The Maximum Matrix Contraction problem : Appendix WORK IN PROGRESS

Dimitri Watel[1,2] and Pierre-Louis Poirion[1,3]

[1] CEDRIC-CNAM, 292 rue du faubourg Saint Martin, 75003, Paris, FRANCE
[2] ENSIIE, 1 Square de la résistance, Evry, FRANCE dimitri.watel@ensiie.fr,
[3] ENSTA Paristech pierre-louis.poirion@ensta-paristech.fr

Abstract. In this paper, we introduce the *Maximum Matrix Contraction problem*, where we aim to contract as much as possible a binary matrix in order to maximize its density. We study the complexity and the polynomial approximability of the problem. Especially, we prove this problem to be NP-Complete and that every algorithm solving this problem is at most a $2\sqrt{n}$-approximation algorithm where n is the number of ones in the matrix. We then focus on efficient algorithms to solve the problem: an integer linear program and three heuristics.
Keywords: Complexity, Approximation algorithm, Linear Programming

This document contains the appendix of a paper accepted at the conference ISCO 2016, this appendix could not be added to the camera-ready version due to lack of space. This document is presently incomplete, some explanations are missing, but it should be completed soon.

A An instance with an $O(\sqrt{n})$ gap between the worst and the best solution.

Theorem **??** of Section **??** proves a default $2\sqrt{n}$ upper bound of the approximation ratio for every algorithm returning a maximal solution.

We give, in this appendix, in Figure **??**, an instance in which the ratio between an optimal density and the lowest density of a maximal solution is $O(\sqrt{n})$.

1															
	1														
		1													
			1												
				1											
					1										
						1									
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1		1		1		1			1						
										1					
1		1		1		1					1				
												1			
1		1		1		1							1		
														1	
1		1		1		1	1	1							1

Fig. 1: An example of grid in which the gap between an optimal solution and the worst maximal solution is $O(\sqrt{n})$.

1								
1	1							
	1	1						
1		1	1					
1								
1	1	1	1		1			
1	1	1	1		1	1		
1	1	1	1			1	1	
1	1	1	1	1	1		1	1

Fig. 2: An optimal solution of the instance given in Figure 1. The density of this solution is $O(n^2)$.

1	1	1	1	1	1	1			
1									
1									
1		1		1		1			1
									1
1		1		1		1			1
									1
1		1		1		1			1
									1
1		1		1		1	1	1	1

Fig. 3: A solution of the instance given in Figure 1 for which the density is $O(n)$.

