Formal Methods for Critical Systems:
A verified implementation of nested procedures*

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Joint work with:

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Julia Lawall\textsuperscript{3}

1. CNAM / Cedric / CPR team
2. INRIA / Gallium team
3. UPMC / LIP6 / Whisper team

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Formal Methods for Critical Systems
Formal Methods for Critical Systems

based on a mathematical formalism
Formal Methods for Critical Systems

Based on a mathematical formalism

Life-critical or safety-critical

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Formal Methods for Critical Systems

- based on a mathematical formalism
- life-critical or safety-critical
- embedded systems
Formal Methods for Critical Systems

- based on a mathematical formalism
- life-critical or safety-critical
- embedded systems

Formal methods are about:
- formal specifications
- mathematical proofs of properties
Formal Methods for Critical Systems

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- formal specifications
- mathematical proofs of properties

Based on a mathematical formalism
Life-critical or safety-critical
Embedded systems

Machine-checked
Machine-checked mathematical proofs

You might want to prove:

- some safety and security properties of your system
- the full correctness of your implementation with respect to its specification
- only the partial correctness of your implementation (no buffer overflow, for instance)

In any case, you need a **formal specification** of your system.
Machine-checked mathematical proofs

You might want to prove:

- some safety and security properties of your system
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In any case, you need a formal specification of your system.

Of course, testing is still allowed and a formal specification is also required in this case (when mixing tests and proofs).
Formal Methods: logics and tools

**First-order logics**

- partial correctness

**Higher-order logics**

- full correctness

**Specialized logics**

- specific properties
Formal Methods: logics and tools

- **Higher-order logics**
  - Expressive
  - Full correctness
  - Proof assistants

- **First-order logics**
  - Decidable
  - Partial correctness
  - Provers and solvers

- **Specialized logics**
  - Automatic
  - Specific properties
  - Model checkers

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Formal Methods: logics and tools

- **Higher-order logics**
  - full correctness
  - Proof assistants

- **First-order logics**
  - partial correctness
  - Provers and solvers

- **Specialized logics**
  - specific properties
  - Model checkers

- **Program logics**
Limits of formal methods

“The correspondence between our formal models of programs and the actual behavior of real systems is limited by three factors:

- the behavior of the programming language,
- the operating system,
- and the underlying hardware.

For safety-critical systems, these limitations are crucially important and we cannot assume that a program is correct just because it has been proved.”

Seven Myths of Formal Methods
Anthony Hall, Praxis Sytems, September 1990
Two success stories about formal methods

- **The seL4 project** developed at NICTA (SSRG).
  - seL4 is a formally-verified microkernel
  - Developed since 2006.
  - First public release in 2011 (open source since 2014).

- **The CompCert project** developed at INRIA (Gallium team).
  - CompCert is a formally-verified C compiler
  - Developed since 2004.
The seL4 project

seL4 is a high-performance general-purpose microkernel, that provides address spaces, threads, IPC and authorisation capabilities

- Formal proof of correctness down to binary level
- Developed for ARM and Intel processors
- The fastest existing microkernel (faster than L4)
- 10,000 lines of code
- 200,000 lines of proof
- About 30 person.years
The CompCert project

A formally-verified optimizing standard C compiler

- Formal proof of correctness down to binary level
- Developed for PowerPC, ARM and Intel processors
- Generated code only 20% slower than gcc -O2
- 15,000 lines of code
- 100,000 lines of proof
- About 6 person.years
Proof Architecture

Specification

\textit{correctness}

Implementation
Proof Architecture

Specification

Prototype

Implementation

\textit{correctness}

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Proof Architecture

Specification

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Implementation

proof assistant
(Isabelle/HOL, Coq, ...)

correctness

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“pure” language
(Haskell, pure ML, pure Prolog, ...)

correctness

correctness
Proof Architecture

Specification

proof assistant (Isabelle/HOL, Coq, ...)

Prototype

“pure” language (Haskell, pure ML, pure Prolog, ...)

Implementation

mainstream language (C, Ada, ...)

correctness

correctness
Proof Architecture: seL4

- Specification
  \[\text{correctness}\]
  \[\text{Prototype}\]
  \[\text{correctness}\]
  Implementation
Proof Architecture: seL4

- Specification
  - correctness
  - Proof assistant: Isabelle/HOL
- Prototype
  - correctness
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Proof Architecture: seL4

**Specification**

correctness

**Prototype**

"pure" language: Haskell

**Implementation**

proof assistant: Isabelle/HOL

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Proof Architecture: seL4

- **Specification**
  - Proof assistant: Isabelle/HOL

- **Prototype**
  - "pure" language: Haskell

- **Implementation**
  - Mainstream language: C (compiled with gcc)

Correctness connections:
- Specification → Prototype
- Prototype → Implementation
Proof Architecture: seL4

