Sensitivity analysis of FBMC signals to Non Linear phase distortion

Hanen Bouhadda^{1,2}, Hmaied Shaiek¹, Yahia Medjahdi¹, Daniel Roviras¹, Rafik Zayani², and Ridha Bouallegue²

¹CNAM CEDRIC/LAETITIA - 292 rue Saint-Martin - 75003 Paris, France 2 INNOV'COM- SUP'COM- Carthage University - Tunis, Tunisia bouhadda.hanene@supcom.rnu.tn, (hmaied.shaiek,yahia.medjahdi,daniel.roviras)@cnam.fr rafik.zayani@gmail.com ridha.bouallegue@supcom.rnu.tn

*Abstract***—In this paper, we present a theoretical analysis of the impact of High Power Amplifier (HPA) Non-Linear Distortion (NLD) on the Bit Error Rate (BER) of multicarrier techniques. For the aim of this study, the Saleh's model was chosen for NL HPA. Two multicarrier schemes are considered: the classical Orthogonal Frequency Division Multiplexing (OFDM) and the Filter Bank based Multi-Carrier (FBMC). According to the Bussgang theorem, the in-band NLD is modeled as a complex gain and an independent additive noise term for both modulations. The BER performance of OFDM and FBMC modulations, transmitting over an Additive Withe Gaussian Noise (AWGN) and Rayleigh channels, is theoretically evaluated and compared to simulation results. We have established that OFDM and FBMC modulations show the same performances, in terms of BER, when just amplitude distortion is induced by the HPA. However, FBMC system is more sensitive to phase distortions when no correction is adopted at the receiver. This behavior has been explained by the difference between the probability distribution of the intrinsic interference present in both OFDM and FBMC modulations.**

*Keywords***— FBMC, OFDM, HPA, Non-linear distortion, BER.**

I. INTRODUCTION

4G systems such as 3GPP-LTE are based on OFDM modulations with Cyclic Prefix (CP) offering better robustness over multi-path fading channel. However, the use of CP induces loss in spectral efficiency. FBMC modulations are potential promising candidates for next generation systems [1] as well as 5G systems [2]. In fact, the good frequency localization of the prototype filters [3], [4], [5], used in FBMC offers him the robustness to several impairments such as timing misalignment between users [6].

The key-idea of multicarrier techniques is to divide the high data rate stream into N low-rate streams that are transmitted over N orthogonal subcarriers. Considering high values of N and according to the central limit theorem, the superposition of these independent streams, leads to a complex Gaussian signal. For this reason OFDM and FBMC exhibit a large Peak-to-Average Power Ratios (PAPR) [7], [8], i.e. large fluctuations in their signals envelope and they are very sensitive to NLD caused by HPA.

The main objective of this paper is to study the BER performance in the presence of NL HPA for both OFDM and FBMC systems. A theoretical characterization of NLD on OFDM systems has been proposed in [9]. The authors focused on the impact of the amplitude distortions induced by three HPA models : the Soft Envelope Limiter (SEL), the Solid State Power Amplifier (SSPA) and the travelling Wave Tube Amplifier (TWTA). In [10] the author investigated the impact of the NLD induced by the HPA on out of band spectral re-growth of FBMC modulated signals.

The aim of this paper is to evaluate the in-band performances of the OFDM/FBMC modulations when a NL HPA is used and its originality consists of the analysis introduced showing that FBMC signal is more sensitive to phase distortion than OFDM one. This analysis is validated by theoretical and simulation results with multilevel MQAM modulation over AWGN and Rayleigh channels.

The rest of this paper is organized as follows: section II describes the system model where we consider the Saleh's model for the NL HPA, with amplitude (AM/AM) and phase (AM/PM) distortions. Section III presents the theoretical model and the estimation of the NLD. In section IV, we develop a theoretical analysis of the BER of OFDM/FBMC systems in presence of NL HPA. The obtained BER expressions are evaluated through various simulation scenarios which are presented in section V. Finally, section VI gives the conclusion.

II. SYSTEM MODEL

We consider, in this paper, an OFDM/FBMC transceiver with NL HPA as shown in figure 1.

A. HPA model

We consider the Saleh's model for the NL HPA [11]. We can underline here that the analysis made in this work remains valid for all memoryless HPA models.

