} x_{i, e, p} \tag{1}
\]

s.t. \[
\sum_{i \in I} \sum_{e \in E} \sum_{p \in P} x_{i, e, p} \leq 1 \quad \forall v \in V \tag{2}
\]

\[
H_v x_{i, e, p} \leq \Pi_{i, e, p} \quad \forall v \in V, i \in I, e \in E, p \in P \tag{3}
\]

![Fig. 1. Illustration of both loading decks of a wagon i.](image-url)
\[
\sum_{i \in I} \sum_{e \in \mathcal{E}} W_i x_{i\text{top}} \leq \bar{W} \quad \forall i \in I, e \in \mathcal{E}
\]

(4)

\[
\sum_{i \in I} c_i x_{i\text{top}} \leq 1 \quad \forall i \in I, e \in \mathcal{E}, p \in \mathcal{P}
\]

(5)

\[x_{i\text{top}} \in \{0, 1\} \quad \forall v \in \mathcal{V}, i \in I, e \in \mathcal{E}, p \in \mathcal{P}.
\]

(6)

The objective function (1) maximizes the number of loaded vehicles. Constraint (2) ensures that each vehicle is assigned to at most one position. Varying heights of positions are considered by restrictions (3). With the inclusion of (4), the adherence to the maximal weight of \(\bar{W}\) for each loading deck is ensured. The position capacity is modeled by constraints (5). Finally, the domain of the decision variables is defined by (6). The model includes 2 \(V \cdot I \cdot P\) variables in total as well as \(V + 2I \cdot (V \cdot P + 1 + P)\) constraints.

3.3. Three-index formulation of the MTP

In order to reformulate the four-index formulation, the position index \(p\) is removed from the decision variables \(x_{i\text{top}}\). Each position is associated with a specific maximal height. The elimination of the position index \(p\) can be achieved by reformulating height constraints (3) as knapsack-type constraints for each loading deck. This can be achieved by grouping vehicle heights in height categories and imposing constraints on the allowed capacity for each height category. Then, an additional position index becomes obsolete as height and position capacities are implicitly handled by the additional knapsack constraints. The use of three-fold indexed decision variables considerably reduces the number of decision variables and provides a stronger LP relaxation than the original four-index formulation.

The reformulation relies on the idea of grouping vehicles into height categories and imposing constraints on the number of vehicles belonging to each height category. Vehicles can be grouped into four different height categories \(h \in \mathcal{H} = \{1, 2, 3, 4\}\), where \(\mathcal{H}\) denotes the index set of all height categories. Different maximum heights for each height category are given by \(\bar{H}_h\) as defined in (7) and (8) and \(\bar{H}_{h,R} = 0\) for both vehicle roof width categories \(R \in \{0, 1\}\). The maximum height for position \((i, e, p)\) is given by \(\bar{H}_{i, p, h}\), and the following height structure holds for all wagons \(i \in I\) and roof width categories \(R \in \{0, 1\}\) (see Fig. 1). All positions on lower loading decks provide the same maximum height as the fifth position on the upper loading deck irrespective of vehicle roof width category \(R\):

\[
\bar{H}_{1,0} = \ldots = \bar{H}_{1,5,0} = \bar{H}_{1,5,1} = \ldots = \bar{H}_{1,5,5,1} = \bar{H}_{1,5,5,0} = \bar{H}_{1,5,5,1}.
\]

(7)

On the upper loading deck, positions can be ordered such that maximum heights are in an increasing order irrespective of vehicle roof width category \(R\):

\[
\bar{H}_{1,0} = \bar{H}_{1,1} = \bar{H}_{1,1,0} = \bar{H}_{1,1,1} = \ldots = \bar{H}_{1,5,1} = \bar{H}_{1,5,0} = \bar{H}_{1,5,1} \leq \bar{H}_{1,5,2} = \bar{H}_{1,5,3} = \bar{H}_{1,5,4} = \bar{H}_{1,5,5,1} = \bar{H}_{1,5,5,0} = \bar{H}_{1,5,5,1}.
\]

(8)

With the preceding notation the set \(\mathcal{H}_h \subset \mathcal{V}\) of vehicles belonging to height category \(h\) is given by

\[\mathcal{H}_h = \{v \in \mathcal{V} | \bar{H}_{h,R} - 1 \leq H_v \leq \bar{H}_{h,R}\} \quad \text{for all } h \in \mathcal{H}.
\]

Let \(\beta_h\) denote the number of positions belonging to the same height category \(h \in \mathcal{H}\) on a single upper loading deck. Note that \(\beta_h\) does not depend on the vehicle roof width. The roof width \(R\) of a given vehicle \(v\) only influences the assigned height category of this vehicle. Hence, the maximum number of vehicles belonging to category \(h\) which can be loaded onto an upper loading deck – ignoring all other vehicles from different height categories – is given by the sum of all positions offering at least a maximum height of \(\bar{H}_{h,R}\):

\[
\beta_h = \sum_{h' \in \mathcal{H}} \sum_{\bar{H}_{h,R} \leq \bar{H}_{h',R} \leq \bar{H}_{h,R}} \beta_{h'} = \sum_{h' \in \mathcal{H}} \sum_{\bar{H}_{h,R} \leq \bar{H}_{h',R} \leq \bar{H}_{h,R}} \beta_{h'}.
\]

