Erratum to: "C. Bentz, M.-C. Costa, F. Roupin. Maximum integer multiflow and minimum multicut problems in two-sided uniform grid graphs [Journal of Discrete Algorithms 5 (2007) 36–54]"

The proof of Lemma 21 is buggy. Indeed, at the end of the proof, we use Lemma 11 on a grid $(\mathcal{G}, \hat{\mathcal{N}})$ that is not two-sided (since both l_1 and (u_m, t_1) lie on t_1).

The proof of this lemma can be corrected by adapting the proof of Lemma 11. However, this is quite tedious. In order to obtain a simpler way to correct it, we first give an alternative proof to Lemma 11. This proof is due to **Guyslain Naves**, to whom we are very grateful.

The idea is to use a "local argument" to prove that, if X is a connected subset violating the cut condition, then there exists a rectangle X' that contains X and also violates this condition. The proof that X reaches both the uppermost and the lowermost lines is unchanged, but the rest of the proof can be modified in order to deal with all the cases ($\lambda > 1$ and $\rho < n$, $\lambda > 1$ and $\rho = n$, $\lambda = 1$ and $\rho < n$, $\lambda = 1$ and $\rho = n$) at the same time.

So, assume X violates the cut condition, is connected, and reaches the uppermost and lowermost lines. If X is not a rectangle, then there exist two consecutive lines i and i' (i.e., $i = i' + 1$ or $i' = i + 1$) and two consecutive columns j and j' (i.e., $j = j' + 1$ or $j' = j + 1$) such that $g(i, j)$, $g(i', j') \in X$ and $g(i', j) \notin X$ (where $g(i, j)$ is the vertex of the grid lying on line i and column j). The idea is to prove that we can add $g(i', j)$ to X and still have a set violating the cut condition. (This will allow us to conclude that X can be transformed into a rectangle, and the rest of the proof will be the same as in our paper.) Let $X^+ = X \cup \{g(i', j)\}$.

- If $g(i', j)$ does not lie on one of the four border lines of the grid, then $U(\delta_G(X^+)) = U(\delta_G(X))$ and $D(\delta_{\mathcal{N}}(X^+)) = D(\delta_{\mathcal{N}}(X)) \Rightarrow$ Ok.
- If $g(i', j)$ lies on one of the two vertical border lines of the grid (but not on a corner), then $U(\delta_G(X^+)) = U(\delta_G(X)) - U_h -$capacity of a vertical edge e_1 + capacity of another vertical edge $e_2 \leq U(\delta_G(X))$ (because, from (d), either $U_h = U_v$ or all the vertical edges are valued by U_v) and $D(\delta_{\mathcal{N}}(X^+)) = D(\delta_{\mathcal{N}}(X)) \Rightarrow$ Ok.

- If $g(i', j)$ lies on one of the two horizontal border lines of the grid (but not on a corner), then $U(\delta_G(X^+)) = U(\delta_G(X)) - U_h - U_v + U_h = U(\delta_G(X)) - U_v$ and $D(\delta_{\mathcal{N}}(X^+)) \geq D(\delta_{\mathcal{N}}(X))$ − a demand $\geq D(\delta_{\mathcal{N}}(X)) - U_v \Rightarrow$ Ok.

- If $g(i', j)$ lies on a corner, then $U(\delta_G(X^+)) = U(\delta_G(X)) - U_h -$ capacity of a vertical edge e_1 and $D(\delta_{\mathcal{N}}(X^+)) \geq D(\delta_{\mathcal{N}}(X))$ − a single demand $\geq D(\delta_{\mathcal{N}}(X)) - U_v \Rightarrow$ Ok (because, from (d), either $U_h = U_v$ or all the vertical edges are valued by U_v).

This concludes the alternative proof of Lemma 11. Now, let us show how to adapt this proof in order to prove Lemma 21.

We have $U(e) = c$ for each horizontal edge e, except the ones on the lowermost line that lie on the left of t_1 (their capacity is $c - 1$). We have $U(e) = c$ for each vertical edge that lies neither on the leftmost column nor on the rightmost column. Eventually, we have $U(u_{2i}, u_{2i+1}) = c - 1$ for each $i \in \{1, \ldots, \frac{m}{2} - 1\}$, and $U(u'_{2i-1}, u'_{2i}) = c - 1$ for each $i \in \{1, \ldots, \frac{m}{2}\}$. The capacity of any other vertical edge lying on the leftmost and rightmost columns is c. We also have $D(l_k) = c$ for each net $l_k \neq l_1$ and $D(l_1) = c - 1$.

