Reflexivity/irreflexivity

"A binary relation R over a set X is reflexive if every element of X is related to itself. Formally, this may be written $\forall x \in X : x \, R \, x$"[1]. A binary relation is called irreflexive if it doesn't relate any element to itself[1].

Let $I = [I^-, I^+]$ be imprecise time intervals. Based on the definitions of our proposed temporal relations, it holds that:

$$\text{Before}(I, I) = \text{Precedes}(I^{+(E)}, I^{-(1)}) = 0 \quad (1)$$

$$\text{After}(I, I) = \text{Before}(I, I) = 0 \quad (2)$$

$$\text{Meets}(I, I) = \text{Same}(I^{+(1)}, I^{-(1)}) \wedge \text{Same}(I^{+(E)}, I^{-(B)}) = 0 \quad (3)$$

$$\text{Met-by}(I, I) = \text{Meets}(I, I) = 0 \quad (4)$$

$$\text{Overlaps}(I, I) = \text{Precedes}(I^{-(B)}, I^{-(1)}) \wedge \text{Precedes}(I^{-(B)}, I^{+(1)}) \wedge \text{Precedes}(I^{+(E)}, I^{+(1)}) = 0 \quad (5)$$

$$\text{Overlapped-by}(I, I) = \text{Overlaps}(I, I) = 0 \quad (6)$$

$$\text{Starts}(I, I) = \text{Same}(I^{-(1)}, I^{-(1)}) \wedge \text{Same}(I^{-(B)}, I^{-(B)}) \wedge \text{Precedes}(I^{+(E)}, I^{+(1)}) = 0 \quad (7)$$

$$\text{Started-by}(I, I) = \text{Starts}(I, I) = 0 \quad (8)$$

$$\text{During}(I, I) = \text{Precedes}(I^{-(B)}, I^{-(1)}) \wedge \text{Precedes}(I^{+(E)}, I^{+(1)}) = 0 \quad (9)$$

$$\text{Contains}(I, I)) = \text{During}(I, I) = 0 \quad (10)$$

$$\text{Ends}(I, I) = \text{Precedes}(I^{-(B)}, I^{-(1)}) \wedge \text{Same}(I^{+(1)}, I^{+(1)}) \wedge \text{Same}(I^{+(E)}, I^{+(E)}) = 0 \quad (11)$$

$$\text{Ended-by}(I, I) = \text{Ends}(I, I) = 0 \quad (12)$$

$$\text{Before}_{(k)}^{(\alpha, \beta)}(I, I) = \text{Precedes}_{(k)}^{(\alpha, \beta)}(I^{+(E)}, I^{-(1)}) = 0 \quad (13)$$

$$\text{After}_{(k)}^{(\alpha, \beta)}(I, I) = \text{Before}_{(k)}^{(\alpha, \beta)}(I, I) = 0 \quad (14)$$

$$\text{Overlaps}_{(k)}^{(\alpha, \beta)}(I, I) = \text{Precedes}\,(I^{-(B)}, I^{-(1)}) \wedge \text{Precedes}_{(k)}^{(\alpha, \beta)}(I^{-(B)}, I^{+(1)}) \wedge \text{Precedes}_{(k)}^{(\alpha, \beta)}(I^{+(E)}, I^{+(1)}) = 0 \quad (15)$$

$$\text{Overlapped-by}_{(k)}^{(\alpha, \beta)}(I, I) = \text{Overlaps}_{(k)}^{(\alpha, \beta)}(I, I) = 0 \quad (16)$$

$$\text{Starts}_{(k)}^{(\alpha, \beta)}(I, I) = \text{Same}(I^{-(1)}, J^{-(1)}) \wedge \text{Same}(I^{-(B)}, J^{-(B)}) \wedge \text{Precedes}_{(k)}^{(\alpha, \beta)}(I^{+(E)}, J^{+(1)}) = 0 \quad (17)$$

$$\text{Started-by}_{(k)}^{(\alpha, \beta)}(I, I) = \text{Starts}_{(k)}^{(\alpha, \beta)}(I, I) = 0 \quad (18)$$

$$\text{During}_{(k)}^{(\alpha, \beta)}(I, I) = \text{Precedes}_{(k)}^{(\alpha, \beta)}(I^{-(B)}, I^{-(1)}) \wedge \text{Precedes}(I^{+(E)}, I^{+(1)}) = 0 \quad (19)$$