Thus, the number of feasible positions on an upper loading deck for a vehicle belonging to category \(h \in \mathcal{H}\) is given by \(\beta_h\). Eliminating the position index from the decision variables requires the transformation of height constraints (3) into capacity constraints for the entire loading deck. Position capacities are rivalrous, meaning that the same position can only be used once. Thus, \(\beta_h\) overestimates the free capacity of a height category if a vehicle from a larger height category is considered for the same loading deck as shown in the following. Loading vehicles \(v_1, v_2 \in \mathcal{V}\), \(v_1 \neq v_2\), belonging to different height categories \(h_1, h_2 \in \mathcal{H}\) with \(h_1 < h_2\), reduces the capacity available for vehicle \(v_1\): The capacity coefficient \(\beta_{h_1}\) for height category \(h_1\) includes positions allowing the loading of higher vehicles than \(v_1\). The presence of a vehicle from category \(h_2\) reduces the capacity available for \(v_1\) by \(C_{v_{1,h_1}}\). Extending this idea by accounting for all vehicles competing for the same positions leads to the following constraints for upper loading decks (\(e = 1\))

\[
\sum_{v \in \mathcal{V}} c_v x_{v\text{top}} \leq a_h \quad \forall h \in \mathcal{H}.
\]

(9)

Here, the position index \(p\) has already been dropped from the decision variables \(x_{v\text{top}}\). As position capacity constraints (5) are implicitly included in constraints (9), they can be excluded from the reformulation. The validity of the combined height and capacity constraints (9) relies on the fact, that the vehicle capacity parameter \(C_{v}\) only takes values in \{1, \ldots, 4\}. Consider the inequality \(x + \frac{1}{2} y + z \leq a\) where \(x \in \mathbb{N}_0\) gives the number of cars, \(y \in \mathbb{N}_0\) denotes the number of motorcycles with sidecar, \(z \in \mathbb{N}_0\) states the number of motorcycles and \(a\) is a positive integer. In this case, each feasible solution \((x, y, z)\) can be written as positive integer combination of \(8\) loading patterns (each represented by a vector \(\#T_1, \#T_2, \#T_3\) containing the number of vehicles of type \(T_e\) for a single position:

\[
\begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix} = \lambda_1 \begin{pmatrix}
  1 \\
  0 \\
  0
\end{pmatrix} + \lambda_2 \begin{pmatrix}
  0 \\
  2 \\
  1
\end{pmatrix} + \lambda_3 \begin{pmatrix}
  0 \\
  1 \\
  1
\end{pmatrix} + \lambda_4 \begin{pmatrix}
  0 \\
  0 \\
  1
\end{pmatrix} + \lambda_5 \begin{pmatrix}
  0 \\
  1 \\
  0
\end{pmatrix} + \lambda_6 \begin{pmatrix}
  0 \\
  0 \\
  2
\end{pmatrix} + \lambda_7 \begin{pmatrix}
  2 \\
  0 \\
  0
\end{pmatrix} + \lambda_8 \begin{pmatrix}
  0 \\
  1 \\
  0
\end{pmatrix}.
\]

(10)

with \(\sum_{k=1}^{8} \lambda_k = a\) and \(\lambda_k \in \mathbb{N}_0\) for all \(1 \leq k \leq 8\). Thus, each feasible solution for (9) can be decomposed into \(a\) feasible loading patterns for a single position. Note that this decomposition does not apply if \(C_v\) has other realizations, e.g. \(C_v \in \{1, 2, 3\}\).

Due to the fact that upper loading decks provide greater maximum heights than lower loading decks, the set of vehicles can be split into two disjoint sets of low and high vehicles. The set of high vehicles which can only be placed on the upper loading deck (\(e = 1\)) is denoted by \(V_{\text{hl}} = \{v \in \mathcal{V} | \mathcal{H}_h\}\). Then, the set of low vehicles which can be placed on both loading decks is given by \(V_{\text{ll}} = V_1 \cup V_2\) (thus \(V = V_1 \cup V_2\)). On the lower loading deck (\(e = 2\), all positions offer the same maximal height. Hence, only a single height category exists which provides a capacity of \(P\). Restricting the set of vehicles to \(V_2\) on lower loading decks allows to substitute constraints (3) and (5) by

\[
\sum_{v \in V_2} c_v x_{v\text{top}} \leq P.
\]
Theorem 3.1. Height constraints (3) as well as capacity constraints (5) on the upper loading deck are equivalent to constraints (9) and (10) in the sense, that each integral feasible solution of (9) and (10) can be transformed into an integral solution satisfying constraints (3) and (5) and vice versa.

Substituting (3) and (5) by (9) and (10) allows dropping of the position index p from the decision variables on both loading decks. Hence, the MTP-3Idx reads as

$$\text{max } \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij}$$

subject to:

$$\sum_{j \in J} x_{ij} \leq 1 \quad \forall \ell \in \mathcal{V}_1$$

$$\sum_{i \in I} x_{ij} \leq 1 \quad \forall \ell \in \mathcal{V}_2$$

$$\sum_{\ell \in \mathcal{V}} x_{\ell j} \leq W \quad \forall i \in I, \forall e \in \mathcal{E}$$

$$\sum_{\ell \in \mathcal{V}} c_{\ell j} x_{\ell j} \leq d_h \quad \forall h \in H, \forall i \in I$$

$$\sum_{\ell \in \mathcal{V}} c_{\ell j} x_{\ell j} \leq P \quad \forall i \in I$$

$$x_{\ell j} \in \{0, 1\} \quad \forall \ell \in \mathcal{V}, \forall i \in I, \forall e \in \mathcal{E}.$$  

The objective function (11) maximizes the number of loaded vehicles. Constraints (12) and (13) assure that each vehicle is assigned to at most one loading deck. The weight capacity for each loading deck is modeled by (14). Constraints (15) and (16) assure that maximal heights as well as position capacity limits are fulfilled for both loading decks. Finally, constraints (17) state the domain of the decision variables. The reformulation leads to saving $2|\mathcal{V}|(P - 1)$ variables compared to the original four-index formulation. The MTP contains a multidimensional knapsack problem with at least two constraints as a special case, if a single wagon with a single loading deck is considered. It is known that the multi-dimensional knapsack problem is NP-hard [11] if two or more knapsack constraints are present. So, the MTP is NP-hard too. The next theorem analyses and compares the strength of both LP relaxations.

