

# ON SEMIPARAMETRIC MODE REGRESSION ESTIMATION

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Abstract It has been found that, for a variety of probability distributions, there is a surprising linear relation between mode, mean and median. In this paper, the relation between mode, mean and median regression functions is assumed to follow a simple parametric model. We propose a semiparametric conditional mode (mode regression) estimation for an unknown (unimodal) conditional distribution function in the context of regression model, so that any  $m$ -step-ahead mean and median forecasts can then be substituted into the resultant model to deliver  $m$ -step-ahead mode prediction. In the semiparametric model, Least-Squared Estimator (LSE) for the parameters of this model and the simultaneous estimation of the unknown mean and median regression functions by the local linear kernel method are combined to infer the parametric and nonparametric components of the proposed model. The asymptotic normality of these estimators is derived, and the asymptotic distribution of the parameter estimates is also given and is shown to follow usual parametric rates in spite of the presence of the nonparametric component in the model. These results are applied to obtain a data-based test for the dependence of mode regression over mean and median regression under a regression model.

***Keywords** : Asymptotic normality, hypothesis testing, local linear kernel estimate, mode, prediction, rate of convergence, semiparametric regression*