Apprentissage, réseaux de neurones et modèles graphiques
(RCP209)
Neural Networks and Deep Learning

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8 Weeks on Deep Learning and Structured Prediction

- Week 1-5: Deep Learning
- Week 6-8: Structured Prediction and Applications
Outline

1. Context
2. Neural Networks
3. Training Deep Neural Networks
Big Data

- Superabundance of data: images, videos, audio, text, use traces, etc

  BBC: 2.4M videos
  Facebook: 350B images
  100M monitoring cameras
  1B each day

- Obvious need to access, search, or classify these data: **Recognition**

- Huge number of applications: mobile visual search, robotics, autonomous driving, augmented reality, medical imaging etc

- Leading track in major ML/CV conferences during the last decade
Recognition and classification

- Classification: assign a given data to a given set of predefined classes
- Recognition much more general than classification, e.g.
  - Object Localization in images
  - Ranking for document indexing
  - Sequence prediction for text, speech, audio, etc
- Many tasks can be cast as classification problems
  $\Rightarrow$ importance of classification
Focus on Visual Recognition: Perceiving Visual World

- Visual Recognition: archetype of low-level signal understanding
- Supposed to be a master class problem in the early 80’s
- Certainly the most impacted topic by deep learning

- Scene categorization
- Object localization
- Context & Attribute recognition
- Rough 3D layout, depth ordering
- Rich description of scene, e.g. sentences
Recognition of low-level signals

Challenge: filling the semantic gap

What we perceive vs What a computer sees

• Illumination variations
• View-point variations
• Deformable objects
• intra-class variance
• etc

⇒ How to design "good" intermediate representation?
Deep Learning (DL) & Recognition of low-level signals

- DL: breakthrough for the recognition of low-level signal data
- Before DL: handcrafted intermediate representations for each task
  - Needs expertise (PhD level) in each field
  - Weak level of semantics in the representation

VISION

```
<image of car>
```

SIFT/HOG → K-Means/pooling → classifier → “car”

fixed → unsupervised → supervised

SPEECH

```
<waveform image>
```

MFCC → Mixture of Gaussians → classifier → ‘d e p’

fixed → unsupervised → supervised

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Deep Learning (DL) & Recognition of low-level signals

- DL: breakthrough for the recognition of low-level signal data
- Since DL: automatically **learning intermediate representations**
  - Outstanding experimental performances >> handcrafted features
  - Able to learn high level intermediate representations
  - Common learning methodology ⇒ field independent, no expertise

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Deep Learning (DL) & Representation Learning

- DL: breakthrough for representation learning
  - Automatically learning intermediate levels of representation

- Ex: Natural language Processing (NLP)

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@Socher
Outline

1. Context
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The Formal Neuron: 1943 [MP43]

- Basis of Neural Networks
- Input: vector $x \in \mathbb{R}^m$, i.e. $x = \{x_i\}_{i \in \{1,2,\ldots,m\}}$
- Neuron output $\hat{y} \in \mathbb{R}$: scalar
The Formal Neuron: 1943 [MP43]

- Mapping from $x$ to $\hat{y}$:
  1. Linear (affine) mapping: $s = w^T x + b$
  2. Non-linear activation function: $f: \hat{y} = f(s)$
The Formal Neuron: Linear Mapping

- Linear (affine) mapping: \( s = w^T x + b = \sum_{i=1}^{m} w_i x_i + b \)
  - \( w \): normal vector to an hyperplane in \( \mathbb{R}^m \) ⇒ linear boundary
  - \( b \): bias, shift the hyperplane position

2D hyperplane: line

\[ w^T x + b = 0 \]

3D hyperplane: plane
The Formal Neuron: Activation Function

- $\hat{y} = f(\mathbf{w}^T \mathbf{x} + b)$, \( f \) activation function
  - Popular \( f \) choices: step, sigmoid, tanh

- Step (Heaviside) function: 
  
  \[
  H(z) = \begin{cases} 
  1 & \text{if } z \geq 0 \\
  0 & \text{otherwise} 
  \end{cases}
  \]
Step function: Connection to Biological Neurons

- **Formal neuron, step activation** $H$: $\hat{y} = H(w^T x + b)$
  - $\hat{y} = 1$ (activated) $\iff w^T x \geq -b$
  - $\hat{y} = 0$ (unactivated) $\iff w^T x < -b$