Theorem 3.2. The three-index formulation of the MTP yields a stronger LP relaxation than the four-index formulation.

The proof of Theorem 3.2 is given in Appendix A.

An important special case concerns the loading of large groups of motorcycles which occurs especially during summer times. The described constraints allow the loading of cars and motorcycles. If a single lower loading deck solely contains motorcycles, a more compact loading mode can be applied allowing the usage of up to one more position. This can be implemented by introducing an additional binary decision variable $\delta_i \in \{0, 1\}$ ($i \in I$) which may take the value 1 if and only if no cars are loaded onto loading deck (1, 2). The following inequalities assure that variables $\delta_i$ can only take the value 1 if the corresponding lower loading deck does not contain cars:

$$x_{\ell j} + \delta_i \leq 1 \quad \forall \ell \in \mathcal{T}_1 \cap \mathcal{V}_2, \forall i \in I$$  

$$\sum_{\ell \in \mathcal{V}_2} c_{\ell j} x_{\ell j} - \delta_i \leq P \quad \forall i \in I.$$  

Including both sets of constraints (18) and (19) into the model MTP-3Idx instead of constraints (16) allows the use of one additional position on the lower loading deck.

Due to the light weight of motorcycles, the total weight capacity of a loading deck allows the loading of arbitrarily chosen subsets of motorcycles as long as the position capacity is not exceeded. This can be used to decompose these instances into $1 + |\mathcal{T}_2|/P + 1$ ordinary MTP instances by consecutively fixing the heaviest motorcycles onto arbitrary lower loading decks until the total position capacity is reached. In this case, the problem size reduces in each step and no additional variables are required. Clearly, the solution of the resulting problem instances can be done in parallel. Each feasible solution to one of the instances is a primal bound for the original formulation. Hence, early termination of instances is possible if the upper bound of a particular instance is larger or equal to the best primal bound. Computational experiments indicate that the variable fixing approach outperforms the δ-variable approach in many instances.

The following section introduces a third problem formulation by exploiting the special structure of the coefficient matrix and proposes an extended formulation which can be solved by a branch-and-price algorithm.

3.4. Extended formulation

Previous model formulations suffer from high problem symmetry. Let $x_{\text{sep}}$ denote a feasible solution of the MTP-4Idx formulation. Then, each permutation of wagon indexes $\sigma : I \to I$ also leads to a feasible solution $x_{\text{sep}}^{\text{new}}$. The same holds for positions $p \in P$ offering identical maximal heights on the same loading deck. In particular, this is the case on all lower loading decks and on some positions of the upper loading deck. Even the three-index formulation is highly symmetric regarding the wagon index i, but eliminates the symmetry concerning positions offering the same maximal heights. Due to this inherent symmetry, a standard branch-and-bound solution process is usually slowed down.

A common way to eliminate the symmetry is to use Dantzig-Wolfe (DW) reformulation leading to a column generation (CG) approach [12]. DW decomposition is applied to separate the problem into a master problem (MP) and into one or more sub-problems (SUB). The MP coordinates the overall structure of the problem while sub-problems generate solutions, i.e. columns in the master problem with negative reduced costs if existent. Instead of enumerating the entire solution space of the sub-problems – which can lead to exponentially many variables – column generation (see for example [13]) is a well-known iterative procedure working only on a subset of all variables of the master problem. In each iteration, one or more sub-problems are used to identify the necessity of further variables and to generate them when needed. Applying column generation within a branch-and-bound procedure is referred to as branch-and-price. In the following, it is shown how to reformulate the MTP by applying Dantzig-Wolfe decomposition.

Both formulations of the MTP inherit a single bordered block-diagonal structure as depicted in Fig. 2 which allows the application of DW decomposition. The master problem contains set packing constraints which assure that each vehicle is loaded at most once. Sub-problems are multi-dimensional knapsack problems which assure that all technical constraints are fulfilled for each transportation wagon. A feasible solution of a sub-problem is referred to as a loading pattern. Loading patterns are defined as subsets of vehicles which are loaded onto the same wagon and fulfill all technical constraints. Hence, sub-problems generate new loading patterns to improve the actual objective function value. The original formulation of the MTP uses loading patterns as decision variables. In contrast, the master problem in the extended
formulation operates on a different variable space by incorporating loading patterns as input parameters and deciding about their use. This leads to a different problem representation and eliminates symmetry when applying appropriate branching rules during the branch-and-bound process. Fig. 3 summarizes the structure of our developed B&P algorithm.

In the following, the main components of the B&P algorithm as shown in Fig. 3 are discussed: the master problem is stated in Section 3.4.1 and the corresponding pricing sub-problems are given in Section 3.4.2. Finally, the used branching rule is described in Section 3.4.3.