$$\text{Contains}_{(k)}^{(\alpha, \beta)}(I, I) = \text{During}_{(k)}^{(\alpha, \beta)}(I, I) = 0 \quad (20)$$

$$\text{Ends}_{(k)}^{(\alpha, \beta)}(I, I) = \text{Precedes}_{(k)}^{(\alpha, \beta)}(I^{-(B)}, I^{-(1)}) \wedge \text{Same}(I^{+(1)}, I^{+(1)}) \wedge \text{Same}(I^{+(E)}, I^{+(E)}) = 0 \quad (21)$$

$$\text{Ended-by}_{(k)}^{(\alpha, \beta)}(I, I) = \text{Ends}_{(k)}^{(\alpha, \beta)}(I, I) = 0 \quad (22)$$

[1] https://en.wikipedia.org/wiki/Reflexive_relation

From the equations {(1)....(23)}, we deduce that the temporal relations {“Before”, “After”, “Meets”, “Met-by”, “Overlaps”, “Overlapped-by”, “Starts”, “Started-by”, “During”, “Contains”, “Ends”, “Ended-by”, “Before(k) (α, β)”, “After(k) (α, β)”, “Overlaps(k) (α, β)”, “Overlapped-by(k) (α, β)”, “Starts(k) (α, β)”, “Started-by(k) (α, β)”, “During(k) (α, β)”, “Contains(k) (α, β)”, “Ends(k) (α, β)” and “Ended-by(k) (α, β)”} are irreflexive.

We have also:

$$\text{Equals}(I, I) = \text{Same}(I^{-(1)}, I^{-(1)}) \wedge \text{Same}(I^{-(B)}, I^{-(B)}) \wedge \text{Same}(I^{+(1)}, I^{+(1)}) \wedge \text{Same}(I^{+(E)}, I^{+(E)}) = 1 \quad (23)$$

From the equation (23), we deduce that the relation “Equals” is reflexive.

Symmetry/asymmetry

Let $I = [I^{-}, I^{+}]$ and $J = [J^{-}, J^{+}]$ be imprecise time intervals. Based on the definitions of our proposed temporal relations, it holds that:

$$\text{Before}(I, J) \text{ and } \text{Before}(J, I) \Rightarrow \text{Precedes}(I^{+(E)}, J^{-(1)}) \text{ and } \text{Precedes}(J^{+(E)}, I^{-(1)}) \Rightarrow I = J \quad (1)$$

$$\text{After}(I, J) \text{ and } \text{After}(J, I) \Rightarrow \text{Precedes}(J^{+(E)}, I^{-(1)}) \text{ and } \text{Precedes}(I^{+(E)}, J^{-(1)}) \Rightarrow I = J \quad (2)$$

$$\text{Meets}(I, J) \text{ and } \text{Meets}(J, I) \Rightarrow \text{Min}(\text{Same}(I^{+(1)}, J^{-(1)}) \wedge \text{Same}(I^{+(E)}, J^{-(B)})) \text{ and } \text{Min}(\text{Same}(J^{+(1)}, I^{-(1)}) \wedge \text{Same}(J^{+(E)}, I^{-(B)})) \Rightarrow I = J \quad (3)$$

$$\text{Met-by}(I, J) \text{ and } \text{Met-by}(J, I) \Rightarrow \text{Min}(\text{Same}(J^{+(1)}, I^{-(1)}) \wedge \text{Same}(J^{+(E)}, I^{-(B)})) \text{ and } \text{Min}(\text{Same}(I^{+(1)}, J^{-(1)}) \wedge \text{Same}(I^{+(E)}, J^{-(B)})) \Rightarrow I = J \quad (4)$$

$$\text{Overlaps}(I, J) \text{ and } \text{Overlaps}(J, I) \Rightarrow \text{Min}(\text{Precedes}(I^{-(B)}, J^{-(1)}) \wedge \text{Precedes}(J^{-(B)}, I^{+(1)}) \wedge \text{Precedes}(I^{+(E)}, J^{+(1)})) \text{ and } \text{Min}(\text{Precedes}(J^{-(B)}, I^{-(1)}) \wedge \text{Precedes}(I^{-(B)}, J^{+(1)}) \wedge \text{Precedes}(J^{+(E)}, I^{+(1)})) \Rightarrow I = J \quad (5)$$