- **Biological Neurons**: output activated
  $\iff$ input weighted by synaptic weight $\geq$ threshold

$\hat{y}$
**Sigmoid Activation Function**

- Neuron output $\hat{y} = f(w^T x + b)$, $f$ activation function
- Sigmoid: $\sigma(z) = (1 + e^{-az})^{-1}$

- $a \uparrow$: more similar to step function (step: $a \to \infty$)
- Sigmoid: linear and saturating regimes
The Formal neuron: Application to Binary Classification

- Binary Classification: label input $x$ as belonging to class 1 or 0
- Neuron output with sigmoid: $\hat{y} = \frac{1}{1 + e^{-a(w^\top x + b)}}$
- Sigmoid: probabilistic interpretation $\Rightarrow \hat{y} \sim P(1/x)$
  - Input $x$ classified as 1 if $P(1/x) > 0.5 \iff w^\top x + b > 0$
  - Input $x$ classified as 0 if $P(1/x) < 0.5 \iff w^\top x + b < 0$
  $\Rightarrow \text{sign}(w^\top x + b)$: linear boundary decision in input space!
The Formal neuron: Toy Example for Binary Classification

- 2d example: \( m = 2, \ x = \{x_1, x_2\} \in [-5; 5] \times [-5; 5] \)
- Linear mapping: \( w = [1; 1] \) and \( b = -2 \)
- Result of linear mapping: \( s = w^T x + b \)
The Formal neuron: Toy Example for Binary Classification

- 2d example: $m = 2$, $x = \{x_1, x_2\} \in [-5; 5] \times [-5; 5]$
- Linear mapping: $w = [1; 1]$ and $b = -2$
- Result of linear mapping: $s = w^T x + b$
- Sigmoid activation function: $\hat{y} = \left(1 + e^{-a(w^T x + b)}\right)^{-1}$, $a = 10$
The Formal neuron: Toy Example for Binary Classification

- 2d example: \( m = 2, \ x = \{x_1, x_2\} \in [-5; 5] \times [-5; 5] \)
- Linear mapping: \( w = [1; 1] \) and \( b = -2 \)
- Result of linear mapping: \( s = w^T x + b \)
- Sigmoid activation function: \( \hat{y} = \left(1 + e^{-a(w^T x + b)}\right)^{-1}, \ a = 1 \)
The Formal neuron: Toy Example for Binary Classification

- 2d example: $m = 2$, $x = \{x_1, x_2\} \in [-5; 5] \times [-5; 5]
- Linear mapping: $w = [1; 1]$ and $b = -2$
- Result of linear mapping: $s = w^T x + b$
- Sigmoid activation function: $\hat{y} = \left(1 + e^{-a(w^T x + b)}\right)^{-1}$, $a = 0.1$
From Formal Neuron to Neural Networks

- **Formal Neuron:**
  1. A single scalar output
  2. Linear decision boundary for binary classification

- **Single scalar output:** limited for several tasks
  - Ex: multi-class classification, e.g. MNIST or CIFAR

![Diagram of neural network with inputs, weights, summation and bias, activation, and output.](image-url)
Perceptron and Multi-Class Classification

- **Formal Neuron**: limited to binary classification

- **Multi-Class Classification**: use several output neurons instead of a single one! ⇒ *Perceptron*

- Input $x$ in $\mathbb{R}^m$

- Output neuron $\hat{y}_1$ is a formal neuron:
  - Linear (affine) mapping: $s_1 = w_1^T x + b_1$
  - Non-linear activation function: $f$: $\hat{y}_1 = f(s_1)$

- Linear mapping parameters:
  - $w_1 = \{w_{11}, ..., w_{m1}\} \in \mathbb{R}^m$
  - $b_1 \in \mathbb{R}$
Perceptron and Multi-Class Classification

- **Input** $x$ in $\mathbb{R}^m$
- **Output neuron** $\hat{y}_k$ is a formal neuron:
  - Linear (affine) mapping: $s_k = w_k^T x + b_k$
  - Non-linear activation function: $f$: $\hat{y}_k = f(s_k)$
- **Linear mapping parameters:**
  - $w_k = \{w_{1k}, \ldots, w_{mk}\} \in \mathbb{R}^m$
  - $b_k \in \mathbb{R}$

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Perceptron and Multi-Class Classification

