Multiscale Fama-French model: application to the French market

Anyssa Trimech
Computational Mathematics Laboratory, Higher Institute of Finances and Tax Systems, Sousse University, Sousse, Tunisia

Hedi Kortas
Computational Mathematics Laboratory, Higher Institute of Transport and Logistics, Sousse University, Sousse, Tunisia

Salwa Benammou
Computational Mathematics Laboratory, Faculty of Law, Economic, and Political Sciences, Sousse University, Sousse, Tunisia, and

Samir Benammou
Computational Mathematics Laboratory, Faculty of Sciences, Monastir University, Monastir, Tunisia

Abstract

Purpose – The purpose of this paper is to discuss a multiscale pricing model for the French stock market by combining wavelet analysis and Fama-French three-factor model. The objective is to examine the relationship between stock returns and Fama-French risk factors at different time-scales.

Design/methodology/approach – Exploiting the scale separation property inherent to the maximal overlap discrete wavelet transform, the data set are decomposed into components associated with different time-scales. This wavelet-based decomposition scheme allows the three Fama-French models to be tested over different investments periods.

Findings – The obtained results show that the explanatory power of the Fama-French three-factor model becomes stronger as the wavelet scale increases. Besides, the relationship between the portfolio returns and the risk factors (i.e. the market, size and value factors) depends significantly upon the considered time-horizon.

Practical implications – The proposed methodology offers investors the opportunity to construct dynamic portfolio management strategies by taking into account the multiscale nature of risk and return. Moreover, it gives a new insight to fund rating and fund selection issues in relation to heterogeneous investments periods.

Originality/value – The paper uses wavelets as a relatively new and powerful tool for statistical analysis that allows a new understanding of pricing models. The paper will be of interest not only for academics in the field of asset pricing but also for fund managers and financial market investors.

Keywords Stock markets, France, Capital asset pricing model, Investment appraisal

Paper type Research paper

Introduction

The main concern of investors is to maximize the profitability of their investments and at the same time to minimize the associated risk. The return-risk tradeoff is a key issue in finance that has lead to the development of several asset-pricing models.

The Journal of Risk Finance
Vol. 10 No. 2, 2009
pp. 179-192

1526-5943
DOI 10.1108/15265940910938251

Amongst the most extensively used is the capital asset pricing model (CAPM) (Sharpe, 1964; Lintner, 1965). The CAPM is a general equilibrium market model built up to investigate the relationship between risk and expected returns. It takes into consideration the asset's sensitivity to non-diversifiable risk also referred to as systematic risk or market risk. Nevertheless, CAPM's assumption of a single risk factor explaining expected returns has been criticized. Fama and French (1992, 1993, 1995, 1996, 1998) proposed an alternative pricing model which incorporates three factors as proxies for risk: the market, the size, and the value factors.

Although the Fama-French model has been adopted both by most practitioners and academics in financial issues pertaining to portfolio management, capital budgeting, and performance evaluation, the three-factor model suffers from many drawbacks. In fact, while financial data exhibit multiscaling, i.e. are a combination of different multi-horizon dynamics, the Fama-French model is a single scale model that studies the relation between risk factors and expected returns on a global scale investment horizon.

Furthermore, most studies neglect the assets' long-term holding period focusing mainly on short-term analysis though it is crucial to take into account the tendency of some investors to hold stocks over the long-run.

Recently, there has been a substantial interest in a set of basis functions called "wavelets". Wavelets are mathematical functions that map data into different frequency components. Compared to standard Fourier analysis, they have the advantage of being localized in the time domain as well as in the frequency domain. Via a multiscale (or multiresolution) approach, wavelets decompose a given time series into different dynamics evolving at various time scales. This allows one to describe financial data over different investment horizons.

The objective of this paper is to apply the wavelet multi-resolution analysis framework to the three Fama-French model in the order to model the relationship between stock returns and risk factors at different time scales.

This work is organized as follows: section one presents a brief theoretical background on the Fama-French three-factor model. Basic concepts of wavelet theory are outlined in section two. Then, a multiscale three-factor model is described in section three. Data description are given in section four. Sections five and six present some empirical results of the single and multiscale models. Finally, we end this paper with concluding remarks.

Fama-French three-factor model

Fama and French (1993, 1995, 1996) argue factors describing "value" (book-to-market equity ratio) and "size" to be the most relevant factors, in addition to market risk, for explaining and capturing the cross-sectional variation in average stock returns. To take these risks into account, they used the excess market returns (Mkt) as market risk of the CAPM and they constructed two factors to address size risk and to deal with value risk.

The Fama-French three-factor model is as follows:

$$R_{it} - R_{\mathrm{ft}} = \alpha_i + \beta_i \mathrm{Mkt}_t + \gamma_i \mathrm{SMB}_t + \delta_i \mathrm{HML}_t + \varepsilon_{it}.$$

In the equation below, Mkt mimics the "market premium" and represents the excess market returns: $(R_{\mathrm{mt}} - R_{\mathrm{ft}})$ where R_{mt} and R_{ft} are, respectively, the market return and the free rate.