### 3.4.1. Master problem

As already mentioned before, upper loading decks provide greater maximum heights than lower loading decks which allows to split the set of vehicles into disjoint sets of high \( V_1 \) and low \( V_2 \) vehicles (thus \( V = V_1 \cup V_2 \)). The master problem consists of all loading patterns for all loading decks \( e \in \mathcal{E} \). The goal is to select subsets of loading patterns such that the number of included vehicles is maximized and each vehicle is contained in at most one loading pattern. Since each loading pattern corresponds to a vehicle, the resulting model is called restricted master problem (RMP). Additional loading patterns – appearing as columns in the master problem – are only generated when needed. Let \( k_e \) denote the current number of feasible loading patterns for loading deck \( e \in \mathcal{E} \). The \( k \)-th loading pattern for loading deck \( e \in \mathcal{E} \) is described by the binary parameter \( x_{e,k} \) which is equal to one if this loading pattern contains vehicle \( v \). The binary decision variables \( x_{e,k} \) for all \( e \in \mathcal{E} \) and \( k \in k_e=[1, \ldots, k_e] \) model the selection of loading patterns and are defined as follows:

\[
x_{e,k} = \begin{cases} 
1 & \text{if loading pattern } k \text{ is selected for a loading deck } e \in \mathcal{E} \\
0 & \text{else}
\end{cases}
\]

The corresponding sub-problems, which ensure that loading patterns comply with all restrictions, coincide for upper respectively lower loading decks. Thus, sub-problems can be aggregated which reduces the number of sub-problems to only two different multidimensional knapsack sub-problems (SUB_1 and SUB_2). Hence, the resulting RMP can be stated as follows:

\[
\begin{align*}
&\text{max} \quad \sum_{e \in \mathcal{E}} \left( \sum_{v \in V_1} x_{e,k} \lambda_{e}^1 \right) + \sum_{e \in \mathcal{E}} \left( \sum_{v \in V_2} x_{e,k} \lambda_{e}^2 \right) \\
&\text{s.t.} \quad \sum_{e \in \mathcal{E}} x_{e,k} \lambda_{e}^1 \leq 1 \quad \forall v \in V_1 \\
&\quad \sum_{e \in \mathcal{E}} x_{e,k} \lambda_{e}^1 + \sum_{e \in \mathcal{E}} x_{e,k} \lambda_{e}^2 \leq 1 \quad \forall v \in V_2 \\
&\quad \sum_{e \in \mathcal{E}} x_{e,k} \leq 1 \quad \forall e \in \mathcal{E} \\
&\quad x_{e,k} \in [0, 1] \quad \forall e \in \mathcal{E}, k \in k_e
\end{align*}
\]

The RMP contains set packing constraints (12) and (13) of the MTP ensuring that each vehicle is included in at most one of the selected loading patterns. The set of vehicles is split into two disjoint sets of vehicles which can only be placed onto an upper loading deck and of vehicles which can be placed on both loading decks. A vehicle \( v \) belonging to the set of large vehicles \( V_1 \) can only be present in a loading pattern for an upper loading deck. It is sufficient to restrict the corresponding set packing constraints (21) to the variables \( x_{e,k} \). Small vehicles \( v \in V_2 \) can be placed on both loading decks and appear in loading patterns for upper as well as lower loading decks. Hence, set packing constraints (22) must contain decision variables for selecting loading patterns on both loading decks. In order to ensure that the number of available

\[\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{Decomposition of the MTP (bordered block-diagonal structure).}
\end{figure}\]

\[\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{Flow chart of the Branch & Price approach.}
\end{figure}\]
wagons is not exceeded, at most \(l\) different loading patterns may be chosen for each loading deck. This is modeled by constraints (23). Finally, (24) define the domain of the decision variables. In the following, the integrality constraints (24) of the RMP will be relaxed. Then, the dual variables of constraints (21) and (22) are denoted by \(\pi_v\) for all vehicles \(v \in \mathcal{V}_1 \cup \mathcal{V}_2\) and the duals of constraints (23) are denoted by \(\mu_e\) for all loading decks \(e \in \mathcal{E}\).

### 3.4.2. Pricing problems

All wagons of a motorail train are of the same type and thus subject to the same set of constraints. A feasible solution for a given wagon is a subset of vehicles fulfilling the multidimensional knapsack constraints (14)–(16). The identical structure of all wagons allows to aggregate all sub-problems to a single multidimensional knapsack problem (MKP). The aggregation of subproblems eliminates the symmetry regarding different transportation wagons. Each wagon consists of two loading decks which differ regarding the height structure of offered positions. All upper loading decks are subject to the same constraints. The same holds for all lower loading decks. A feasible solution for the entire wagon is a subset of vehicles fulfilling the multidimensional constraints (14)–(16).

A feasible solution of a sub-problem corresponds to a loading pattern which is encoded by the binary decision variables \(\xi_v \in \{0, 1\}\) for all \(v \in \mathcal{V}\). The variables \(\xi_v\) indicate whether vehicle \(v\) is included in the current loading pattern or not. The goal of each sub-problem is to generate a loading pattern which improves the current objective value of the RMP or if no such exists, to prove the optimality of the current RMP solution. Therefore, the sub-problems use the dual variables \(\pi_v\) whereas \(v \in \mathcal{V}\), and \(\mu_e\) with \(e \in \mathcal{E}\) as objective function coefficients. A loading pattern with negative reduced costs corresponds to a sub-problem solution with objective value < 0 and enters the basis of the RMP in the next iteration.