- Input $x$ in $\mathbb{R}^m (1 \times m)$, output $\hat{y}$: concatenation of $K$ formal neurons
- Linear (affine) mapping $\sim$ matrix multiplication: $s = xW + b$
  - $W$ matrix of size $m \times K$ - columns are $w_k$
  - $b$: bias vector - size $1 \times K$
- Element-wise non-linear activation: $\hat{y} = f(s)$
Perceptron and Multi-Class Classification

- **Soft-max Activation:**
  \[
  \hat{y}_k = f(s_k) = \frac{e^{s_k}}{\sum_{k'=1}^{K} e^{s_{k'}}}
  \]

- **Probabilistic interpretation for multi-class classification:**
  - Each output neuron \( \Leftrightarrow \) class
  - \( \hat{y}_k \sim P(k|x, w) \)

  \( \Rightarrow \) Logistic Regression (LR) Model!
2d Toy Example for Multi-Class Classification

- \( x = \{x_1, x_2\} \in [-5; 5] \times [-5; 5] \), \( \hat{y} \): 3 outputs (classes)

Linear mapping for each class:
\[ s_k = w_k^\top x + b_k \]

Soft-max output:
\[ P(k/x, W) \]
2d Toy Example for Multi-Class Classification

- \( x = \{x_1, x_2\} \in [-5; 5] \times [-5; 5], \hat{y}: 3 \) outputs (classes)

Soft-max output:
\[ P(k/x, W) \]

Class Prediction:
\[ k^* = \arg \max_k P(k/x, W) \]
Beyond Linear Classification

X-OR Problem

- Logistic Regression (LR): NN with 1 input layer & 1 output layer
- LR: limited to linear decision boundaries
- X-OR: NOT 1 and 2 OR NOT 2 AND 1
  - X-OR: Non linear decision function
Beyond Linear Classification

- LR: limited to linear boundaries
- **Solution**: add a layer!

- Input $x$ in $\mathbb{R}^m$, e.g. $m = 4$
- Output $\hat{y}$ in $\mathbb{R}^K$ ($K$ # classes), e.g. $K = 2$
- **Hidden layer** $h$ in $\mathbb{R}^L$
Multi-Layer Perceptron

- **Hidden layer** $h$: $x$ projection to a new space $\mathbb{R}^L$
- Neural Net with $\geq 1$ hidden layer: Multi-Layer Perceptron (MLP)
- $h$: intermediate representations of $x$ for classification $\hat{y}$: $h = f(xW + b)$
- **Mapping from** $x$ **to** $\hat{y}$: non-linear boundary! $\Rightarrow$ activation $f$ crucial!
Deep Neural Networks

- Adding more hidden layers: Deep Neural Networks (DNN) ⇒ Basis of Deep Learning
- Each layer $h^l$ projects layer $h^{l-1}$ into a new space
- Gradually learning intermediate representations useful for the task
• Deep Neural Networks: applicable to classification problems with non-linear decision boundaries

• Visualize prediction from fixed model parameters

• Reverse problem: **Supervised Learning**
Outline

1. Context
2. Neural Networks
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Training Multi-Layer Perceptron (MLP)

- Input $x$, output $y$
- A parametrized model $x \Rightarrow y$: $f_w(x_i) = \hat{y}_i$
- Supervised context: training set $\mathcal{A} = \{(x_i, y_i^*)\}_{i \in \{1, 2, \ldots, N\}}$
  - A loss function $\ell(\hat{y}_i, y_i^*)$ for each annotated pair $(x_i, y_i^*)$
- Assumptions: parameters $w \in \mathbb{R}^d$ continuous, $\mathcal{L}$ differentiable
- Gradient $\nabla_w = \frac{\partial \mathcal{L}}{\partial w}$: steepest direction to decrease loss $\mathcal{L}$
MLP Training

- **Gradient descent algorithm:**
  - Initialize parameters $\mathbf{w}$
  - Update: $\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} - \eta \frac{\partial \mathcal{L}}{\partial \mathbf{w}}$
  - Until convergence, e.g. $\|\nabla \mathcal{L}\|^2 \approx 0$
Gradient Descent

Update rule: \[ w^{(t+1)} = w^{(t)} - \eta \frac{\partial L}{\partial w} \] \( \eta \) learning rate

- Convergence ensured ? \( \Rightarrow \) provided a "well chosen" learning rate \( \eta \)
Gradient Descent

Update rule: \[ w^{(t+1)} = w^{(t)} - \eta \frac{\partial L}{\partial w} \]