SMB which stands for small-minus big, is designed to measure the additional return that investors have historically received by investing in stocks of companies with relatively small market capitalization. This additional return is often referred to as the "size premium".

HML which is short for high-minus low, has been constructed to measure the "value premium" provided to investors for investing in companies with high book-to-market values (essentially, the value placed on the company by accountants as a ratio relative to the book equity (BE) divided by the market equity (BM) of the company, commonly expressed as BE/BM.

The quantities β_i, γ_i and δ_i are the factor sensitivities of the portfolio i to the state variables obtained from the multiple regression of the excess return of this portfolio, designed by $(R_{\mathrm{it}} - R_{\mathrm{ft}})$ where R_{it} is his associated return on Mkt, SMB and HML, respectively. α_i is the abnormal return of the portfolio i.

The Fama-French three factors are constructed on the basis of six value-weighted portfolios based on size and book-to-market equity ratio (BE/BM). As a first step, all stocks are ordered and divided into two groups formed on size: group small and group big, which are indicated, respectively, by S and B. The size breakpoint for a given year is the median market capitalization at the end of June of that year. Secondly, three groups of stocks are built up on the ratio of book-to-market. The ratio BE/BM for year t is the BE for the fiscal year ending in $(t-1)$ divided by the market equity (ME) for December of $(t-1)$. The BE/BM breakpoints are the 30th and the 70th percentiles corresponding to the groups indicated by low (L), medium (M) and high (H). As a final step, the six value-weight portfolios formed at the end of each June are the intersections of the two size and the three BE/BM-based portfolios. For each obtained portfolio, which is hereafter labeled SL, SM, SH, BL, BM, or BH, we calculate the corresponding returns denoted by R_{SL}, R_{SM}, R_{SH}, R_{BL}, R_{BM}, and R_{BH}.

The methodology for constructing the size and the risk factors is as follows: SMB is the difference between the monthly average return on the three small portfolios and the monthly average return on the three big portfolios:

$$\mathrm{SMB} = \bar{R}_{\mathrm{S}} - \bar{R}_{\mathrm{B}} = \frac{1}{3}[R_{\mathrm{SL}} + R_{\mathrm{SM}} + R_{\mathrm{SH}}] - \frac{1}{3}[R_{\mathrm{BL}} + R_{\mathrm{BM}} + R_{\mathrm{BH}}].$$

HML is the difference between the monthly average return on the two portfolios within the high group and the monthly average return on the two portfolios with low BE/BM ratio:

$$\mathrm{HML} = \bar{R}_{\mathrm{H}} - \bar{R}_{\mathrm{L}} = \frac{1}{2}[R_{\mathrm{SH}} + R_{\mathrm{BH}}] - \frac{1}{2}[R_{\mathrm{SL}} + R_{\mathrm{BL}}].$$

Wavelet analysis

The wavelet transform is a time-scale representation that describes the time evolution of a given signal on a scale-by-scale basis. It is similar to standard Fourier transform. Yet, the infinite support sine and cosine basis functions are substituted by wavelet functions that are, actually, dilated and/or translated versions of a unique basic function.

Wavelet functions can be considered as special filters that have particular characteristics. Actually, the wavelet transform can be defined in terms of a length-L

high-pass wavelet filter $\{h_l, l = 0, \ldots, L-1\}$ and its associated low-pass scaling filter $\{g_l, l = 0, \ldots, L-1\}$ linked by the "quadrature mirror" relation:

$$g_l = (-1)^{l+1} h_{(L-1)-l} \quad \text{with } l = 0, \ldots, L-1.$$

There are various families of wavelet bases. Daubechies (1992) constructed a special class of wavelet filters of even length L that improve the frequency-domain characteristics of the Haar wavelet. The Daubechies wavelet filters, denoted by $D(L)$are orthogonal, compactly supported, possess different degrees of smoothness and admit a number of vanishing moments.

Let $\mathbf{X}$ be a length N vector containing a real-valued time series $\{X_t, t = 0, \ldots, N-1\}$ and let J_0 a positive integer. The maximal overlap discrete wavelet transform (MODWT) of level J_0 maps the input data vector $\mathbf{X}$ from the time domain to the wavelet time-scale domain yielding $J_0 + 1$ vectors: $\tilde{\mathbf{w}}_1, \tilde{\mathbf{w}}_2, \ldots, \tilde{\mathbf{w}}_{J_0}$ and $\tilde{\mathbf{v}}_{J_0}$, each of dimension N. Here, $\tilde{\mathbf{w}}_j, j = 1, \ldots, J_0$ are vectors composed of the MODWT wavelet coefficients associated with changes on scale $\tau_j = 2^{j-1}$ while $\mathbf{v}_J$ contains the MODWT scaling coefficients associated with averages on scale $\iota_{J_0} = 2^{J_0}$. With matrix notation, the MODWT coefficients are obtained via: $\mathbf{w}_j = \tilde{\mathbf{W}}_j \mathbf{X}, \quad j = 1, \ldots, J_0$ and $\mathbf{v}_J = \tilde{\mathbf{V}}_{J_0} X$ where the matrices $\tilde{\mathbf{W}}_j$ and $\tilde{\mathbf{V}}_{J_0}$ contain the MODWT wavelet and scaling filter coefficients.