According to Saleh's model, the AM/AM and AM/PM conversion characteristics are expressed as follows :

$$
A(\rho(t)) = A_{sat}^2 \frac{\rho(t)}{\rho(t)^2 + A_{sat}^2}
$$
 (1)

$$
\phi(\rho(t)) = \varphi_0 \frac{\rho(t)^2}{\rho(t)^2 + A_{sat}^2} \tag{2}
$$

where :

- A_{sat} is the HPA input saturation level,
- $\rho(t)$ is the input signal modulus,
- φ_0 is the phase distortion introduced by the HPA.

Fig. 1: The transmission system model with FBMC/OFDM modulations.

The AM/AM and AM/PM characteristics cause distortions on the constellation scheme and affect the spectral efficiency, which degrade the system performance. The signal at the HPA output can be written as:

$$
u(t) = A(\rho(t)) \exp(j\phi(\rho(t))) \exp(j\varphi(t))
$$

= $S(\rho(t)) \exp(\varphi(t))$ (3)

where :

- $\varphi(t)$ is the signal input phase,
- $S(\rho)$ is the complex soft envelop of the amplified signal.

In practice, to avoid or at least to reduce the effects of nonlinearities, the HPA is operated at a given Input Back-Off (IBO) from its saturation level. This parameter is defined as :

$$
IBO = 10 \log_{10} \left(\frac{A_{sat}^2}{P_{v_{in}}} \right)
$$
 (4)

where :

• $P_{v_{in}}$ is the signal input average power.

B. Introduction to FBMC system

The FBMC technique consists in transmitting Offset Quadrature Amplitude Modulation (OQAM) data symbols instead of conventional QAM ones [3], [12], where the in-phase and the quadrature components are time staggered by half a symbol period, $T/2$. Accordingly, the baseband continuoustime model of the FBMC transmitted signal can be defined as follows [3] :

$$
i(t) = \sum_{m=0}^{N-1} \sum_{n=-\infty}^{+\infty} a_{m,n} h(t - nT/2) e^{j\frac{2\pi}{T}mt} e^{j\varphi_{m,n}}
$$
 (5)

where :

- N is the number of subcarriers.
- $h(t)$ is the prototype filter impulse response,
- $a_{m,n}$ are real-valued symbols.

The phase term $\varphi_{m,n}$ is given by :

$$
\varphi_{m,n} = \frac{\pi}{2}(m+n) - \pi mn
$$

Considering the shifted versions of $h(t)$ in time and frequency noted :

$$
\gamma_{m,n}(t) = h(t - nT/2)e^{j\frac{2\pi}{T}mt}e^{j\varphi_{m,n}}
$$
(6)

We can rewrite equation (5) as follows,

$$
i(t) = \sum_{m=0}^{N-1} \sum_{n=-\infty}^{+\infty} a_{m,n} \gamma_{m,n}(t)
$$
 (7)

In a distortion-free noise-less channel, the demodulated signal y_{m_0,n_0} at time instant n_0 and subcarrier m_0 is given by :

$$
y_{m_0,n_0} = \langle i(t), \gamma_{m_0,n_0}(t) \rangle = \int_{-\infty}^{+\infty} i(t) \gamma_{m_0,n_0}^*(t) dt
$$

\n
$$
= \sum_{n=-\infty}^{+\infty} \sum_{m=0}^{N-1} a_{m,n} \int_{-\infty}^{+\infty} \gamma_{m,n}(t) \gamma_{m_0,n_0}^*(t) dt
$$

\n
$$
= a_{m_0,n_0} + \sum_n \sum_{m} \int_{(m,n)\neq(m_0,n_0)-\infty}^{+\infty} \gamma_{m,n}(t) \gamma_{m_0,n_0}^*(t) dt
$$

\n(8)

where :

- $\gamma^*_{m_0,n_0}(t)$ is the complex conjugate of $\gamma_{m_0,n_0}(t)$,
- $\langle ., . \rangle$ stands for the inner product.

According to [5], the prototype filter is designed such that the intrinsic interference term is orthogonal to the useful symbol i.e. is purly imaginary.

$$
ju_{m_0,n_0} = \sum_{m \neq m_0} \sum_{n \neq n_0} a_{m,n} \int_{-\infty}^{+\infty} \gamma_{m,n}(t) \gamma_{m_0,n_0}^*(t) dt \qquad (9)
$$

Considering the PHYDYAS prototype filter proposed in [5], the coefficients Ψ_{m_0,n_0} are given in Table I.

Consequently, a perfect reconstruction of the transmitted real symbols $a_{m,n}$ is obtained by taking the real part (OQAM decision) of the demodulated signal y_{m_0,n_0} .