- **Global minimum?**
  \[ \Rightarrow \text{convex a) vs non convex b) loss } L(w) \]

\[ L(w) \]

- **Convex function**
  - Initial weight
  - Gradient
  - Global cost minimum

- **Non convex function**
  - Starting pt.
  - Local minima
  - Global minima
Supervised Learning: Multi-Class Classification

- Logistic Regression for multi-class classification
- \( s_i = x_i W + b \)
- Soft-Max (SM): \( \hat{y}_k \sim P(k/x_i, W, b) = \frac{e^{s_k}}{\sum_{k'=1}^{K} e^{s_k'}} \)
- Supervised loss function: \( \mathcal{L}(W, b) = \frac{1}{N} \sum_{i=1}^{N} \ell(\hat{y}_i, y^*_i) \)

1. \( y \in \{1; 2; \ldots; K\} \)
2. \( \hat{y}_i = \arg \max_k P(k/x_i, W, b) \)
3. \( \ell_{0/1}(\hat{y}_i, y^*_i) = \begin{cases} 1 & \text{if } \hat{y}_i \neq y^*_i \\ 0 & \text{otherwise} \end{cases} : 0/1 \text{ loss} \)
Logistic Regression Training Formulation

- Input $x_i$, ground truth output supervision $y_i^*$
- One hot-encoding for $y_i^*$:

$$y_{c,i}^* = \begin{cases} 
1 & \text{if } c \text{ is the ground truth class for } x_i \\
0 & \text{otherwise}
\end{cases}$$
Logistic Regression Training Formulation

- Loss function: multi-class Cross-Entropy (CE) $\ell_{CE}$
- $\ell_{CE}$: Kullback-Leibler divergence between $y_i^*$ and $\hat{y}_i$

$$\ell_{CE}(\hat{y}_i, y_i^*) = KL(y_i^*, \hat{y}_i) = - \sum_{c=1}^{K} y_{c,i}^* \log(\hat{y}_{c,i}) = -\log(\hat{y}_{c^*,i})$$

- △ KL asymmetric: $KL(\hat{y}_i, y_i^*) \neq KL(y_i^*, \hat{y}_i)$

$y_i^* = \begin{bmatrix} 1.0 \\ 0.0 \\ 0.0 \end{bmatrix}$ $\hat{y}_i = \begin{bmatrix} 0.80 \\ 0.15 \\ 0.05 \end{bmatrix}$

$$KL(y_i^*, \hat{y}_i) = -\log(\hat{y}_{c^*,i}) = -\log(0.8) \approx 0.22$$
Logistic Regression Training

- $\mathcal{L}_{CE}(W, b) = \frac{1}{N} \sum_{i=1}^{N} \ell_{CE}(\hat{y}_i, y^*_i) = -\frac{1}{N} \sum_{i=1}^{N} \log(\hat{y}_{c^*,i})$

- $\ell_{CE}$ smooth convex upper bound of $\ell_{0/1}$
  $\Rightarrow$ gradient descent optimization

- Gradient descent: $W^{(t+1)} = W^{(t)} - \eta \frac{\partial \mathcal{L}_{CE}}{\partial W}$  
  $b^{(t+1)} = b^{(t)} - \eta \frac{\partial \mathcal{L}_{CE}}{\partial b}$

- **MAIN CHALLENGE:** computing $\frac{\partial \mathcal{L}_{CE}}{\partial W} = \frac{1}{N} \sum_{i=1}^{N} \frac{\partial \ell_{CE}}{\partial W}$ ?

  $\Rightarrow$ **Key Property:** chain rule $\frac{\partial x}{\partial z} = \frac{\partial x}{\partial y} \frac{\partial y}{\partial z}$

  $\Rightarrow$ Backpropagation of gradient error!
Chain Rule

\[
\frac{\partial l}{\partial x} = \frac{\partial l}{\partial y} \frac{\partial y}{\partial x}
\]

- Logistic regression:

\[
\frac{\partial CE}{\partial W} = \frac{\partial CE}{\partial \hat{y}_i} \frac{\partial \hat{y}_i}{\partial s_i} \frac{\partial s_i}{\partial W}
\]
Logistic Regression Training: Backpropagation

\[
\frac{\partial \ell_{CE}}{\partial W} = \frac{\partial \ell_{CE}}{\partial \hat{y}_i} \frac{\partial \hat{y}_i}{\partial s_i} \frac{\partial s_i}{\partial W}, \quad \ell_{CE}(\hat{y}_i, y_i^\star) = -\log(\hat{y}_{c^\star,i}) \Rightarrow \text{Update for 1 example:}
\]