$$\text{Overlapped-by}(I, J) \text{ and } \text{Overlapped-by}(J, I) \Rightarrow \text{Min}(\text{Precedes}(J^{-(B)}, I^{-(1)}) \wedge \text{Precedes}(I^{-(B)}, J^{+(1)}) \wedge \text{Precedes}(J^{+(E)}, I^{+(1)})) \text{ and } \text{Min}(\text{Precedes}(I^{-(B)}, J^{-(1)}) \wedge \text{Precedes}(J^{-(B)}, I^{+(1)}) \wedge \text{Precedes}(I^{+(E)}, J^{+(1)})) \Rightarrow I = J \quad (6)$$

$$\text{Starts}(I, J) \text{ and } \text{Starts}(J, I) \Rightarrow \text{Min}(\text{Same}(I^{-(1)}, J^{-(1)}) \wedge \text{Same}(I^{-(B)}, J^{-(B)}) \wedge \text{Precedes}(I^{+(E)}, J^{+(1)})) \text{ and } \text{Min}(\text{Same}(J^{-(1)}, I^{-(1)}) \wedge \text{Same}(J^{-(B)}, I^{-(B)}) \wedge \text{Precedes}(J^{+(E)}, I^{+(1)})) \Rightarrow I = J \quad (7)$$

$$\text{Started-by}(I, J) \text{ and } \text{Started-by}(J, I) \Rightarrow \text{Min}(\text{Same}(J^{-(1)}, I^{-(1)}) \wedge \text{Same}(J^{-(B)}, I^{-(B)}) \wedge \text{Precedes}(J^{+(E)}, I^{+(1)})) \text{ and } \text{Min}(\text{Same}(I^{-(1)}, J^{-(1)}) \wedge \text{Same}(I^{-(B)}, J^{-(B)}) \wedge \text{Precedes}(I^{+(E)}, J^{+(1)})) \Rightarrow I = J \quad (8)$$

$$\text{During}(I, J) \text{ and } \text{During}(J, I) \Rightarrow \text{Min}(\text{Precedes}(J^{-(B)}, I^{-(1)}) \wedge \text{Precedes}(I^{+(E)}, J^{+(1)})) \text{ and } \text{Min}(\text{Precedes}(I^{-(B)}, J^{-(1)}) \wedge \text{Precedes}(J^{+(E)}, I^{+(1)})) \Rightarrow I = J \quad (9)$$

$$\text{Contains}(I, J) \text{ and } \text{Contains}(J, I) \Rightarrow \text{Min}(\text{Precedes}(I^{-(B)}, J^{-(1)}) \wedge \text{Precedes}(J^{+(E)}, I^{+(1)})) \text{ and } \text{Min}(\text{Precedes}(J^{-(B)}, I^{-(1)}) \wedge \text{Precedes}(I^{+(E)}, J^{+(1)})) \Rightarrow I = J \quad (10)$$

$$\text{Ends}(I, J) \text{ and } \text{Ends}(J, I) \Rightarrow \text{Min}(\text{Precedes}(J^{-(B)}, I^{-(1)}) \wedge \text{Same}(I^{+(1)}, J^{+(1)}) \wedge \text{Same}(I^{+(E)}, J^{+(E)})) \text{ and } \text{Min}(\text{Precedes}(I^{-(B)}, J^{-(1)}) \wedge \text{Same}(J^{+(1)}, I^{+(1)}) \wedge \text{Same}(J^{+(E)}, I^{+(E)})) \Rightarrow I = J \quad (11)$$

$$\text{Ended-by}(I, J) \text{ and } \text{Ended-by}(J, I) \Rightarrow \text{Min}(\text{Precedes}(I^{-(B)}, J^{-(1)}) \wedge \text{Same}(J^{+(1)}, I^{+(1)}) \wedge \text{Same}(J^{+(E)}, I^{+(E)})) \text{ and } \text{Min}(\text{Precedes}(J^{-(B)}, I^{-(1)}) \wedge \text{Same}(I^{+(1)}, J^{+(1)}) \wedge \text{Same}(I^{+(E)}, J^{+(E)})) \Rightarrow I = J \quad (12)$$

