

FROM LINEAR TO SEMIDEFINITE PROGRAMMING : AN ALGORITHM TO OBTAIN SEMIDEFINITE RELAXATIONS FOR BIVALENT QUADRATIC PROBLEMS

FRÉDÉRIC ROUPIN

*CEDRIC-Institut d'Informatique d'Entreprise,
18 allée Jean Rostand 91025 Evry cedex, France
e-mail : roupin@iie.cnam.fr*

ABSTRACT. In this paper, we present a simple algorithm to obtain mechanically SDP relaxations for any quadratic or linear program with bivalent variables, starting from an existing linear relaxation of the considered combinatorial problem. A significant advantage of our approach is that we obtain an improvement on the linear relaxation we start from. Moreover, we can take into account all the existing theoretical and practical experience accumulated in the linear approach. After presenting the rules to treat each type of constraint, we describe our algorithm, and then apply it to obtain semidefinite relaxations for three classical combinatorial problems: the κ -CLUSTER problem, the Quadratic Assignment Problem, and the Constrained-Memory Allocation Problem. We show that we obtain better SDP relaxations than the previous ones, and we report computational experiments for the three problems.

Keywords : Semidefinite Programming, Quadratic Assignment Problem, κ -CLUSTER problem, Constrained-Memory Allocation Problem.

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