- \[
\frac{\partial \ell_{CE}}{\partial \hat{y}_i} = -\frac{1}{\hat{y}_{c^\star,i}} = -\frac{1}{\hat{y}_i} \odot \delta_{c,c^\star}
\]
- \[
\frac{\partial \ell_{CE}}{\partial s_i} = \hat{y}_i - y_i^\star = \delta_i^y
\]
- \[
\frac{\partial \ell_{CE}}{\partial W} = x_i^T \delta_i^y
\]
Logistic Regression Training: Backpropagation

- Whole dataset: data matrix $X \ (N \times m)$, label matrix $\hat{Y}, \ Y^* \ (N \times K)$
- $L_{CE}(W, b) = -\frac{1}{N} \sum_{i=1}^{N} \log(\hat{y}_{c^*,i}), \ \frac{\partial L_{CE}}{\partial W} = \frac{\partial L_{CE}}{\partial \hat{Y}} \frac{\partial \hat{Y}}{\partial S} \frac{\partial S}{\partial W}$

$X \ (N,m)$ \hspace{1cm} $S \ (N,K)$ \hspace{1cm} $\hat{Y} \ (N,K)$ \hspace{1cm} $L_{CE}(\hat{Y},Y^*)$

- $\frac{\partial L_{CE}}{\partial S} = \hat{Y} - Y^* = \Delta^y$
- $\frac{\partial L_{CE}}{\partial W} = X^T \Delta^y$
Perceptron Training: Backpropagation

- Perceptron vs Logistic Regression: adding hidden layer (sigmoid)
- **Goal:** Train parameters $W^y$ and $W^h$ (+bias) with Backpropagation
  
  \[ \frac{\partial L_{CE}}{\partial W^y} = \frac{1}{N} \sum_{i=1}^{N} \frac{\partial \ell_{CE}}{\partial W^y} \quad \text{and} \quad \frac{\partial L_{CE}}{\partial W^h} = \frac{1}{N} \sum_{i=1}^{N} \frac{\partial \ell_{CE}}{\partial W^h} \]

- Last hidden layer $\sim$ Logistic Regression
- First hidden layer: 
  \[ \frac{\partial \ell_{CE}}{\partial W^h} = x_i^T \frac{\partial \ell_{CE}}{\partial u_i} \implies \text{computing} \quad \frac{\partial \ell_{CE}}{\partial u_i} = \delta_i^h \]
Neural Nets

Perceptron Training: Backpropagation

- Computing $\frac{\partial \ell_{CE}}{\partial u_i} = \delta^h_i \Rightarrow$ use chain rule: $\frac{\partial \ell_{CE}}{\partial u_i} = \frac{\partial \ell_{CE}}{\partial v_i} \frac{\partial v_i}{\partial h_i} \frac{\partial h_i}{\partial u_i}$
- ... Leading to: $\frac{\partial \ell_{CE}}{\partial u_i} = \delta^h_i = \delta^y_i W^y \circ \sigma'(h_i) = \delta^y_i W^y \circ (h_i \circ (1 - h_i))$
Deep Neural Network Training: Backpropagation

- Multi-Layer Perceptron (MLP): adding more hidden layers
- Backpropagation update ~ Perceptron: assuming $\frac{\partial L}{\partial u_{i+1}} = \Delta^{i+1}$ known

\[
\frac{\partial L}{\partial W^{i+1}} = H_i^T \Delta^{i+1}
\]

Computing $\frac{\partial L}{\partial U_i} = \Delta^i$ ($= \Delta^{i+1}^T W^{i+1} \odot H_i \odot (1 - H_i)$ sigmoid)

\[
\frac{\partial L}{\partial W^i} = H_{i-1}^T \Delta^{h_i}
\]
Neural Network Training: Optimization Issues

- Classification loss over training set (vectorized $w$, $b$ ignored):
  \[
  \mathcal{L}_{CE}(w) = \frac{1}{N} \sum_{i=1}^{N} \ell_{CE}(\hat{y}_i, y^*_i) = -\frac{1}{N} \sum_{i=1}^{N} \log(\hat{y}_{c^*,i})
  \]