The sub-problem for upper loading decks needs to consider all low and high vehicles \(v \in \mathcal{V}\) in order to generate new loading patterns. Using the reformulation of the height constraints as shown in Theorem 3.1, the sub-problem for upper loading decks \(\text{SUB}_1\) reads as:

\[
\max \sum_{v \in \mathcal{V}} (1 - \pi_v) \xi_v - \mu_1 \\
\text{s.t.} \sum_{v \in \mathcal{V}} C_{hv} \xi_v \leq a_h \quad \forall h \in \mathcal{H} \\
\sum_{v \in \mathcal{V}} W_v \xi_v \leq W \\
\xi_v \in \{0, 1\} \quad \forall v \in \mathcal{V}
\]

(25)–(28)

Vehicles belonging to the subset \(\mathcal{V}_1\) cannot be loaded on the lower loading deck. The problem size is reduced by only considering set \(\mathcal{V}_2\). As described above, focusing on the subset \(\mathcal{V}_2\) allows dropping height constraints (3). Then, the lower loading deck sub-problem becomes:

\[
\max \sum_{v \in \mathcal{V}_2} (1 - \pi_v) \xi_v - \mu_2 \\
\text{s.t.} \sum_{v \in \mathcal{V}_2} C_{hv} \xi_v \leq P \\
\sum_{v \in \mathcal{V}_2} W_v \xi_v \leq W
\]

(29)–(31)

Both sub-problems do not reveal the integrality property, thus CG provides tighter bounds than the LP relaxation of the MTP-3dx. Additional constraints for large groups of motorcycles can easily be implemented in the pricing problems. In contrast to formulations in Section 3.3 loading deck indices (e) as well as wagon indices (i) have to be eliminated from the decision variables.

### 3.4.3. Branching

Branching needs to be applied if a solution of the RMP is fractional and no improving column can be found during the pricing step. The choice of the branching rule strongly affects the performance of the B&P approach and may even reintroduce symmetry. B&P literature suggests to branch on original variables instead of branching on master problem variables [14]. Branching on the \(\xi_v\) variables would lead to an unbalanced branching-tree, due to the \(\xi_v = 0\) branch which only excludes one loading scheme out of exponentially many. Branching on original variables is always an alternative, but reintroduces symmetries in the branching process when using aggregated sub-problems [15]. To overcome mentioned drawbacks, specialized branching rules have been introduced [16,17]. The well-known Ryan/Foster (RF) branching rule [18], which has shown to perform well with set partitioning constraints [16], does not suffer from the symmetry problem but causes slight changes in the sub-problems. Instead of branching on single vehicles or on a single loading scheme, the idea behind RF branching is to choose two different vehicles which are either forced to be loaded on separate loading decks or which are forced to be loaded together. If the selected vehicles, say \(v, v'\), are forced to be loaded onto different loading decks all loading patterns containing both vehicles \(v, v'\) are excluded:

\[
\sum_{e \in \mathcal{E}} \lambda_e \xi_v + \sum_{e \in \mathcal{E}} \lambda_e \xi_{v'} = 0.
\]

(32)

In order to include the branching decision in the sub-problems to avoid the generation of a loading scheme containing both vehicles \(v\) and \(v'\), the constraint

\[
\xi_v + \xi_{v'} \leq 1
\]

is included in both sub-problems. Otherwise, the selected vehicles \(v\) and \(v'\) are forced to be loaded together:

\[
\sum_{e \in \mathcal{E}} \lambda_e \xi_v + \sum_{e \in \mathcal{E}} \lambda_e \xi_{v'} = 1.
\]

Again both sub-problems require to add the constraint

\[
\xi_v - \xi_{v'} = 0
\]

in order to avoid the generation of a loading scheme containing only one of both vehicles. As all sub-problems are solved as integer linear programs by a MIP-solver, additional (linear) constraints can easily be implemented. The next section shows the use of the developed reformulations in the context of order acceptance.

### 3.5. Reformulations of the motorail capacity problem

The MCP aims at providing decision support for the order acceptance management of motorail companies in case of partly unknown vehicle weights during the booking process. The goal is to identify the maximal weight for a pre-specified artificial vehicle, in the following called dummy vehicle, on the condition that all other vehicles in a given set \(\mathcal{V}\) can be loaded as well. The dummy vehicle is characterized by its height \(H'\) and its capacity demand \(C'\). Due to the described grouping of height categories, the roof
width can be set to 0 without loss of generality. Note that the weight of the dummy is a decision variable whose value needs to be maximized. It is assumed that the exact weight of a booking request is unknown at the point of the acceptance decision and becomes visible to the order acceptance system not until accepting the order. Instead, only an interval of the true weight – encoded by the submitted booking class – is initially visible to the order acceptance system. The MCP, introduced as a variant of the MTP-4Idx, is used for calculating the maximal weight of prespecified dummy vehicles such that all already accepted vehicles can be loaded. Thus, the optimization problem does not incorporate any uncertainty. Based on the obtained maximal weights and historical data sets showing the weight distribution within a booking class, the risk of accepting such an order request can be empirically assessed in a second step. Optimization based proactive order acceptance management is thus achieved by blocking booking categories involving too high risks in advance. This kind of order acceptance problem differs from typically studied problems due to short respond times in a very constrained environment. Previously introduced optimization models suffer from poor computational behavior. The following briefly describes the original representation of the MCP and shows how to reformulate the problem as a series of two MTP instances which significantly improves the computational performance.

The original formulation of the MCP requires the introduction of two new decision variables. The position \( (i, e, p) \) of the dummy vehicle is indicated by the binary variables \( y_{iep} \) while the weight of the dummy vehicle is given by the continuous decision variables \( w_{iep} \). With these definitions, the MCP can be represented as a mixed-integer linear program (see [1]) and reads as

\[
\begin{align*}
\max & \sum_{i \in I} \sum_{e \in E} \sum_{p \in P} w_{iep} y_{iep} \\
\text{s.t.} & \sum_{i \in I} \sum_{e \in E} \sum_{p \in P} x_{iep} = 1 \quad \forall v \in V \\
& \sum_{i \in I} \sum_{e \in E} \sum_{p \in P} y_{iep} \leq 1 \\
& \zeta_{iep} \leq M \cdot y_{iep} \quad \forall i \in I, e \in E, p \in P \\
& \sum_{p \in P} \left( W_{i} x_{iep} + \zeta_{iep} \right) \leq W_{i} \quad \forall i \in I, e \in E \\
& H^{e} y_{iep} \leq H_{i,e,p} \quad \forall i \in I, e \in E, p \in P \\
& \sum_{v \in V} c_{v} x_{iep} + c^{d} y_{iep} \leq 1 \quad \forall i \in I, e \in E, p \in P \\
& \zeta_{iep} \geq 0 \quad \forall i \in I, e \in E, p \in P. 
\end{align*}
\]