In actual fact, matrices $\tilde{\mathbf{W}}_j$ and $\tilde{\mathbf{V}}_{J_0}$ are not explicitly constructed and the MODWT is performed via a pyramidal algorithm.

Renormalizing the wavelet filters $\{h_l\}_{l=0}^{L-1}$ and $\{g_l\}_{l=0}^{L-1}$ as follows: $\tilde{h}_l = h_l / 2^{l/2}$ and $\tilde{g}_l = g_l / 2^{l/2}$ and (starting with) setting $X_t = \tilde{w}_{0,t}$, the MODWT coefficients are obtained through the following filtering steps:

$$\tilde{w}_{j,t} = \sum_{l=0}^{L-1} \tilde{h}_l \tilde{v}_{j-1,t-2^{j-1}l \bmod N} \text{ and } \tilde{v}_{j,t} = \sum_{l=0}^{L-1} \tilde{g}_l \tilde{v}_{j-1,t-2^{j-1}l \bmod N}, \quad t = 0, \ldots, N-1.$$

If we write $\{w_{j,t} : t = 0, \ldots, N_j - 1\}$ as $\tilde{\mathbf{w}}_j$ and $\{v_{j,t} : t = 0, \ldots, N_j - 1\}$ as $\tilde{\mathbf{v}}_j$, then, after J_0 iterations, the pyramidal algorithm yields $\tilde{\mathbf{w}}_1, \tilde{\mathbf{w}}_2, \ldots, \tilde{\mathbf{w}}_{J_0}$ and $\tilde{\mathbf{v}}_{J_0}$.

It should be stressed that we can get an additive decomposition of a time series $\{X_t, t = 0, \ldots, N-1\}$. In fact, we have:

$$X = \sum_{j=1}^{J_0} \tilde{\mathbf{W}}_j^{\mathrm{T}} \mathbf{w}_j + \tilde{\mathbf{V}}_{J_0}^{\mathrm{T}} \mathbf{v}_{J_0} = \sum_{j=1}^{J_0} \tilde{d}_j + \tilde{s}_{J_0}.$$

The latter equation defines a multiresolution analysis (MRA) of X. Whereas the jth level wavelet detail $\tilde{d}_j = \tilde{\mathbf{W}}_j^{\mathrm{T}} \mathbf{w}_j$ describes changes at a scale $\tau_j = 2^{j-1}$ corresponding to frequency bands $[1/2^{j+1}, 1/2^j]$, $\tilde{s}_{J_0}$ is referred to as the smooth and is associated with averages at a scale $\iota_{J_0} = 2^{J_0}$. The smooth represents the trend component of a time series, while the wavelet details capture deviations from that trend.

It is noteworthy that each wavelet detail $\tilde{d}_j$ correspond to a frequency band $[1/2^{j+1}, 1/2^j]$ and thus to $[2^j, 2^{j+1}]$ time period, e.g. for monthly data, the level-one wavelet detail $\tilde{d}_1$ is associated with oscillations of two to four months period.

When performing the MODWT filtering algorithms for finite length vectors of observations, we assume that the signal is periodic with period N. This raises the problem of boundary effects affecting the wavelet coefficients near the beginning or the end of the time series, i.e. at "the boundaries". Within the MODWT set-up, Percival and Walden (2000) have shown that the coefficients affected by the boundary are $\tilde{w}_{j,t}$ and $\tilde{v}_{j,t}$, where $t = 0, K, \min\{L_j - 2, N\}$ and $L_j = (2^j - 1)(L - 1) + 1$.

A key concept in wavelet theory is the wavelet covariance $\gamma_X(\tau_j)$, which decomposes the covariance between two stochastic processes on scale-by-scale basis.

Formally, let $X_t = (x_{1,t}, x_{2,t})$ be a length N bivariate stochastic process and let $w_{j,t} = (w_{1,j,t}, w_{2,j,t})$ be the wavelet coefficients issued from the wavelet transform of $x_{1,t}$ and $x_{2,t}$ at a resolution scale τ_j. It should be stressed here that each wavelet-coefficient process is derived by implementing the wavelet transform for each series in X_t one at a time. The wavelet covariance at scale τ_j is then given by:

$$\gamma_X(\tau_j) = \frac{1}{2\tau_j} \operatorname{cov}(w_{1,j,t}, w_{2,j,t}).$$

An unbiased estimator of the wavelet covariance is constructed using the MODWT as follows:

$$\hat{\gamma}_X(\tau_j) = \frac{1}{\tilde{N}_j} \sum_{l=L_j-1}^{N-1} \tilde{w}_{1,j,t} \tilde{w}_{2,j,t},$$

where $\tilde{N}_j = N - L_j + 1$ is the number of non-boundary wavelet coefficients at level j and $\tilde{w}_{1,j,t}$ and $\tilde{w}_{2,j,t}$ are the MODWT coefficients of the bivariate process X_t.