We want to prove that, if there exists in this grid a set violating the cut condition, then there exists a vertical strip that also violates this condition. (Since obviously there exists no such vertical strip, this will prove Lemma 21.) So, we define X, i' and j as previously, and follow the alternative proof of Lemma 11. We first have to prove that X reaches both the lowermost and the uppermost lines. Assume this is not the case:

- If X reaches neither the lowermost nor the uppermost line, then we have $\delta_{\mathcal{N}}(X) = \emptyset \Rightarrow U(\delta_G(X)) \geq 0 = D(\delta_{\mathcal{N}}(X))$.

- If $\lambda > 1$ or $\rho < n$ (or if $\lambda = 1$ and $\rho = n$ and $\delta_G(X)$ contains at least one horizontal edge), then X contains at most $\rho - \lambda + 1$ vertical edges and one horizontal edge, whereas $\delta_{\mathcal{N}}(X)$ contains at most $\rho - \lambda + 1$ (demand) edges. We have $U(\delta_G(X)) \geq (\rho - \lambda + 1)c - 2 + (c - 1) \geq (\rho - \lambda + 1)c$ (since $c \geq 3$) and $D(\delta_{\mathcal{N}}(X)) \leq (\rho - \lambda + 1)c$. Hence, X does not violate the cut condition.

- Otherwise, $\delta_G(X)$ is a horizontal strip, and $U(\delta_G(X)) = D(\delta_{\mathcal{N}}(X)) = Kc - 1$. Hence, X does not violate the cut condition.

We get a contradiction. Therefore, X reaches both the lowermost and the uppermost lines (and can be assumed to be connected). Now:

- If $g(i', j)$ does not lie on one of the four border lines of the grid, then the proof is the same as in Lemma 11.

- If $g(i', j)$ lies on one of the two vertical border lines of the grid (but not on a corner), then $U(\delta_G(X^+)) = U(\delta_G(X))$ − capacity of a horizontal edge e_1 − capacity of a vertical edge e_2 + capacity of another vertical edge $e_3 \leq U(\delta_G(X)) - (c-1) - (c-1) + c < U(\delta_G(X))$ (since $c > 2$) and $D(\delta_{\mathcal{N}}(X^+)) = D(\delta_{\mathcal{N}}(X)) \Rightarrow$ Ok.

- If $g(i', j)$ lies on one of the two horizontal border lines of the grid (but not on a corner), then we have three cases. If $i' = 1$, then the proof is similar to the one of Lemma 11. If $i' = n$ and $j < j'$, then $U(\delta_G(X^+)) \leq U(\delta_G(X)) - c[-c + c]$(or $[-(c-1) + (c-1)]$) $\leq U(\delta_G(X)) - c$ and $D(\delta_{\mathcal{N}}(X^+)) \geq D(\delta_{\mathcal{N}}(X))-$a demand $\geq D(\delta_{\mathcal{N}}(X)) - c \Rightarrow$ Ok. If $i' = n$ and $j' < j$, then either $U(\delta_G(X^+)) \leq U(\delta_G(X)) - c - (c-1) + c \leq U(\delta_G(X)) - (c-1)$ and $D(\delta_{\mathcal{N}}(X^+)) \geq D(\delta_{\mathcal{N}}(X)) - D(l_1) \geq D(\delta_{\mathcal{N}}(X)) - (c-1)$ (if t_1 lies on j), or $U(\delta_G(X^+)) \leq U(\delta_G(X)) - c[-(c-1) + (c-1)]$(or $[-c + c]$) $\leq U(\delta_G(X)) - c$ and $D(\delta_{\mathcal{N}}(X^+)) \geq D(\delta_{\mathcal{N}}(X)) - c$ (otherwise) $\Rightarrow$ Ok.

- If $g(i', j)$ lies on a corner of the grid, then either $U(\delta_G(X^+)) = U(\delta_G(X)) - c - (c-1) < U(\delta_G(X)) - c$ and $D(\delta_{\mathcal{N}}(X^+)) \geq D(\delta_{\mathcal{N}}(X)) - c$ (if $t_1 \neq u_m^{'}$), or $U(\delta_G(X^+)) = U(\delta_G(X)) - (c-1) - (c-1) < U(\delta_G(X)) - (c-1)$ and $D(\delta_{\mathcal{N}}(X^+)) \geq D(\delta_{\mathcal{N}}(X)) - (c-1)$ (if $t_1 = u_m^{'}$) $\Rightarrow$ Ok.