The objective function maximizes the weight \( (33) \) of the dummy vehicle. Set partitioning constraints \( (34) \) assure that all already accepted vehicles are loaded onto the train. Set packing constraints \( (35) \) ensure that the dummy vehicle is assigned to at most a single position. Constraints \( (36) \) guarantee that the dummy vehicles’ weight can only take a value larger than zero at its assigned position \( y_{iep} \), where the parameter \( M \) is to be chosen sufficiently big. Weight constraints for each loading deck are given by \( (37) \). Constraints \( (38) \) make sure that the assigned position of the dummy vehicle provides sufficient height capacity while constraints \( (39) \) ensure that position capacities are fulfilled. Finally, constraints \( (40) \) provide the domain of decision variables.

The reformulation of the MCP relies on the following observation: let \( (i, e, p) \) denote the optimal position of the dummy vehicle. Then, the weight of the dummy is at maximum on deck \( (i, e) \) under the condition that all other vehicles are loaded as well. The weight of the dummy vehicle is given by \( W_{i} - \sum_{v \in V} \sum_{p \in P} W_{v} x_{v_{iep}} \) which is equivalent to minimizing \( \sum_{v \in V} \sum_{p \in P} W_{v} x_{v_{iep}} \) in the four-index formulation. Due to the inherent symmetry, the wagon index \( i \in I \) can be chosen arbitrarily. Hence, if the capacity requirements of the dummy vehicle are adequately included in the constraints of loading deck \( (i, e) \), the MCP can be solved by a series of at most two MTP-4Idx instances. In case of a dummy vehicle which can only be loaded onto an upper loading deck, we set \( e = 1 \) and solve only one modified MTP instance. Otherwise the dummy vehicle can be placed on both decks and two models – one for each loading deck \( e \in E \) – need to be solved. Taking the minimum of the objective values for both model runs and subtracting this value from \( W_{i} \) gives the optimal objective value of the MCP. As the objective function minimizes the weight loaded onto a given loading deck, set packing constraints \( (2) \) have to be changed to set partitioning constraints. Additionally, capacity constraints \( (4) \) and \( (5) \) for the particular loading deck \( (i, e) \) have to be reduced by the required capacity of the dummy vehicle.

The same ideas can be applied in the three-index formulation of the MTP. Again, let \( i \in I \) be chosen arbitrarily. Then, the objective is to minimize \( \sum_{v \in V} \sum_{p \in P} W_{v} x_{v_{iep}} \) the weight of vehicles on loading deck \( (i, e) \). Again, the number of required instances to be solved depends on the height of the dummy vehicle. In case of a high dummy vehicle setting \( e = 1 \) suffices, otherwise two instances have to be solved. Additionally, the capacity for loading the dummy vehicle needs to be blocked on the particular loading deck. This can be achieved for an upper loading deck by reducing the right hand side of capacity constraints \( (15) \) for all \( h \in H \) with \( H_{i,h,1} \geq H^{e} \) by the constant \( C^{d} \). In case of lower loading decks, only a single height category exists which needs to be reduced by the constant \( C^{d} \). Additionally, set packing constraints \( (12) \) and \( (13) \) have to be changed to set partitioning constraints.

If the extended formulation is used, all changes are in analogy to the three-index formulation. The mentioned changes only affect one particular sub-problem. Aggregating sub-problems is still possible, but the modified sub-problem needs to be excluded from the others and solved separately. Results of computational experiments with all presented models using real-world data are shown in the next section.

4. Computational results

The proposed MTP reformulations as well as both of the original formulations – MTP and MCP – are compared on the basis of real-world instances provided by DB Fernverkehr AG. All MIP-formulations are solved by SCIP 3.0.2 (see [19]) using SOLEX 1.7.2 (see [20]) as LP solver. The Branch-and-Price approach is solved by GCC 1.11 (see [21]) which is included in the SCIP Optimization Suite. GCC is a generic branch-and-price solver which is fairly easy to handle and competitive to branch-and-price implementations developed from scratch. After submitting the desired decomposition and customizing branching rules, GCC coordinates column generation and branching automatically. In addition, several internal heuristics help to speed up the solution process. We have configured GCC according to the branch-and-price algorithm described in Section 3.4. All experiments are conducted on virtualized Linux workstations using two high frequency Intel Xeon E5-2670v2 processor cores running at 2.5 GHz with 7.5 GB of RAM. In all instances, a time limit of 1500 s is imposed for each model.
4.1. Test instances

A set of 100 test instances provided by DB Fernverkehr AG is used to evaluate the model performance. All test instances are grouped into two subsets from different fields of application in order to evaluate both the MTP and the MCP models. Instances of the first group originate from motorail terminals and are used to test the performance of the MTP model formulations. It is distinguished between medium size instances containing three wagons and large instances consisting of seven wagons. The first group is split into 14 medium and 25 large size instances. The second group of instances represents different states of the order acceptance process and is used for testing the four different MCP formulations as well as the three MTP formulations. Again, the second group is split into 35 medium and 26 large size instances. In all instances, maximum heights for each wagon are identically chosen in standard configuration for the German market as depicted in Fig. 4. Thus, each wagon consists of two loading decks offering at most five car positions and in special cases six positions for motorcycles. In addition, a strict weight limit of 7500 kg holds for all loading decks. All MTP-instances feature high capacity utilization with less than 10% of free weight or position capacity. MCP-instances simulate the order acceptance process and are thus featuring lower capacity utilization. The considered instances differ regarding the number of vehicles and their characteristics, e.g. height or weight. Fig. 5 summarizes details for all test instances.