$$\text{Before}_{(k)}^{(\alpha, \beta)}(I, J) \text{ and } \text{Before}_{(k)}^{(\alpha, \beta)}(J, I) \Rightarrow \text{Precedes}_{(k)}^{(\alpha, \beta)}(I^{+(E)}, J^{-(1)}) \text{ and } \text{Precedes}_{(k)}^{(\alpha, \beta)}(J^{+(E)}, I^{-(1)}) \Rightarrow I = J \quad (13)$$

$$\text{After}_{(k)}^{(\alpha, \beta)}(I, J) \text{ and } \text{Before}(J, I) \Rightarrow \text{Precedes}_{(k)}^{(\alpha, \beta)}(J^{+(E)}, I^{-(1)}) \text{ and } \text{Precedes}_{(k)}^{(\alpha, \beta)}(I^{+(E)}, J^{-(1)}) \Rightarrow I = J \quad (14)$$

$Overlaps_{(k)}^{(\alpha,\beta)}(I, J)$ and $Overlaps_{(k)}^{(\alpha,\beta)}(J, I) \Rightarrow Min(Precedes^{(\alpha,\beta)}(I^{-(B)}, J^{-(1)}) \wedge Precedes^{(\alpha,\beta)}(J^{-(B)}, I^{+(1)}) \wedge Precedes_{(k)}^{(\alpha,\beta)}(I^{+(E)}, J^{+(1)}))$ and $Min(Precedes^{(\alpha,\beta)}(J^{-(B)}, I^{-(1)}) \wedge Precedes^{(\alpha,\beta)}(I^{-(B)}, J^{+(1)}) \wedge Precedes_{(k)}^{(\alpha,\beta)}(J^{+(E)}, I^{+(1)})) \Rightarrow I = J$ (15)

$Overlapped\text{-}by_{(k)}^{(\alpha,\beta)}(I, J)$ and $Overlapped\text{-}by_{(k)}^{(\alpha,\beta)}(J, I) \Rightarrow Min(Precedes^{(\alpha,\beta)}(J^{-(B)}, I^{-(1)}) \wedge Precedes^{(\alpha,\beta)}(I^{-(B)}, J^{+(1)}) \wedge Precedes_{(k)}^{(\alpha,\beta)}(J^{+(E)}, I^{+(1)}))$ and $Min(Precedes^{(\alpha,\beta)}(I^{-(B)}, J^{-(1)}) \wedge Precedes^{(\alpha,\beta)}(J^{-(B)}, I^{+(1)}) \wedge Precedes_{(k)}^{(\alpha,\beta)}(I^{+(E)}, J^{+(1)})) \Rightarrow I = J$ (16)

$Starts_{(k)}^{(\alpha,\beta)}(I, J)$ and $Starts_{(k)}^{(\alpha,\beta)}(J, I) \Rightarrow Min(Same(I^{-(1)}, J^{-(1)}) \wedge Same(I^{-(B)}, J^{-(B)}) \wedge Precedes_{(k)}^{(\alpha,\beta)}(I^{+(E)}, J^{+(1)}))$ and $Min(Same(J^{-(1)}, I^{-(1)}) \wedge Same(J^{-(B)}, I^{-(B)}) \wedge Precedes_{(k)}^{(\alpha,\beta)}(J^{+(E)}, I^{+(1)})) \Rightarrow I = J$ (17)

$Started\text{-}by_{(k)}^{(\alpha,\beta)}(I, J)$ and $Started\text{-}by_{(k)}^{(\alpha,\beta)}(J, I) \Rightarrow Min(Same(J^{-(1)}, I^{-(1)}) \wedge Same(J^{-(B)}, I^{-(B)}) \wedge Precedes_{(k)}^{(\alpha,\beta)}(J^{+(E)}, I^{+(1)}))$ and $Min(Same(I^{-(1)}, J^{-(1)}) \wedge Same(I^{-(B)}, J^{-(B)}) \wedge Precedes_{(k)}^{(\alpha,\beta)}(I^{+(E)}, J^{+(1)})) \Rightarrow I = J$ (18)

$During_{(k)}^{(\alpha,\beta)}(I, J)$ and $During_{(k)}^{(\alpha,\beta)}(J, I) \Rightarrow Min(Precedes_{(k)}^{(\alpha,\beta)}(J^{-(B)}, I^{-(1)}) \wedge Precedes^{(\alpha,\beta)}(I^{+(E)}, J^{+(1)}))$ and $Min(Precedes_{(k)}^{(\alpha,\beta)}(I^{-(B)}, J^{-(1)}) \wedge Precedes^{(\alpha,\beta)}(J^{+(E)}, I^{+(1)})) \Rightarrow I = J$ (19)