TABLE I: Transmultiplexer impulse response

	n_0-3	$n_0 - 2$	$n_0 - 1$	n_0	n_0+1	n_0+2 n_0+3	
m_0-1	0.043i	0.125j	0.206i	0.239 i	0.206 <i>i</i>	$0.125j$ $0.043j$	
m ₀	$-0.067i$		$-0.564i$		0.564 i		0.067i
$m_0 + 1$	0.043 <i>i</i>	$-0.125j$	0.206j		$-0.239j$ $-0.206j$ $-0.125j$ $0.043j$		

TABLE II: Comparison between estimated values of Kand σ_d^2 for both nonlinearly amplified OFDM and FBMC signals

III. NON-LINEAR DISTORTION MODELING

By Considering a large number of subcarriers N , the input signal $i(t)$ is assumed to be a complex Gaussian process. Thus, the amplified signal can be written as:

$$
u(t) = Ki(t) + d(t)
$$
\n(10)

where :

- $d(t)$ is a zero-mean additive white noise which is uncorrelated with $i(t)$,
- K is a complex gain with modulus |K| and phase ϕ_K that can be expressed as [9]:

$$
K = \frac{1}{2} \mathbb{E} \left[\frac{\partial S(\rho)}{\partial \rho} + \frac{S(\rho)}{\rho} \right] \tag{11}
$$

where:

- E is the expectation operator.
- ρ is the modulus of $i(t)$.

The variance σ_d^2 of the NL distortion $d(t)$ is given by the following equation:

$$
\sigma_d^2 = \mathbb{E}(|d_n|^2) - |K|^2 \mathbb{E}(i_n^2)
$$

=
$$
\mathbb{E}(|S(\rho)|^2) - |K|^2 \mathbb{E}(\rho^2)
$$
 (12)

In Table II, we compare the NLD parameter values given by equations (11) and (12) for both OFDM and FBMC. Considering the HPA model defined by the conversion characteristics given by equations (1). The estimation of the parameters K and σ_d^2 is made for several values of subcarriers N with a $\varphi_0 = \pi/3$ the comparaison is made for two values of *IBO* of 3dB and 6dB.

Results illustrated in Table II show that for sufficiently high number of subcarriers N, the OFDM and FBMC modulated signals, which are considered as gaussian processes (according to the central limit theorem), can be modelled by the same NLD parameters K and σ_d^2 when they are passed through a nonlinear HPA.

Fig. 2: OFDM Constellation, IBO=6dB, $\varphi_0 = \pi/3$

IV. THEORETICAL PERFORMANCE ANALYSIS OF NLD ON OFDM/FBMC SIGNALS

The received signal after HPA and channel filtering can be expressed, using the notation of figure 1, as :

$$
z(t) = h_c(t) \otimes (Ki(t) + d(t)) + w(t)
$$

= $i(t) \otimes [Kh_c(t)] + d(t) \otimes h_c(t) + w(t)$ (13)

where :

- $h_c(t)$ is the channel impulse response,
- ⊗ stands to the convolution product operator.

Looking at equation (13), it's clear that the effect of the NL factor K will be taken into account during frequency equalization at the receiver side $(h_c(t))$ and K will be estimated jointly). Nevertheless, in order to stress the impact of phase rotation on both OFDM and FBMC performances, we will present in paragraph IV-A the analysis of the BER when the channel is AWGN and when no correction is made for the NL factor K . In the paragraph IV-B, we will evaluate theoretical BER in AWGN channel with correction of the phase rotation related to the factor K . The last paragraph of this section is dedicated to BER analysis of OFDM/FBMC modulations, after phase correction, with Rayleigh channel.

A. Sensitivity of OFDM/FBMC to phase error

For simplicity reasons we will conduct our analysis for 4QAM modulated symbols. The extension of this analysis to M-QAM $(M > 4)$ is also possible.

1) OFDM case: The output constellation after HPA and OFDM demodulation is presented on figure 2. The distance d (called d_{OFDM} in this case) is a function of the NLD induced by the HPA. It represents the distance projected on real axis of the received signal points affected by the phase rotation. The distance d is equal to:

Fig. 3: FBMC Constellation, IBO=6dB, $\varphi_0 = \pi/3$

$$
d_{OFDM} = \text{Re}\left((a_{m,n}^I + ja_{m,n}^Q)|K|\exp(j\phi_k)\right) \tag{14}
$$

where :

• $a_{m,n}^I$ and $a_{m,n}^Q$ denote respectively the in-phase and the quadrature components of the transmitted complex symbol.

