The shortest multipaths problem in a capacitated dense channel

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Given a graph $G = (V, E)$ with integral capacities on the edges and K pairs (source s_k, sink t_k) of *terminal* vertices, the *shortest multipaths problem* (SMPP) consists of linking each pair (s_k, t_k) by a path with respect to the capacity constraints, such that the sum of the lengths of the K paths is minimized [2]. In particular, this problem has applications in the design of grid-like VLSI circuits. An instance of SMPP is called *feasible* if there exists a way of routing the K paths without violating any capacity constraint, i.e., if there exists at least one feasible solution. SMPP is $\mathcal{NP}$-hard in planar graphs, even if all the terminals lie on the outer face and the maximum degree is four [1]. Moreover, in grids, the problem of deciding whether all the nets can be routed is polynomial-time solvable if all the terminals lie on the boundary [4] and NP-complete otherwise [5].

A *dense channel* is a rectilinear grid with K vertical lines, in which all the terminals are distinct, all the sources lie on the uppermost horizontal line and all the sinks on the lowermost one. Assume that each horizontal (resp. vertical) edge of the grid is valued by the same positive integer C_h (resp. C_v). Formann et al. study the case of a dense channel where $C_h = C_v = 1$ [3]: in this case, it follows from a result of Frank in [4] that the problem has no solution (i.e., no instance is feasible), unless either all the nets can be routed vertically (trivial case) or there is an additional vertical line on the right or on the left of the grid. Thus, for the non trivial case, the authors consider a grid with $K + 1$ vertical lines and prove that SMPP is polynomial-time solvable.

In this communication, we give a greedy polynomial-time algorithm to solve SMPP in dense channels when C_v and C_h are arbitrary (except for the case $C_h = C_v = 1$, settled in [3]). The main feature of this simple algorithm is that it routes each pair (s_k, t_k) along a shortest path, and thus the total length is minimum. We first solve a *base case* ($C_h = 2$ and $C_v = 1$), and then detail all the other cases and show how they can be settled.

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