$Contains_{(k)}^{(\alpha,\beta)}(I, J)$ and $Contains_{(k)}^{(\alpha,\beta)}(J, I) \Rightarrow Min(Precedes_{(k)}^{(\alpha,\beta)}(I^{-(B)}, J^{-(1)}) \wedge Precedes^{(\alpha,\beta)}(J^{+(E)}, I^{+(1)}))$ and $Min(Precedes_{(k)}^{(\alpha,\beta)}(J^{-(B)}, I^{-(1)}) \wedge Precedes^{(\alpha,\beta)}(I^{+(E)}, J^{+(1)})) \Rightarrow I = J$ (20)

$Ends_{(k)}^{(\alpha,\beta)}(I, J)$ and $Ends_{(k)}^{(\alpha,\beta)}(J, I) \Rightarrow Min(Precedes_{(k)}^{(\alpha,\beta)}(J^{-(B)}, I^{-(1)}) \wedge Same(I^{+(1)}, J^{+(1)}) \wedge Same(I^{+(E)}, J^{+(E)}))$ and $Min(Precedes_{(k)}^{(\alpha,\beta)}(I^{-(B)}, J^{-(1)}) \wedge Same(J^{+(1)}, I^{+(1)}) \wedge Same(J^{+(E)}, I^{+(E)})) \Rightarrow I = J$ (21)

$Ended\text{-}by_{(k)}^{(\alpha,\beta)}(I, J)$ and $Ended\text{-}by_{(k)}^{(\alpha,\beta)}(J, I) \Rightarrow Min(Precedes_{(k)}^{(\alpha,\beta)}(I^{-(B)}, J^{-(1)}) \wedge Same(J^{+(1)}, I^{+(1)}) \wedge Same(J^{+(E)}, I^{+(E)}))$ and $Min(Precedes_{(k)}^{(\alpha,\beta)}(J^{-(B)}, I^{-(1)}) \wedge Same(I^{+(1)}, J^{+(1)}) \wedge Same(I^{+(E)}, J^{+(E)})) \Rightarrow I = J$ (22)

From the equations {(1)....(22)}, we conclude that the temporal relations {"Before", "After", "Meets", "Met-by", "Overlaps", "Overlapped-by", "Starts", "Started-by", "During", "Contains", "Ends", "Ended-by", "$Before_{(k)}^{(\alpha,\beta)}$", "$After_{(k)}^{(\alpha,\beta)}$", "$Overlaps_{(k)}^{(\alpha,\beta)}$", "$Overlapped\text{-}by_{(k)}^{(\alpha,\beta)}$", "$Starts_{(k)}^{(\alpha,\beta)}$", "$Started\text{-}by_{(k)}^{(\alpha,\beta)}$", "$During_{(k)}^{(\alpha,\beta)}$", "$Contains_{(k)}^{(\alpha,\beta)}$", "$Ends_{(k)}^{(\alpha,\beta)}$" and "$Ended\text{-}by_{(k)}^{(\alpha,\beta)}$"} are asymmetric.

We have also:

$Min(Same(I^{-(1)}, J^{-(1)}) \wedge Same(I^{-(B)}, J^{-(B)}) \wedge Same(I^{+(1)}, J^{+(1)}) \wedge Same(I^{+(E)}, J^{+(E)})) = Min(Same(J^{-(1)}, I^{-(1)}) \wedge Same(J^{-(B)}, I^{-(B)}) \wedge Same(J^{+(1)}, I^{+(1)}) \wedge Same(J^{+(E)}, I^{+(E)})) \Rightarrow Equals(I, J) = Equals(J, I)$ (23)

From the equation (23), we deduce that the relation "Equals" is symmetric.