Equivalently, we can define the MODWT-unbiased wavelet correlation by normalizing the unbiased wavelet covariance in the following way:

$$\hat{\rho}_X(\tau_j) = \frac{\hat{\gamma}_X(\tau_j)}{\sqrt{\hat{v}_1^2 \hat{v}_2^2}},$$

where $\hat{v}_i^2 = (1/\tilde{N}_j) \sum_{t=L_j-1}^{N-1} \tilde{w}_{j,t}^2, \quad i = 1, 2$ are the unbiased wavelet variances for $x_{1,t}$ and $x_{2,t}$, respectively.

Multiscale Fama-French three-factor model

The proposed methodology, which was initiated by Kim and In (2006, 2007), entails two steps. First, we perform a MODWT-based MRA curtailed at a resolution level J_0 for the series relative to excess portfolio return, Mkt, SMB and HML. Every series can be then expressed as the sum of the smooth component and the wavelet details. As a second step and at each resolution scale $\tau_j = 2^{j-1}$, we consider the following time series regression scheme hereafter called the multiscale Fama-French three-factor model:

$$R_{it}(\tau_j) - R_{\mathrm{ft}}(\tau_j) = \alpha_i(\tau_j) + \beta_i(\tau_j)\mathrm{Mkt}_t(\tau_j) + \gamma_i(\tau_j)\mathrm{SMB}_t(\tau_j) + \delta_i(\tau_j)\mathrm{HML}_t(\tau_j) + \varepsilon_{it}(\tau_j),$$

where $R_{it}(\tau_j), j = 1, \ldots, J_0$, is the excess return over the risk free rate on a buy-and-hold portfolio i at time t and scale $\tau_j = 2^{j-1}$. $\mathrm{Mkt}_t(\tau_j)$, $\mathrm{SMB}_t(\tau_j)$ and $\mathrm{HML}_t(\tau_j)$ are the Fama-French risk factors at time t and scale $\tau_j = 2^{j-1}$. The intercept $\alpha_i(\tau_j)$ is the measure of the abnormal performance or pricing error at scale τ_j. The residual $\varepsilon_{it}(\tau_j)$, is a zero mean abnormal portfolio return unexplained by common risk factors at scale τ_j.

$\beta_i(\tau_j)$, $\gamma_i(\tau_j)$ and $\delta_i(\tau_j)$ are the assigned loadings at scale τ_j for the market, the size and the value factors, respectively.

The underlying idea behind the suggested approach is that financial time series exhibit "multi scaling phenomenon", i.e. exhibit different dynamics each occurring at a particular time horizon. Exploiting the scale separation characteristics of the wavelet transform, the MRA isolates low-frequency dynamics from high-frequency dynamics resulting in a set of sub-series that evolve at different scales. Hence, investigating the relationship between the risk factors and the portfolio returns over different time scales would allow analyzing the investment strategies and decision making on different time periods.

Data description

Our investigation deals with the French market over the period ranging from January 1985 to October 2006. It is worth mentioning that the start and the end dates are determined by the data availability. The data set are measured at monthly frequencies totalling 256 observations for each variable. The 13-week (91-day) treasury bill rate is used as a risk free rate and the Cac40 as a market index.

Descriptive statistics for the portfolios returns are reported in columns (2:7) of Table I.

All sample means are negatively valued ranging from -0.051 (SL) to -0.045 (SH). Looking at the standard deviation estimation results, we see that the portfolios show almost the same degree of volatility. All analyzed datasets present significant kurtosis values greater than 3. This indicates that the six portfolios are characterized by heavy-tailed leptokurtic distributions. While examining the skewness coefficients estimates, we observe that the return series are negatively skewed as compared to the Gaussian distribution. Deviation from normality is further confirmed by the Jarque-Bera test statistics, which are highly significant at 1 per cent statistical level thus rejecting the null hypothesis of Gaussianity. For the six portfolios, the Box-Pierce test statistics for up to 20th-order serial correlation point towards the existence of linear and non-linear dependencies.

Looking at the summary statistics for the three Fama-French factors given in columns (8:10) of Table I, we see that the statistical results concerning the excess market factor do not differ significantly from those related to the six portfolios in terms of sample mean, standard deviation, skewness, kurtosis and Box-Pierce statistics' values while the Jarque-Bera test rejects the normal distribution at 5 per cent level of statistical significance. However, in contrast to the other datasets, the SMB and the HML variables show positive sample means. Moreover, compared to the excess market returns, the standard deviations for the size and value factors are smaller indicating that these two variables are more volatile. Nevertheless, whereas the descriptive statistics point out that the HML variable is positively skewed and the normality assumption is rejected at the1 per cent level, SMB is slightly negatively skewed with a skewness value of -0.082. In addition, the Jarque-Bera test fails to reject the null hypothesis of Gaussianity.

Table II reports the sample correlations between the three risk factors.