A.1 Adaptation to the LCL algorithm

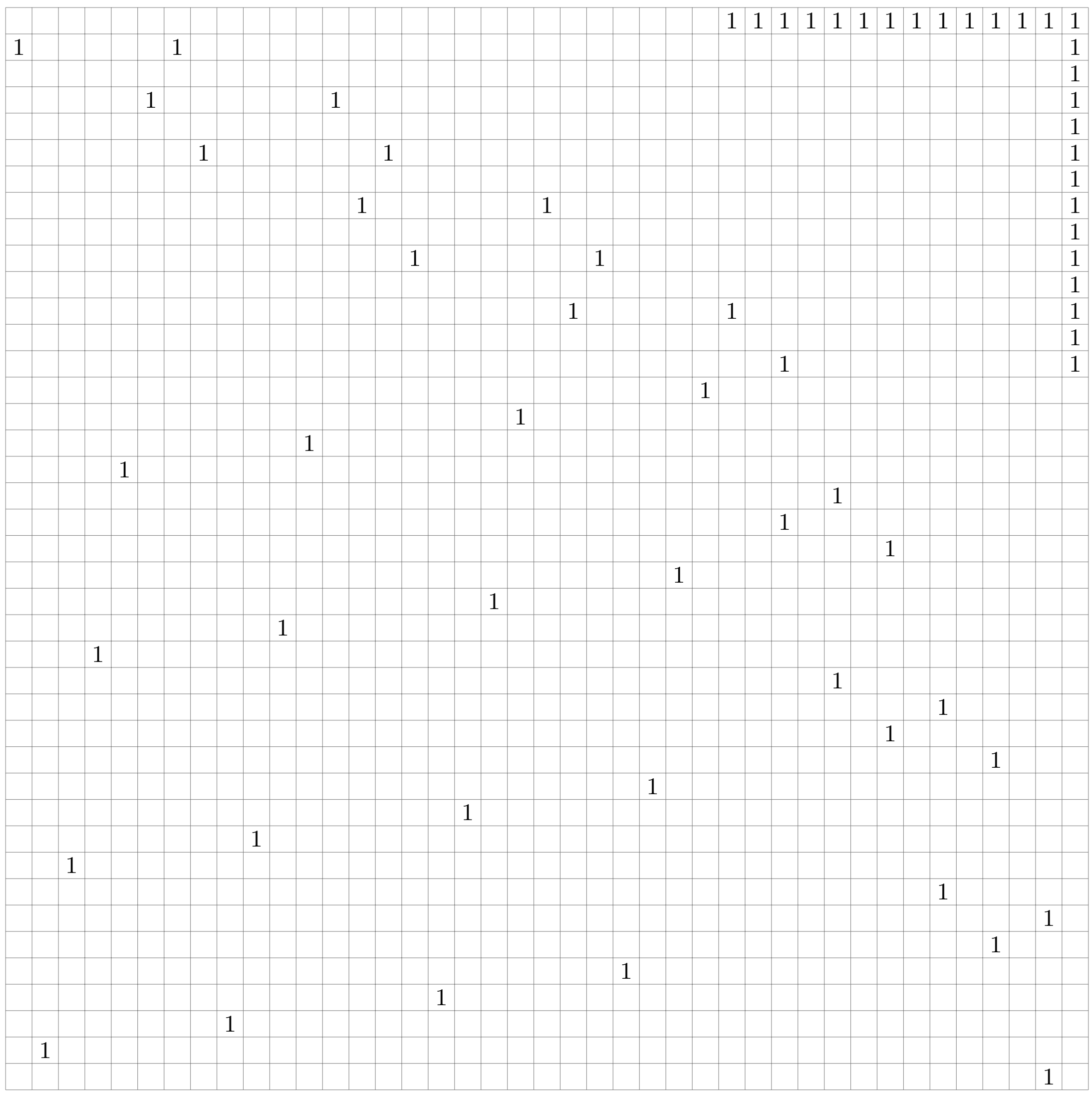

Fig. 4: An example of instance for which an optimal density is $O(\sqrt{n})$ times the density of a solution returned by the LCL algorithm.

				1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1																1
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1	1																1
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	1	1															1
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	1	1															1
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		1	1														1
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			1	1													1
																	1
1	1	1	1			1		1									1
1	1	1	1			1		1		1		1					
1	1	1	1							1		1		1		1	
1	1	1	1											1		1	

Fig. 5: An optimal solution of the instance given in Figure 4. The density of this solution is $O(n)$.

						1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1																		1
																			1
	1	1																	1
																			1
		1	1																1
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			1	1															1
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				1	1														1
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					1	1													1
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1		1		1		1		1											1
								1		1									
1		1		1		1				1		1							
												1		1					
1		1		1		1								1		1			
																1		1	
1		1		1		1												1	

Fig. 6: A solution of the instance given in Figure 4 that may be returned by the LCL algorithm. The density of this solution is $O(n)$.

A.2 Adaptation to the greedy algorithm

													1	1		1	1								
												1		1	1		1								
												1		1	1		1								
												1		1	1		1								
												1		1	1		1								
										1	1	1		1	1		1	1	1						
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																				1	1	1	1	1	1
		1	1			1	1																		1
				1				1												1	1	1	1	1	
																				1	1	1	1	1	1
1	1			1	1			1	1																1
		1				1				1										1	1	1	1	1	
																				1					
1		1		1		1		1		1		1								1					
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1		1		1		1		1		1			1		1										
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1		1		1		1		1		1		1				1									
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1		1		1		1		1		1			1		1										
																1									
1		1		1		1		1		1		1				1									
													1												
1		1		1		1		1		1			1												

Fig. 7: An example of instance for which an optimal density is $O(\sqrt{n})$ times the density of a solution returned by the Greedy algorithm.