- **Specification**
  - proof assistant: Isabelle/HOL
  - correctness

- **Prototype**
  - "pure" language: Haskell
  - generation

- **Implementation**
  - mainstream language: C (compiled with gcc)
  - correctness
Proof Architecture: CompCert

```
<table>
<thead>
<tr>
<th>Specification</th>
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<tbody>
<tr>
<td>correctness</td>
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<tr>
<td>Prototype</td>
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Proof Architecture: CompCert

- Specification
  - correctness
  - Prototype
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proof assistant: Coq
Proof Architecture: CompCert

Specification

Prototype

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“pure” language: pure ML (OCaml)

correctness

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- **Specification**
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- **Implementation**
  - mainstream language: OCaml (native compiler)

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Proof Architecture: CompCert

- Specification
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  - "pure" language: pure ML (OCaml)

- Implementation
  - mainstream language: OCaml (native compiler)
How to prove the correctness of a compiler?

- A compiler translates a source program into a target program.
- The translation is correct if the target program has the same behaviour as the source program.
- Formally, we need some mathematical abstraction of the behaviour (a semantics).
How to prove the correctness of a compiler?

- Let us write \( p \sim p' \) when \( p \) and \( p' \) have the same behaviour.
- Let us call \( \ast \) the translation performed by the compiler.

Let us call \( \ast \) the translation performed by the compiler.

**Correctness:** For *any* source program \( p \),

\[
p \sim p^*\]
How to prove the correctness of a compiler?

There are two options:

- For each program $p$, prove $p \sim p^*$
  - *translation validation* approach [Pnuelli 1998]
  - first-order formulas (mostly automatic)
  - works for a regular compiler (for instance gcc)
  - successfully used in the seL4 project

- Prove $\forall p, p \sim p^*$
  - higher-order formula (requires a proof-assistant)
  - successfully used in the CompCert project
“The CompCert project investigates the formal verification of realistic compilers usable for critical embedded software.

- Such verified compilers come with a mathematical, machine-checked proof that the generated executable code behaves exactly as prescribed by the semantics of the source program.

- By ruling out the possibility of compiler-introduced bugs, verified compilers strengthen the guarantees that can be obtained by applying formal methods to source programs.”
Can you trust your C compiler?

“We created a tool that generates random C programs, and [...] every compiler that we tested has been found to crash and also to silently generate wrong code when presented with valid inputs.”

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“The striking thing about our CompCert results is that the middleend bugs we found in all other compilers are absent. As of early 2011, the under-development version of CompCert is the only compiler we have tested for which Csmith cannot find wrong-code errors.”

Finding and understanding bugs in C compilers.
Yang et al. University of Utah, PLDI 2011.
Critical Systems: programming languages

- Embedded systems are usually developed in C or Ada (with some assembly code)
- Critical systems are developed in subsets of these languages, such as MISTRA C or SPARK Ada.
- Dedicated frameworks also generate either C or (SPARK) Ada source code.
  - B Method
  - SCADE Suite
  - Simulink
Ada: a language designed for embedded systems

- First standardized version in 1983
- Ada is an algol-like language:
  - strong static typing
  - real procedures with proper parameter modes
  - packages (modules)
  - generics
  - support for concurrency
  - support for real-time systems
  - object-oriented (since 1995)
  - support for contracts (since 2012)
Who is using Ada?

Ada is often used in large critical systems:

- **Commercial Aviation:**
  Most Airbus and Boeing airplanes

- **Commercial Rockets:**
  Ariane 4 and 5

- **Railway Transportation:**
  Paris drive-less Metro line 14

- ...
SPARK: a strict subset of Ada

- Developed by ALTRAN Praxis and AdaCore
- Supported by any standard Ada 2012 compiler
- Well-defined subset of Ada designed for Critical Systems
  - Pointers
  - Effects in expressions
  - Parameter-induced aliasing
  - Exception handler

- Static analysis (SPARK tools)
  - to ensure that contracts are met (pre/post conditions)
  - to ensure that runtime checks never fail
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What about a verified SPARK Ada Compiler?

- A large on-going project, in collaboration with AdaCore and SAnToS Lab (Kansas State University), since 2012.

- Current state of the formal specification:
  - small fragment of Ada (similar to C in expressiveness)
  - some runtime checks (overflows)
  - nested procedures

- Unsupported features:
  - packages
  - generics
  - contracts
  - ...
What about a verified SPARK Ada Compiler?

- Current state of the compiler:
  - a SPARK Ada frontend to CompCert
  - lexer and parser from gnat (developed by AdaCore)
  - converter to Coq AST (developed by SAnToS Lab)
  - proof-of-concept compiler (developed by P. Courtieu)
  - nested procedures (work in progress)

- Current state of the proofs:
  - correctness of the compiler (P. Courtieu)