The HML portfolio returns and the excess market returns are positively correlated. In contrast to Fama and French (1993), SMB and market factor have negative correlation. Molay (1999) argues that this negative correlation can be due to the fact

Table I. Summary statistics

	SH	SM	SL	BH	BM	BL	Mkt	SMB	HML
Mean	−0.045	−0.047	−0.051	−0.048	−0.048	−0.049	−0.05	0.0004	0.003
Median	−0.039	−0.038	−0.041	−0.041	−0.039	−0.046	−0.042	0.003	0.001
Maximum	0.206	0.213	0.178	0.149	0.136	0.111	0.138	0.077	0.119
Minimum	−0.309	−0.286	−0.375	−0.3	−0.28	−0.329	−0.325	−0.083	−0.092
SD	0.066	0.061	0.068	0.059	0.062	0.059	0.068	0.027	0.031
Skewness	−0.199	−0.267	−0.569	−0.567	−0.373	−0.4	−0.290	−0.082	0.476
Kurtosis	4.308	4.616	4.883	4.397	3.588	4.469	3.647	3.422	4.36
J-B	19.94***	30.88***	51.63***	34.506***	9.635***	29.85***	8.056**	2.187	29.4***
Q(20)	137.82***	187.14***	78.97***	138.30***	227.14***	191.16***	145.05***	23.38	48.9***

Notes: *, ** and *** indicate rejection significance at: 10, 5 and 1 per cent levels, respectively. J-B is the value of Jarque-Bera test statistics of the return residuals. *Q*(20) is the Box-Pierce test statistics for the return residuals for up to 20th-order serial correlation that market portfolio is value weighted and shows via empirical investigation that when dealing with an equally weighted portfolio, SMB and Mkt become positively correlated. Interestingly, the correlation coefficient between HML and SMB, although small-valued, is negative.

Estimation results for the single-scale Fama-French model

In this section, we present the empirical results drawn from the implementation of the Fama-French model during the period of study. For sake of conciseness, we do not report the constant terms' estimates. Nevertheless, the values of the constant terms are generally not statistically significant at conventional levels. The estimation results are given in the Table III.

First, when investigating the market-factor effect, we see that all estimated market coefficients $\hat{\beta}(\tau_j)$ are statistically significant at the 1 per cent level. The point estimates display only positive values. It should be noticed that the market effect is nearly the same across the constructed portfolios with the coefficients' values lying in the range 0.898 (SM) $-$ 0.917 (SL). Hence, we can state that the market risk is a key variable in capturing the cross-section of average stock returns regardless of the assets forming the portfolios.

For the value risk factor proxied by the explanatory variable HML, empirical results suggest that the book-to-market ratio has a non-negligible impact on the stock returns regardless of the portfolios as the value coefficients are highly significant. In terms of their signs, the results show a clear pattern. In fact, for two small-value stock portfolios (SL and BL) estimated coefficients are negative, whereas for the portfolios characterized by large and medium ratio, HML regression coefficients are of positive signs.

For the size factor represented by SMB, significant positive relationships can be observed for all portfolios. The estimated size effect is more pronounced for small

Table II. Correlation between factors

	Mkt	HML	SMB
Mkt	1.0000	0.1176	−0.0530
HML	0.1176	1.0000	−0.0239
SMB	−0.0530	−0.0239	1.0000

Table III. Single-scale Fama-French estimation results

	SH	SM	SL	BH	BM	BL
MKT	0.916	0.898	0.917	0.914	0.903	0.913
	29.5	28.56	29.72	29.95	29.55	30.45
SMB	1.239	1.125	1.35	0.305	0.215	0.194
	12.94	11.59	14.19	3.24	2.28***	2.1***
HML	0.819	0.316	−0.483	0.586	0.178	−0.112
	9.76	3.71	−5.79	7.1	2.15***	−1.35*
R^2	0.820	0.790	0.804	0.798	0.779	0.784

Notes: *, ** and *** indicate rejection significant at: 10, 5 and 1 per cent levels, respectively portfolios than for big ones. These results are in accordance with the findings of Fama and French (1995) who found that the exposure to size risk is particularly important for small portfolios. Moreover, estimation results show that the sensitivities to size risk for the big portfolios decrease when the book-to-market ratio for theses portfolios decrease.

When looking at the adjusted R^2 ($\bar{R}^2$) values, it is clear that the Fama-French three-factor model captures common variations in stock returns quite well. Actually, for the six portfolios, the $\bar{R}^2$ values vary between 0.779 (BM) and 0.82 (SH) with an average $\bar{R}^2$ of approximately 80 per cent.

Estimation results for the multiscale Fama-French model

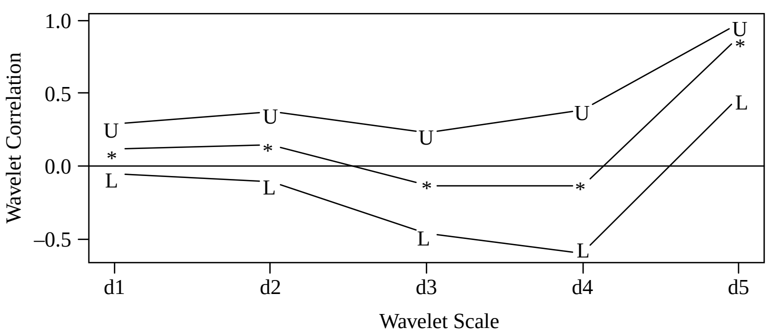
Before reporting the parameters' estimates of the wavelet Fama-French model, we examine the wavelet correlations between the three risk factors over different time-scales. As can be seen from Figure 1, a positive relationship between the market and the HML factors can be observed over scales d_1 (two to four months period) and d_5 (32-64 months period), while for intermediate wavelet resolution levels, a negative relationship exists between the two series.