4.2. Results for MTP-instances

The performance of the three different formulations of the MTP is tested with all 100 real-word instances. The goal is to find a feasible allocation for all vehicles to the given motorail train. This task typically arises during the loading process at motorail terminals and needs to be solved within short time. The performance of the MTP model formulations. It is distinguished between medium size instances containing three wagons and large instances consisting of seven wagons. The first group is split into 14 medium and 25 large size instances. The second group of instances represents different states of the order acceptance process and is used for testing the four different MCP formulations as well as the three MTP formulations. Again, the second group is split into 35 medium and 26 large size instances. In all instances, maximum heights for each wagon are identically chosen in standard configuration for the German market as depicted in Fig. 4. Thus, each wagon consists of two loading decks offering at most five car positions and in special cases six positions for motorcycles. In addition, a strict weight limit of 7500 kg holds for all loading decks. All MTP-instances feature high capacity utilization with less than 10% of free weight or position capacity. MCP-instances simulate the order acceptance process and are thus featuring lower capacity utilization. The considered instances differ regarding the number of vehicles and their characteristics, e.g. height or weight. Fig. 5 summarizes details for all test instances.

4.3. Results for MCP-instances

The performance of the four different formulations of the MCP is evaluated using test sets of 61 real-word instances. In particular,
the original MCP formulation with additional binary and continuous variables is tested against the MTP-based formulations as described in Section 3.5. In each instance, the goal is to determine the maximal weight for an additional vehicle such that all other vehicles can be loaded as well. An adequate formulation should be able to generate high quality solutions within reasonable time, e.g. 1500 s. The quality of a solution is measured by the maximal weight and by the remaining gap if optimality is not proven within the time limit. In practice, maximal weights need to be calculated for each height category. We restrict our experiments to the smallest height category, as we do not expect any further insights when using different height categories for the same instances.

Table 2 summarizes computational results for all MCP-instances. In contrast to the MTP-instances, not all instances are solved to optimality within the given time limit of 1500 s. Row # best obj states the number of instances in which the considered model formulation obtains the best objective value. The original MCP formulation is clearly inferior to the MTP-based formulations regarding solution quality and speed. In medium-size instances less than 50% of all instances are proven to be solved to optimality. The MTP-based four-index formulation shows marginally better performance on medium instances, but only proves optimality in 51% of all cases. The best performance is achieved by the three-
index model and the extended formulation. Surprisingly, the three-index MTP-based formulation slightly outperforms the extended formulation in medium-size instances. While all new formulations achieve the same objective value after 1500 s, the three-index formulation proves optimality in 32 out of 35 instances. In addition, the three-index formulation requires the shortest run times for proving optimality.

Large size instances lead to different results regarding the novel formulations. Again, the original formulation is considerably outperformed by all MTP-based model formulations. In only four instances competitive objectives are generated. Optimality is only proven in a single instance. In addition, reported gaps are at least three times larger than those obtained by the MTP-based models. The four-index formulation shows the worst results of all MTP-based models. Competitive objective values are achieved in six out of 26 instances but optimality is not proven a single instance. The three-index formulation proves optimality in 8 instances and finds solutions with competitive objectives in 9 instances. With a growing number of wagons, the extended formulation shows clear advantages over all other formulations. The MTP-B&P model generates the best objective value in all instances and proves optimality in nearly 70% of all instances. The remaining 30% of instances are solved to an average optimality gap of 0.35%. Again, most of the time is spent in the pricing problems. In contrary to the previous instances, the master problem time has increased to a proportion of 14.6% of the total time.

Computational results highlight the superior performance of the novel model formulations. Although not all proposed model formulations comply with practical requirements in terms of solution quality and speed, they all outperform the original problem formulation [1]. The extended formulation qualifies best for usage in practical applications, due to its superior performance in nearly all test instances.

5. Conclusions

In this paper, two novel formulations of the motorail transportation problem and the motorail capacity problem were developed and analyzed. Original model formulations turn out to provide poor bounds, require long run times and fail to prove optimality in many real-world instances. The new models achieve superior results in most instances and considerably improve run times. Developments made in this paper enable motorail companies to use optimization methods within decision support systems. In particular, our reformulations allow for improving order acceptance decisions within reasonable time and for supporting loading processes at motorail terminals nearly in real-time. Both models are currently implemented in decision support systems with graphical user interfaces running at DB Fernverkehr AG. The MCP is automatically running after each arriving booking request providing suggestions for optimal capacity utilization. At motorail terminals, the MTP model is manually called by terminal staff to calculate feasible loading plans in real-time. Further improvements for motorail terminal optimization concern the inclusion of vehicle shunting operations in the MTP. These extensions may slow down run times as additional binary variables need to be introduced. Further research should address the development of specific solution methods to enable the real-world use of these eligible extensions.

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Appendix A. Proof of Theorem 3.2

Theorem 3.2 states that the model MCP-3Idx yields a tighter LP relaxation than the model MCP-4Idx.