								1	1		1	1							
							1		1	1		1							
							1		1	1		1							
							1		1	1		1							
							1		1	1		1							
					1	1	1		1	1		1	1	1					
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														1					
														1	1	1	1	1	1
	1	1	1	1															1
		1		1										1	1	1	1	1	
														1	1	1	1	1	1
1	1	1	1	1	1														1
	1		1		1									1	1	1	1	1	
														1					
1	1	1	1	1	1		1	1						1					
1	1	1	1	1	1			1		1	1								
1	1	1	1	1	1		1	1			1								
1	1	1	1	1	1			1		1	1								
1	1	1	1	1	1		1	1			1								
1	1	1	1	1	1			1											

Fig. 8: An optimal solution of the instance given in Figure 7. The density of this solution is $O(n)$.

												1	1	1	1								
												1	1	1	1								
												1	1	1	1								
												1	1	1	1								
												1	1	1	1								
										1	1	1	1	1	1	1	1	1					
																		1					
																		1	1	1	1	1	1
		1	1	1		1	1	1										1	1	1	1	1	1
																		1	1	1	1	1	1
1	1	1		1	1	1		1	1	1								1	1	1	1	1	1
																		1					
1		1		1		1		1		1		1						1					
												1											
1		1		1		1		1		1		1		1									
														1									
1		1		1		1		1		1		1		1									
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1		1		1		1		1		1		1		1									
														1									
1		1		1		1		1		1		1		1									
												1											
1		1		1		1		1		1		1											

Fig. 9: A solution of the instance given in Figure 7 that is returned by the greedy algorithm. The density of this solution is $O(n)$.

A.3 Adaptation to the neighborization algorithm

															1	1			1	1									
												1				1	1			1									
												1				1	1			1									
												1				1	1			1									
												1				1	1			1									
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																						1	1	1	1	1	1	1	1
						1	1																						1
		1	1					1																					
				1																		1	1	1	1	1	1	1	
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1		1		1		1		1		1			1																

Fig. 10: An example of instance for which an optimal density is $O(\sqrt{n})$ times the density of a solution returned by the Neighborization algorithm.

										1	1			1	1									
							1				1	1			1									
							1				1	1			1									
							1				1	1			1									
							1				1	1			1									
							1				1	1			1									
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					1	1	1				1	1			1	1	1							
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																	1	1	1	1	1	1	1	1
			1	1																				1
	1	1		1																				
		1															1	1	1	1	1	1	1	
																	1	1	1	1	1	1	1	1
				1	1																			1
		1	1		1																			
1	1		1																					
	1																1	1	1	1	1	1	1	
																	1							
1	1	1	1	1	1				1	1							1							
1	1	1	1	1	1					1			1	1										
1	1	1	1	1	1			1	1					1										
1	1	1	1	1	1				1			1	1											
1	1	1	1	1	1		1	1					1											
1	1	1	1	1	1			1																

Fig. 11: An optimal solution of the instance given in Figure 10. The density of this solution is $O(n)$.

												1	1	1	1									
												1	1	1	1									
												1	1	1	1									
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												1	1	1	1									
												1	1	1	1									
												1	1	1	1									
										1	1	1	1	1	1	1	1							
																	1							
																	1	1	1	1	1	1	1	1
		1	1	1		1	1	1									1	1	1	1	1	1	1	1
																	1	1	1	1	1	1	1	1
1	1	1		1	1	1		1	1	1							1	1	1	1	1	1	1	1
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1		1		1		1		1		1		1					1							
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1		1		1		1		1		1		1		1										
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1		1		1		1		1		1		1		1										
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1		1		1		1		1		1		1		1										
												1												
1		1		1		1		1		1		1												

Fig. 12: A solution of the instance given in Figure 10 that is returned by the neighborization algorithm. The density of this solution is $O(n)$.

B Discussion

We highly believe this instance is the worst case that can happen and that the density of a maximal solution is always higher than $4\sqrt{n}$. The result of Theorem **??** may then possibly be updated to the following conjecture.

Conjecture 1. An algorithm returning any maximal solution of an instance of MMC is a $\sqrt{n}$-approximation.