Looking at Figure 2, the wavelet correlations between the market excess returns and the SMB series increase with increasing resolution level. These correlations are positive and appear to be statistically significant at the shortest scale d_1 and the largest scale d_5.

Figure 3 shows that the wavelet correlation, between the size and the value factors, is an increasing function of the wavelet scale. It should be remarked here that the correlation takes negative values for scales $d_1 - d_2$ corresponding to two to four months and four to eight months periods whereas at the large-scales $d_4 - d_5$, the wavelet correlations are positively valued thus reflecting a positive association between the two factors. In sum, the correlation between the three factors if found to be scale-dependent; the wavelet MRA seems to be a well-adapted framework for studying and investigating time-scale correlation's characteristics.

We note also that the confidence intervals are significantly increased given the amount of variability in the estimated wavelet variances.

Estimation results are presented in Table IV. Let us retain the following remarks. For the excess market return, the estimated coefficients $\hat{\beta}(\tau_j)$ are all statistically significant at usual levels for all portfolios regardless of the resolution scale.

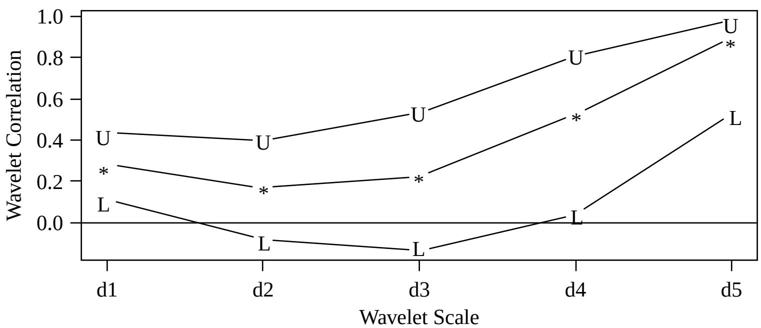


Note: The dashed lines with the characters "L" and "U" indicate the lower and upper bounds for the approximate 95 per cent confidence intervals, respectively

Figure 1. Estimated MODWT-wavelet correlations between the market factor and the HML factor

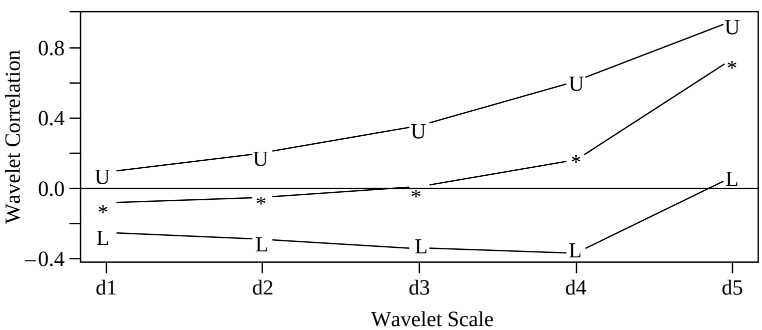
This shows that the excess market return plays a significant important role in explaining the cross-sectional variation of the six analyzed portfolios over the range of time-scales. Nevertheless, it should be stressed here that estimated coefficients show positive values for all scales except for the smallest high-frequency scale d_1 (level 1, covering a period of two to four months). In fact for d_1, we have a negative effect that is almost constant for all the portfolios with coefficients values ranging from -0.39 to -0.45. Another interesting feature here is that the impact of the market factor increases when increasing the multiresolution scale, i.e. with time period. Compared to the results obtained by the single scale Fama-French model, the coefficients in the multi-sale framework are lower-valued reflecting a lower impact of the market factor for all the portfolios.

Looking at the estimated coefficients $\hat{\gamma}(\tau_j)$ and their corresponding t-statistics, we see that SMB factor loadings are significant at the 5 per cent level for all time-scales. Exceptions are the estimates associated with the scale d_1 for the portfolio BH and those related to the intermediate scale d_4 for the BM and BL. It is noteworthy that coefficients $\hat{\gamma}(\tau_j)$ show positive values at the long-term for all portfolios, i.e. at scales d_4 and d_5. These coefficients' values are increasing with the time scales. Nevertheless, when considering the high frequency (short and medium term) d_1, d_2 and d_3 the results



Note: The dashed lines with the characters "L" and "U" indicate the lower and upper bounds for the approximate 95 per cent confidence intervals, respectively

Figure 2. Estimated MODWT-wavelet correlations between the market factor and the SMB factor