  - absence of runtime error (P. Courtieu and SAnToS Lab)
  - nested procedures (this talk)
Architecture of the CompCert C compiler

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Architecture of the CompCert C compiler

Front-end

C source
- external preprocessor
  - Preprocessed C
  - lexing and parsing (*)
- Parse tree
- type-checking and elaboration

CompCert C AST

Back-end

Not verified yet
(*) the parser is formally verified

Pull side effects out of expressions

Clight
- type elimination; simplification of control

C#minor
- stack allocation

Cminor
- instruction selection

CminorSel
- construction of a CFG

RTL
- function inlining
  - tail call optimization
  - constant propagation
  - common subexpression elimination
  - dead code elimination
  - live range splitting
  - register allocation, spilling, reloading

Asm AST
- printing

Asm text
- external assembler

Object file
- external linker

Executable

LTL
- linearization of the CFG

Linear
- layout of the stack frame

Mach
- generation of Asm code

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Architecture of the CompCert C compiler

C source
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      └── CompCert C AST

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  - packages
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![Diagram of CompCert C compiler architecture](image-url)
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- should require several intermediate languages
package Sorting
  with SPARK_Mode
is
  subtype Index is Integer range 1..100;
  type Vector is array (Index) of Integer ;

  procedure Swap(I, J : Index; V: in out Vector)
    with Post => V = V'Old'Update (I => V'Old (J), J => V'Old (I));

  procedure Sort(V : in out Vector)
    with Post => (for all X in V'First + 1 .. V'Last => (V(X - 1) <= V(X)));

end Sorting;
package body Sorting
  with SPARK_Mode
is

  procedure Swap(I, J : Index; V: in out Vector) is
    Aux: Integer ;
  begin
    Aux := V(I);
    V(I) := V(J);
    V(J) := Aux;
  end Swap;

  procedure Sort(V : in out Vector) is
  begin
    -- some code using Swap
  end Sort;

end Sorting;
Implementing Swap as a **nested** procedure

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end Sorting;
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```
Implementations of nested procedures

Several known implementations:

- In functional or object-oriented languages:
  - full-fledged heap-allocated closures
  - more general than nested procedures

- In languages that obey a stack discipline, classical techniques are rather tricky:
  - “static links” (in the P-code machine [Wirth 1966])
  - “displays” [Dijkstra 1961]

Two optimized implementations but no high-level semantics!
We formalized a frame stack as an Abstract Data Type
We formalized a frame stack as an Abstract Data Type

Definition $I := \text{nat} \times \text{nat}$.

Structure $FS \{ V : Type \} :=$

\[
\begin{align*}
S & : Type; \\
\text{empty} & : S; \\
\text{fetch} & : S \rightarrow I \rightarrow \text{option } V; \\
\text{update} & : S \rightarrow I \rightarrow V \rightarrow \text{option } S; \\
\text{top_frame} & : S \rightarrow \text{option (list } V); \\
\text{new_frame} & : S \rightarrow \text{nat} \rightarrow \text{list } V \rightarrow (\text{option } S); \\
\text{clear_frame} & : S \rightarrow S \rightarrow \text{nat} \rightarrow \text{option } S; \\
\text{frame_offset} & : S \rightarrow I \rightarrow \text{option nat}
\end{align*}
\]
We formalized a frame stack as an Abstract Data Type
A verified implementation of nested procedures

- We formalized a frame stack as an Abstract Data Type
- We provided two implementations of this ADT:
  - a simple high-level implementation (our prototype)
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- We provided two implementations of this ADT:
  - a simple high-level implementation (our prototype)
  - an optimized implementation based on “static links”

- We proved in Coq that the optimized implementation is correct with respect to the prototype, by defining a bi-simulation.

- This bi-simulation then gives us for free a strong property called “parametricity” [Reynolds 1983].

- Parametricity in implemented in Coq as a plugin [Keller & Lasson 2012].
A verified implementation of nested procedures

- As a corollary of parametricity, you obtain the following informal property:

  for any programming language, for any semantics relying on the frame stack ADT, the optimized implementation works as expected
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You need to provide the syntax and the semantics of your language, and Coq does the rest:
You get a formal machine-checked proof of this property.
A verified implementation of nested procedures

■ As a corollary of parametricity, you obtain the following informal property:

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\text{for any programming language,}
\]

\[
\text{for any semantics relying on the frame stack ADT,}
\]

\[
\text{the optimized implementation works as expected}
\]

■ You need to provide the syntax and the semantics of your language, and Coq does the rest:
You get a formal machine-checked proof of this property.

■ Some statistics (just for nested procedures)
– 1,000 lines of statement
– 2,000 lines of proof
Future Works

- Full SPARK 2014 support (packages, generics, ...)
- Correctness of SPARK tools (static analysis, contracts, ...)
- Correctness of the OCaml compiler (and its runtime)?
- Correctness of the Coq proof assistant?
- ...

What is now the weakest link in the chain?