Depending on the sign of $a_{m,n}^Q$, the decision distance is rather d^1_{OFDM} $(a^I_{m,n} > 0)$ or d^2_{OFDM} $(a^I_{m,n} < 0)$.

2) FBMC case: With this modulation technique, the transmission of a real symbol on subcarrier m_0 generates a pure imaginary intrinsic interference u_{m_0,n_0} .

However, in the presence of a phase offset (φ_k) , this interference is no longer imaginary. Consequently, by taking the real part of the received signal, we obtain a part of the useful signal distorted by the interference signal (u_{m_0,n_0}) .

Figure 3 shows the 4QAM constellation affected by the HPA and the distance representation in the FBMC case for one realization of neighboring symbols. We note that this constellation is considered before the OQAM demodulation. The FBMC distance d_{FBMC} is given by equation :

$$
d_{FBMC} = |K|(a_{m,n}\cos(\phi_k) - u_{m,n}\sin(\phi_k)) \tag{15}
$$

 $u_{m,n}$ is given by equation (9) and it corresponds to all the possible combinations of adjacent symbols.

3) BER analysis without phase rotation correction: If no correction is made at the receiver side for the multiplicative constant K , the 4QAM BER in an AWGN channel is defined as follows [13] :

$$
BER_{4QAM} = \frac{1}{2} \int_0^{+\infty} \text{erfc}\left(\frac{u}{\sqrt{2T(\sigma_w^2 + \sigma_d^2)}}\right) pdf(u)du
$$
\n(16)

where:

- u is the decision distance.
- $pdf(u)$ is the probability density function of u,
- σ_w^2 is the variance of the AWGN,
- σ_d^2 is the variance of the NL distortion $d(t)$,
- T is the symbol duration.

For the OFDM case, we have seen that the decision distance d takes only two values d^1_{OFDM} and d^2_{OFDM} (depending on the sign of $a_{m,n}^I$). We have then :

$$
BER_{4QAM}^{OFDM} = \frac{1}{2} \operatorname{erfc}\left(\frac{d_{OFDM}^1}{\sqrt{2T(\sigma_w^2 + \sigma_d^2)}}\right) + \frac{1}{2} \operatorname{erfc}\left(\frac{d_{OFDM}^2}{\sqrt{2T(\sigma_w^2 + \sigma_d^2)}}\right)
$$
(17)

For the FBMC case, the decision distance d can take a set of values corresponding to all possible values of $u_{m,n}$ given by equation (9). We have then :

$$
BER_{4QAM}^{FBMC} = \sum_{u_{m,n}} P(u_{m,n}) \text{erfc}\left(\frac{d_{FBMC}}{\sqrt{2T(\sigma_w^2 + \sigma_d^2)}}\right)
$$
 (18)

where:

 $P(u_{m,n})$ is the distribution probability of $u_{m,n}$.

B. BER analysis with phase rotation correction, case of an AWGN channel

As said in the previous paragraph, in a real situation the phase shift of the constellation can be easily estimated and compensated for.

In the case where we assume a perfect estimation of the NLD parameter K (K is jointly estimated with the channel response), we did the correction and the decision distances $d_{OFDM}^1 = d_{OFDM}^2 = d_{FBMC} = 1.$

In this case, we can rewrite the signal at the input of OFDM/FBMC Rx as :

$$
z(t) = i(t) + (d(t) + w(t))\frac{1}{K}
$$
 (19)

Then BER_{4QAM}^{OFDM} and BER_{4QAM}^{FBMC} given by equations (17) and (18) are identical and are given by :

$$
BER_{4QAM}^{OFDM} = BER_{4QAM}^{FBMC} = \text{erfc}\left(\frac{|K|}{\sqrt{2T(\sigma_w^2 + \sigma_d^2)}}\right)
$$
 (20)

C. BER analysis with phase rotation correction, case of a Rayleigh channel

A flat fading channel can be considered as an AWGN with a variable gain, which is considered as a random variable with Rayleigh probability distribution function. So the average BER can be calculated by averaging BER for instantaneous SNR over the distribution of SNR. The nonlinearly amplified signal in a Rayleigh fading channel, with coherent detection, is given by equation (13). Let $\alpha = |h_c|^2$, which is an exponentially distributed random variable with a pdf expressed as

$$
p_{\alpha}(\alpha) = \frac{1}{\Omega} e^{(-\alpha/\Omega)} \tag{21}
$$

where $\Omega = \mathbb{E}[\alpha]$ is the average fading power. Then, the instantaneous Signal-to-Noise Ratio (SNR) at the receiver is expressed by the equation below

$$
\gamma = \gamma_c \frac{\alpha}{\alpha \sigma_d^2 + \sigma_w^2} \tag{22}
$$

where $\gamma_c = \frac{|K|^2 u^2}{4T}$ $rac{u}{4T}$.