Note: The dashed lines with the characters "L" and "U" indicate the lower and upper bounds for the approximate 95 per cent confidence intervals, respectively

Figure 3. Estimated MODWT-wavelet correlations between the HML factor and the SMB factor

Table IV. Multiscale estimation parameters

	d_1				d_2				d_3				d_4				d_5			
	Mkt	SMB	HML	R^2	Mkt	SMB	HML	R^2	Mkt	SMB	HML	R^2	Mkt	SMB	HML	R^2	Mkt	SMB	HML	R^2
SH	−0.445	0.889	0.809	0.266	0.341	0.629	0.192	0.256	0.807	0.566	1.229	0.790	0.898	1.156	0.747	0.861	0.932	1.214	0.674	0.946
	−4.729	9.457	8.601		3.105	5.732	1.752**		9.831	6.893	14.959		9.789	12.605	8.142		14.074	18.333	10.176	
SM	−0.418	0.637	0.188	0.274	0.360	0.622	−0.200	0.310	0.795	0.588	0.532	0.718	0.756	0.877	0.361	0.735	0.646	1.183	0.232	0.917
	−4.355	6.631	1.954**		3.724	6.446	−2.075***		8.668	6.41	5.803		6.969	8.086	3.33		11.247	20.594	4.04	
SL	−0.397	0.889	−0.617	0.309	0.355	0.708	−0.987	0.436	0.744	0.754	−0.072	0.696	0.898	1.498	−0.420	0.819	0.911	1.725	−0.545	0.945
	−3.853	8.629	−5.988		3.395	6.775	−9.447		7.84	7.95	−0.755*		8.324	13.888	−3.896		13.228	25.048	−7.921	
BH	−0.407	−0.190	0.466	0.249	0.328	−0.262	0.107	0.175	0.701	−0.232	0.892	0.585	0.873	0.454	0.666	0.794	0.788	0.679	0.460	0.910
	−4.047	−1.89**	4.637		3.44	−2.753	1.121*		7.189	−2.385***	9.147		8.312	4.324	6.342		12.805	11.031	7.484	
BM	−0.398	−0.206	0.022	0.282	0.414	−0.437	−0.389	0.212	0.882	−0.438	0.606	0.743	0.804	−0.037	0.188	0.771	0.892	0.276	0.220	0.908
	−4.15	−2.148***	0.231*		3.651	−3.853	−3.429		10.954	−5.447	7.529		8.247	−0.38*	1.928**		14.446	4.462	3.568	
BL	−0.455	−0.189	−0.108	0.362	0.314	−0.341	−0.713	0.278	0.764	−0.421	0.192	0.662	0.873	0.113	−0.167	0.867	0.809	0.168	−0.320	0.922
	−5.002	−2.08***	−1.192*		3.234	−3.513	−7.353		9.114	−5.015	2.289***		11.068	1.427*	−2.115***		14.788	3.071	−5.853	

Notes: *, ** and *** indicate rejection significant at: 10, 5 and 1 per cent levels, respectively diverge between small and big portfolios. In fact, while for small stocks, the estimated coefficients are positive showing a positive impact of the SMB factor, they are negatively valued for large portfolios. This confirms the Fama-French findings that small capitalization stocks are more profitable than those with big market capitalization.

When focusing on the HML coefficients estimates, we observe that the book-to-market effect is rejected at the resolution scale d_1 for the portfolios BM and BL, at scale d_2 for the portfolios SH and BH, at scale d_3 for the portfolio SL and BL and at scale d_4 for the portfolio BL and BM as the corresponding loadings' estimates are not statistically significant at conventional levels. Hence, the resolution level d_5 is the only wavelet scale for which all coefficients are significantly different from zero for all portfolios. Apart from these exceptions, all other estimates are highly significant at the 1 per cent level leading to rejection of the null hypothesis $H_0 : \delta_j = 0$ for large investment horizons. However, it is worthy to note here that discrimination can be made between stocks with high book-to-market ratio and those with low book-to-market ratio.

In fact, similar to the results drawn from the unit-scale Fama-French model, the estimated coefficients for the value risk factor HML display positive values for stocks with the highest BE/BM ratio, known as value stocks, whereas these coefficients are negatively valued for stocks with the lowest book-to-market equity ratio, known as growth stocks.

Within the multiscale framework, the goodness-of-fit of the Fama-French three-factor model measured by the adjusted coefficients of determination $\bar{R}^2$ is scale-sensitive. In fact, for the small scales d_1 and d_2, capturing the short-term dynamics (two to four months and four to eight months, respectively), estimated $\bar{R}^2$ are very low-valued ranging from 0.175 for the portfolio BH at scale d_2 to 0.362 for the portfolio BL at the smallest scale d_1. This indicates that the three risk factors possess limited power in explaining cross-sectional variation of the stock returns in the French market. However, for medium- and high-resolution scales, the explanatory power of the Fama-French improves considerably while increasing the investment periods. The adjusted coefficients of determination $\bar{R}^2$ display very high values attaining 0.94 for the largest wavelet scale d_5 associated with the longest investment horizon. This demonstrates the relevance of the three risk factors in explaining the returns of the French stocks.