Proof. Consider an instance containing a single wagon and two vehicles \(v_1, v_2 \in H_4\) with normal roof width belonging to the largest height category, hence \(H_{v_1}, H_{v_2} \in \{H_{3a}, H_{4a}\}\). Let the sum of both vehicle weights satisfy the weight limit for a single loading deck (constraints (4)). Assigning \(v_1\) to the largest position and \(v_2\) to the second largest position as a fraction represents a feasible solution: setting \(x_{v_1,1,1,1} = 1\) and \(x_{v_2,1,1,1} = H_{20}/H_{v_2} > 0\) is feasible for height constraints (3) of the four-index formulation. Each variable transformation must satisfy \(y_{v_1} = \sum_{p, e} x_{v_1, p, e}\) for all \(v \in V\), where \(y_{v_1}\) denote the variables of the three-index formulation. Both vehicles belong to the largest height category \(v_1, v_2 \in H_4\) which implies \(\sum_{v \in S_{v_1}} C_v \sum_{p, e} x_{v_1, p, e} = y_{v_1}\). Hence, no variable transformation fulfills constraints (9) of the three-index formulation.

On the other hand, let \(y_{v_1} \in [0, 1]\) for all \(v \in V, l \in L, e \in E\) denote a feasible solution for the LP-relaxation of the model MTP-3Idx. Let \(S_{v_1} = \{v \in H_l | y_{v_1} > 0\}\) denote the subset of vehicles belonging to height category \(h \in H\) which are (partially) assigned to wagon \(i \in I\) and loading deck \(e \in E\). Further, let \(F_{v_1} = \{p \in P | v \in \{0, 1\} : T_{e,g,v} > 0, g \in \mathbb{N}\} \subset P\) denote the subset of positions on deck \(e\) offering enough space for a vehicle from height category \(h \in H\). Applying the following transformation generates a feasible solution for the LP-relaxation of the model MTP-4Idx. Let \(i \in I\) and \(e \in E\) be fixed. In the following, we will use the sets \(D_p \subset V\) for \(p \in P\) to indicate the vehicles which are (partially) assigned to position \(p\). Additionally, the variables \(r_{vp} \in [0, 1]\) are used to determine the fraction of vehicle \(v\) being assigned to position \(p\). Initialize \(D_p = \emptyset, r_{vp} = 0\) and \(x_{vp} = 0\) for all \(v \in V\) and \(p \in P\). Then, set \(h^* = \max \{h | h \in H\}\) and apply the following procedure: 1. Select \(v \in S_{v_1}^e\).
2. Remove \(v\) from \(S_{v_1}^e\).
3. Select \(p \in F_{v_1}^{e}\) with \(\sum_{v \in D_p} C_v r_{vp} < 1\).
We now show that the resulting transformation is feasible. Assume that $\frac{1 - \sum_{v' \in D_p} C_{v'} rv_p}{C_v}$ leads to a feasible solution for the LP-relaxation of the model MTP-4ldx. Update $x_{viep}$.

5. Set $x_{viep} = rvp$ and $y_{vie} = y_{vie} - rvp$.

6. Update $D_p = D_p \cup \{v\}$.

(a) If $y_{vie} > 0$: go to step 3.

(b) If $y_{vie} = 0$: go to 1.

Constraints (9) assure that no more than $|F^r| = a_{rv} = \sum_{v \in Y_{rv}} y_{rv}$ positions are needed to transform all variables $y_{rv}$ to $x_{viep}$ for all $v \in S_e$ and $p \in F^r$. Constraints (16) guarantee that at most $p$ positions are needed to transform all $y_{rv}$ to $x_{viep}$ for all $v \in S_e$ and $p \in F^r$.

Hence, the procedure terminates with $S_e = \emptyset$. Then, set $h^* = h^* - 1$. While $h^* > 1$ holds, repeat the procedure.

The procedure terminates, after all height categories have been transformed. We now show that the resulting transformation is feasible for the LP-relaxation of the MTP-4ldx:

1. Height constraints (3) are fulfilled as each vehicle $v \in V$ is assigned to positions $p \in F$ which are feasible for its height category: Step 3 of the transformation procedure guarantees that a vehicle $v'$ belonging to height category $h' \in H$ is only assigned to a position $p \in F^r$. By construction $x_{viep} = 0$ holds for all $p \in F^r$. Each position $p \in F^r$ provides at least a maximum height of $H_{i_p} = h$. Therefore, it follows that $H_{i_p} x_{viep} \leq H_{i_p} \leq H_{i_p} \leq \Pi_{i_p}$. For all $p \in F$ and $e \in E$.

2. Let $p \in F$ be fixed. Assume $|D_p| > 1$ and let $v_e \in D_p$ denote the last vehicle being included into set $D_p$. Then, $\sum_{v \in D_p \setminus \{v_e\}} C_{v} y_{vie} \leq 1$ holds per construction and thus if $C_{v_e} y_{vie} > \sum_{v \in D_p \setminus \{v_e\}} C_{v} y_{vie}$.

Otherwise, if $|D_p| \leq 1$ holds, we have

$$\sum_{v \in D_p} C_{v} x_{viep} = \sum_{v \in D_p} C_{v} x_{viep} = \sum_{v \in D_p \setminus \{v_e\}} C_{v} y_{vie} + C_{v_e} \frac{1 - \sum_{v \in D_p \setminus \{v_e\}} C_{v} y_{vie}}{C_{v_e}} = 1.$$

Hence, capacity constraints $\sum_{v \in v} C_{v} x_{viep} \leq 1$ are satisfied for all $p \in F$.

3. Finally, constraints (2) and (4) are fulfilled due to $y_{vie} = \sum_{p \in F} y_{viep}$.

Indices $i$ and $e$ were chosen arbitrarily, hence the transformation leads to a feasible solution for the LP-relaxation of the model MTP-4ldx.  

References