- Gradient descent optimization:
  \[
  w^{(t+1)} = w^{(t)} - \eta \frac{\partial \mathcal{L}_{CE}}{\partial w}(w^{(t)}) = w^{(t)} - \eta \nabla_w^{(t)}
  \]

- Gradient $\nabla_w^{(t)} = \frac{1}{N} \sum_{i=1}^{N} \frac{\partial \ell_{CE}(\hat{y}_i, y^*_i)}{\partial w}(w^{(t)})$ linearly scales wrt:
  - $w$ dimension
  - Training set size

  $\Rightarrow$ Too slow even for moderate dimensionality & dataset size!
**Stochastic Gradient Descent**

- **Solution:** approximate \( \nabla_w^{(t)} = \frac{1}{N} \sum_{i=1}^{N} \frac{\partial \ell_{CE}(\hat{y}_i, y^*_i)}{\partial w} (w^{(t)}) \) with subset of examples

\[ \Rightarrow \text{Stochastic Gradient Descent (SGD)} \]

- Use a single example (online):

\[
\nabla_w^{(t)} \approx \frac{\partial \ell_{CE}(\hat{y}_i, y^*_i)}{\partial w} (w^{(t)})
\]

- Mini-batch: use \( B < N \) examples:

\[
\nabla_w^{(t)} \approx \frac{1}{B} \sum_{i=1}^{B} \frac{\partial \ell_{CE}(\hat{y}_i, y^*_i)}{\partial w} (w^{(t)})
\]
Stochastic Gradient Descent

- **SGD**: approximation of the true Gradient $\nabla_w$!
  - Noisy gradient can lead to bad direction, increase loss
  - **BUT**: much more parameter updates: online $\times N$, mini-batch $\times \frac{N}{B}$
  - **Faster convergence**, at the core of Deep Learning for large scale datasets
Optimization: Learning Rate Decay

- Gradient descent optimization: \( w^{(t+1)} = w^{(t)} - \eta \nabla w^{(t)} \)
- \( \eta \) setup \( \Rightarrow \) open question
- Learning Rate Decay: decrease \( \eta \) during training progress
  - Inverse (time-based) decay: \( \eta_t = \frac{\eta_0}{1 + rt} \), \( r \) decay rate
  - Exponential decay: \( \eta_t = \eta_0 \cdot e^{-\lambda t} \)
  - Step Decay \( \eta_t = \eta_0 \cdot r^{\frac{t}{t_u}} \ldots \)

Exponential Decay \((\eta_0 = 0.1, \lambda = 0.1s)\)  Step Decay \((\eta_0 = 0.1, r = 0.5, t_u = 10)\)
Generalization and Overfitting

- **Learning**: minimizing classification loss $\mathcal{L}_{CE}$ over training set
  - Training set: sample representing data vs labels distributions
  - **Ultimate goal**: train a prediction function with low prediction error on the true (unknown) data distribution

$$\mathcal{L}_{train} = 4, \quad \mathcal{L}_{train} = 9$$

$$\mathcal{L}_{test} = 15, \quad \mathcal{L}_{test} = 13$$

⇒ **Optimization ≠ Machine Learning!**
⇒ **Generalization / Overfitting!**
Regularization

- **Regularization**: improving generalization, *i.e.* test (≠ train) performances
- Structural regularization: add *Prior* $R(w)$ in training objective:
  \[ \mathcal{L}(w) = \mathcal{L}_{CE}(w) + \alpha R(w) \]

- $L^2$ regularization: weight decay, $R(w) = ||w||^2$
  - Commonly used in neural networks
  - Theoretical justifications, generalization bounds (SVM)

- Other possible $R(w)$: $L^1$ regularization, dropout, *etc*
Regularization and hyper-parameters

- **Neural networks**: hyper-parameters to tune:
  - **Training parameters**: learning rate, weight decay, learning rate decay, # epochs, etc
  - **Architectural parameters**: number of layers, number neurones, non-linearity type, etc

- **Hyper-parameters tuning**: ⇒ improve generalization: estimate performances on a validation set

![Graph showing error vs. number of epochs for training and validation sets](chart.png)
Neural networks: Conclusion

- Training issues at several levels: optimization, generalization, cross-validation
- Limits of fully connected layers and Convolutional Neural Nets? ⇒ next course!
Warren S McCulloch and Walter Pitts, *A logical calculus of the ideas immanent in nervous activity*, The bulletin of mathematical biophysics 5 (1943), no. 4, 115–133.