Conclusions

In this paper, a multiscale Fama-French three-factor model has been investigated in the French market. Using a time-scale mapping induced by the MODWT-based MRA, the data set are decomposed into dynamics associated with different time periods. Within the multiscale framework, the relationships between expected returns and the three risk factors have been examined over different investment horizons.

Empirical results show that the multiresolution approach improves the explanatory power of the Fama-French model as compared to the single scale model. It should be stressed here that the highest $\bar{R}^2$ are obtained for the medium- and long-term scales (especially for more than 12 months period). Moreover, we have shown that the risk-sensitivity strongly depends on the time scale as the estimated factor loadings exhibit different values and are of different signs depending on the considered wavelet resolution level. This information was hidden within the single scale set up, i.e. the classical Fama-French model.

Another contribution of this paper is the correlation analysis between the expected returns and the risk factors at different time scales using wavelet techniques.

In view of the obtained results, investors can adopt the wavelet-based pricing scheme to conduct risk-based analysis for heterogeneous investment horizons. The multiscale Fama-French framework also permits to build adequate portfolio allocation strategies *vis-à-vis* different targeted time periods. Besides, it is known that fund ratings play a significant role in fund selection for investors either for an initial subscription or with the objective of readjusting their investments in function of rating upgrades or downgrades. Therefore, time-scale models mentioned above can address the required properties for an optimal rating system by handling the time-varying nature of the risk factors.

As future research directions, we intend to construct a higher-order version of the multiscale Fama-French model. This could be done by incorporating additional pricing factors such as systematic co-skewness and systematic co-kurtosis. This new methodology is expected to further enhance the explanatory power of the model by taking into account the excess kurtosis and the skewness coefficients reported in Table I which are indicative of non-normality.

References

Daubechies, I. (1992), "Ten lectures on wavelets", *CBMS-NSF Regional Conference Series on Applied Mathematics*, Vol. 61, SIAM, Philadelphia, PA.

Fama, E. and French, K. (1992), "The cross-section of expected stock returns", *Journal of Finance*, Vol. 47, pp. 427-65.

Fama, E. and French, K. (1993), "Common risk factors in the returns on stocks and bonds", *Journal of Financial Economics*, Vol. 33, pp. 3-56.

Fama, E. and French, K. (1995), "Size and book-to-market factors in earnings and returns", *Journal of Finance*, Vol. 50, pp. 131-56.

Fama, E. and French, K. (1996), "Multifactor explanations of asset pricing anomalies", *Journal of Finance*, Vol. 41, pp. 55-84.

Fama, E. and French, K. (1998), "Value versus growth: the international evidence", *Journal of Finance*, Vol. 43, pp. 1975-98.

Kim, S. and In, F. (2006), "The relationship between Fama-French three risk factors, industry portfolio returns, and industrial production", available at SSRN: http://ssrn.com/abstract = 891567

Kim, S. and In, F. (2007), "A note on the relationship between Fama-French risk factors and innovations of ICAPM state variables", *Finance Research Letters*, Vol. 4 No. 3, pp. 165-71.

Lintner, J. (1965), "The valuation of risky assets and the selection of risky investments in stock portfolios and capital budgets", *Review of Economics and Statistics*, Vol. 47, pp. 13-37.

Molay, E. (1999), "Le modèle de rentabilitè á trois facteurs de Fama et French (1993): le cas français", papier de recherche, CEROG, Aix-Marseille.

Percival, D.B. and Walden, A.T. (2000), *Wavelet Methods for Time Series Analysis*, Cambridge University, Cambridge.

Sharpe, W. (1964), "Capital asset prices: a theory of market equilibrium under conditions of risk", *Journal of Finance*, Vol. 19, pp. 425-42.

Further reading

Brockwell, P.J. and Davis, R.A. (1991), *Time Series: Theory and Methods*, 2nd ed., Springer, New York, NY.

Gençay, R., Whitcher, B. and Selçuk, F. (2001), *An Introduction to Wavelets and other Filtering Methods in Finance and Economics*, Academic Press, London.

Gençay, R., Whitcher, B. and Selçuk, F. (2003), "Systematic risk and time scales", *Quantitative Finance*, Vol. 3, pp. 108-16.

Gençay, R., Whitcher, B. and Selçuk, F. (2005), "Multiscale systematic risk", *Journal of International Money and Finance*, Vol. 24, pp. 55-70.

Kim, S. and In, F. (2005), "Stock returns and inflation: new evidence from wavelet analysis", *Journal of Empirical Finance*, Vol. 12, pp. 435-44.

Mallat, S. (1989), "A theory for multiresolution signal decomposition: the wavelet representation", *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol. 11 No. 7, pp. 674-93.

Rhaim, N., Benammou, S. and Ben Mabrouk, A. (2007), "Wavelet estimation of systematic risk at different time scales: application to French stock market", *The International Journal of Applied Economics and Finance*, Vol. 1, pp. 113-9.

Corresponding author
Anyssa Trimech can be contacted at: trimech.anyssa@yahoo.fr