Derivation of the pdf of γ

Lemma 1 :

Let $X = \alpha$, be an exponentially distributed random variable, then the SNR can be described as

$$
\gamma = \gamma_c \frac{X}{X \sigma_d^2 + \sigma_w^2} = h(X) \tag{23}
$$

 $h(X)$ is strictly increasing continuously differentiable function with inverse $X = g(\gamma)$. Then

$$
p_{\gamma}(\gamma) = \begin{cases} \frac{\sigma_w^2 \gamma_c}{\Omega(\gamma_c - \sigma_d^2 \gamma)^2} e^{-\frac{\sigma_w^2 \gamma}{\Omega(\gamma_c - \sigma_d^2 \gamma)}}, & \text{if } 0 \le \gamma < \frac{\gamma_c}{\sigma_d^2} \text{ (24)}\\ 0 & \text{otherwise} \end{cases}
$$

By substituting equation (24) on equation (16), the average BER using 4QAM modulation is given by the following equation :

$$
BER_{4QAM} = \frac{1}{2} \int_0^{+\infty} \text{erfc}\left(\sqrt{\left(\frac{u^2}{2T(\sigma_w^2 + \sigma_d^2)}\right)}\right)
$$

$$
\frac{\sigma_w^2 \gamma_c}{\Omega(\gamma_c - \sigma_d^2 \gamma)^2} e^{-\frac{\sigma_w^2 \gamma}{\Omega(\gamma_c - \sigma_d^2 \gamma)}} du
$$
(25)

V. SIMULATION RESULTS

In this section, we present theoretical and simulation results representing the performance comparison, in terms of BER, of OFDM and FBMC systems in presence Saleh's HPA model. We will investigate the cases where the phase rotation is compensated for or not at the receiver side.

For OFDM/FBMC systems with $N = 64$ sub-carriers, the BER is computed by averaging on 6.4×10^7 randomly generated 4 QAM symbols. We recall that transmission is achieved through an AWGN or Rayleigh flat fading channel.

Using equation (17) and equation (18), we compare theoretical and simulation results for both OFDM and FBMC systems. Figure 4 depicts the OFDM and FBMC BER performance considering an IBO of $6dB$ and a 4QAM modulation

Fig. 4: OFDM and FBMC performance comparison, 4QAM, 64 subcarriers, Saleh's HPA model, IBO=6dB, AWGN channel.

scheme. Four scenarios are investigated : a) The linear case (no HPA). b) Only the AM/AM characteristic of the HPA model is considered (i.e, $\varphi_0 = 0$), c) Both AM/AM and AM/PM conversion characteristics are considered (with $\varphi_0 = \pi/3$) without phase error correction, d) Both AM/AM and AM/PM conversion characteristics are considered (with $\varphi_0 = \pi/3$) with phase error correction. This figure shows that the analytical and simulation results are in good mach in the different scenarios. It can be noted that in the cases b) [$\varphi_0 = 0$ (K is a real gain)] and d) $[\varphi_0 = \frac{\pi}{3}$ (with correction of the NL factor K)], FBMC and OFDM reach the same BER performance and that theoretical result of equation (20) is in very good agreement with simulation results. Difference between cases b) and d) is only related to the increase of the NL noise variance σ_d^2 when $\varphi_0 = \frac{\pi}{3}$. for case c), a significant degradation is present for FBMC system compared to OFDM. Even if this case is not very realistic (we have said earlier that correction of the NL factor K will be made jointly with the one tap frequency equalizer), it shows that FBMC modulations are more sensitive to phase estimation errors than OFDM. This is mainly due to the distribution of the intrinsic interference in the FBMC modulation.

