MAXIMUM EDGE DISJOINT PATHS AND MINIMUM UNWEIGHTED MULTICUT PROBLEMS IN GRID GRAPHS

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Let $G = (V, E)$ be an undirected graph and let $\mathcal{L}$ be a list of K pairs (source s_k, sink t_k) of terminal vertices of G (or *nets*). The *maximum edge disjoint paths problem* (MaxEDP) consists in maximizing the number of nets linked by edge disjoint paths. The related *minimum multicut problem* (MinUWMC) is to find a minimum set of edges whose removal separates each pair (s_k, t_k) in an *augmented* graph (i.e. where each terminal vertex is linked to the graph by a unique edge, as in [4]).

Both problems are NP-hard even in planar graphs [1]. MaxEDP defined in rectilinear grids where any vertex can be a terminal is also NP-hard [3]. However, A. Frank gives in [2] necessary and sufficient conditions for the existence of K edge disjoint paths when the terminals are *two-sided* (i.e. they are all distinct and lie on the uppermost and lowermost lines of the grid): thus, solving MaxEDP is equivalent to removing the minimum number of nets in order to fulfill Frank's conditions. We prove that this can be done by solving a polynomial number of linear programs having totally unimodular constraints matrices. Then, we show that, in two-sided augmented grids, MinUWMC is polynomial time solvable via linear programming, by using a duality relationship with a *continuous multiflow problem*. As a by-product, the gap between the optimal values of MaxEDP and MinUWMC is proved to be at most one.

References

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