To further illustrate the effect of nonlinearity in the case of Rayleigh channel, the BER performance of FBMC, taking 4QAM as the modulation scheme and a NL HPA with an IBO of 6dB, is shown in figure 5. The Rayleigh channel was assumed to be a slowly-varying flat-fading one at a rate slower than the symbol rate. Based on equations (22), (25) and simulation results shown in figure 5, we note that the BER, for relatively low SNR (i.e. $E_b/\sigma_w^2 < 20dB$), is very close to the BER performance of the Rayleigh with a linear HPA. This phenomenon is due to the fact that the residual degradation of the BER, which is caused by the NL HPA after the corresponding phase correction, is negligible compared to the AWGN interference (i.e. $\sigma_d \ll \sigma_w$). Indeed, the SNR has the same distribution as the Rayleigh one. Whereas, at high values of SNR (i.e. $E_b/\sigma_w^2 > 45dB$) σ_w is negligible and the

Fig. 5: BER performance of FBMC, 4QAM, 64 subcarriers, Saleh's HPA model, $\varphi_0 = \pi/3$, IBO=6dB, Rayleigh channel.

SNR tends to a constant $(\gamma \rightarrow \frac{E_b}{\sigma_d^2})$ leading then to a constant BER

VI. CONCLUSION

In this paper, we have studied the impact of in-band NLD caused by HPA on both OFDM and FBMC systems over AWGN and Rayleigh channels. For the HPA, we used the Saleh's model inducing amplitude distortion (AM/AM) and phase distortion (AM/PM). A theoretical approach was proposed to evaluate the BER performance for these modulation techniques. This approch is based on the fact that the inband non-linear distortion is modeled with a complex gain and an uncorrelated additive white Gaussian noise. We have demonstrated that when only the amplitude of the modulated signals is distorted by the HPA, OFDM and FBMC exhibit the same performance in terms of BER. However, FBMC system is shown to be more sensitive to phase distortions than OFDM one. Simulation and theoretical results are in agreement. Moreover, we have proved that the phase error sensitivity of FBMC is directly related to the intrinsic interference term introduced by this modulation.

ACKNOWLEDGEMENT

This work is supported by the European project EM-PhAtiC (ICT- 318362) and the PHC-Utique C3 project (code 12G1414).

REFERENCES

- [1] B. F. Boroujeny, *OFDM versus Filter Bank Multicarrier*, IEEE Signal Processing Magazine, 2011.
- [2] G. Wunder et. al., "[5gnow] : Challenging the lte design paradigms of orthonality and synchronicity," in *IEEE VTC Spring*, Dresden, Germany, Jun. 2013.
- [3] P. Siohan, C. Siclet, and N. Lacaille, "Analysis and design of ofdm/oqam systems based on filter bank theory," *IEEE Transactions on Signal Processing*, vol. 50, pp. 1170–1183, May 2002.
- [4] Y. Dandach and P. Siohan, *Design Method of OFDM/OQAM systems using a weighted time frequency localization criterion*, 18th European Signal Processing Conference, EUSIPCO, August 2010.
- [5] M. G. Bellanger, "Spicification and design of prototype filter for filter bank based multicarrier transmission," in *IEEE International Confrence on Acoustic, Speech and Signal Processing*, May 2001, pp. 2417–2420.
- [6] Y. Medjahdi, M. Terre, D. Le Ruyet, D. Roviras, and A. Dziri, *The impact of timing synchronization errors on the performance of OFDM/FBMC system*, vol. 10, IEEE Transactions on Wireless Communications, August 2011.
- [7] Z. Kolar and P. Horvath, "Physical layer consideration for radio cognitive: Modulation technique," in *IEEE VTC Spring*, May 2011.
- [8] M. I. Abdullah, M. Z. Mahmud, M. S. Hossain, and M. N. Islam, *Comparetive study of PAPR Reduction techniques in OFDM*, vol. 1, ARPN Journal of systems and software, November 2011.
- [9] D. Dardari, V. Tralli, and A. Vaccari, "A theoretical characterization of nonlinear distortion effects in ofdm systems," *IEEE transactions on Communications*, pp. 1755–1764, October 2000.
- [10] M. Khodjet, C. Saber, D. Roviras, and Y. Medjahdi, "Multi-carrier interference evaluation with jointly non-linear amplification and timing error," in *IEEE VTC Spring*, May 2011.
- [11] A. Saleh, "Frequency-independent and frequency-dependent nonlinear models of twt amplifiers," *IEEE Transactions on Communications*, vol. 29, November 1981.
- [12] H.Bolski, *Orthogonal frequency division multiplexing based on offset QAM*, Advances in Gabor Analysis, 2003.
- [13] J. G. Proakis, *Digital Communications*, 4th ed. New York:McGrawHill, 2001.