Transitivity

Let $I = [I^-, I^+]$, $J = [J^-, J^+]$ and $K = [K^-, K^+]$ be imprecise time intervals. Based on the definitions of our proposed temporal relations, it holds that:

Before(I, J) and Before(J, K) $\Rightarrow$ Precedes $(I^{+(E)}, J^{-(1)})$ and Precedes $(J^{+(E)}, K^{-(1)}) \Rightarrow$ Precedes $(I^{+(E)}, K^{-(1)}) \Rightarrow$ Before(I, K) (1)

After(I, J) and After(J, K) $\Rightarrow$ Precedes $(J^{+(E)}, I^{-(1)})$ and Precedes $(K^{+(E)}, J^{-(1)}) \Rightarrow$ Precedes $(K^{+(E)}, I^{-(1)}) \Rightarrow$ After(I, K) (2)

Overlaps(I, J) and Overlaps(J, K) $\Rightarrow$ Min (Precedes $(I^{-(B)}, J^{-(1)}) \wedge$ Precedes $(J^{-(B)}, I^{+(1)}) \wedge$ Precedes $(I^{+(E)}, J^{+(1)})$) and Min (Precedes $(J^{-(B)}, K^{-(1)}) \wedge$ Precedes $(K^{-(B)}, J^{+(1)}) \wedge$ Precedes $(J^{+(E)}, K^{+(1)})) \Rightarrow$ Min (Precedes $(I^{-(B)}, K^{-(1)}) \wedge$ Precedes $(K^{-(B)}, I^{+(1)}) \wedge$ Precedes $(I^{+(E)}, K^{+(1)})) \Rightarrow$ Overlaps(I, K) (3)

Overlapped-by(I, J) and Overlapped-by(J, K) => Min (Precedes ($J^{-(B)}$, $I^{-(1)}$) ∧ Precedes ($I^{-(B)}$, $J^{+(1)}$) ∧ Precedes ($J^{+(E)}$, $I^{+(1)}$)) and Min (Precedes ($K^{-(B)}$, $J^{-(1)}$) ∧ Precedes ($J^{-(B)}$, $K^{+(1)}$) ∧ Precedes ($K^{+(E)}$, $J^{+(1)}$)) => Min (Precedes ($K^{-(B)}$, $I^{-(1)}$) ∧ Precedes ($I^{-(B)}$, $K^{+(1)}$) ∧ Precedes ($K^{+(E)}$, $I^{+(1)}$)) => Overlapped-by(I, K) (4)

Starts(I, J) and Starts(J, K) => Min (Same ($I^{-(1)}$, $J^{-(1)}$) ∧ Same ($I^{-(B)}$, $J^{-(B)}$) ∧ Precedes ($I^{+(E)}$, $J^{+(1)}$)) and Min (Same ($J^{-(1)}$, $K^{-(1)}$) ∧ Same ($J^{-(B)}$, $K^{-(B)}$) ∧ Precedes ($J^{+(E)}$, $K^{+(1)}$)) => Min (Same ($I^{-(1)}$, $K^{-(1)}$) ∧ Same ($I^{-(B)}$, $K^{-(B)}$) ∧ Precedes ($I^{+(E)}$, $K^{+(1)}$)) => Starts(I, K) (5)

Started-by(I, J) and Started-by(J, K) => Min (Same ($J^{-(1)}$, $I^{-(1)}$) ∧ Same ($J^{-(B)}$, $I^{-(B)}$) ∧ Precedes ($J^{+(E)}$, $I^{+(1)}$)) and Min (Same ($K^{-(1)}$, $J^{-(1)}$) ∧ Same ($K^{-(B)}$, $J^{-(B)}$) ∧ Precedes ($K^{+(E)}$, $J^{+(1)}$)) => Min (Same ($K^{-(1)}$, $I^{-(1)}$) ∧ Same ($K^{-(B)}$, $I^{-(B)}$) ∧ Precedes ($K^{+(E)}$, $I^{+(1)}$)) => Started-by(I, K) (6)

During(I, J) and During(J, K) => Min (Precedes ($J^{-(B)}$, $I^{-(1)}$) ∧ Precedes ($I^{+(E)}$, $J^{+(1)}$)) and Min (Precedes ($K^{-(B)}$, $J^{-(1)}$) ∧ Precedes ($J^{+(E)}$, $K^{+(1)}$)) => Min (Precedes ($K^{-(B)}$, $I^{-(1)}$) ∧ Precedes ($I^{+(E)}$, $K^{+(1)}$)) => During(I, K) (7)

Contains(I, J) and Contains(J, K) => Min (Precedes ($I^{-(B)}$, $J^{-(1)}$) ∧ Precedes ($J^{+(E)}$, $I^{+(1)}$)) and Min (Precedes ($J^{-(B)}$, $K^{-(1)}$) ∧ Precedes ($K^{+(E)}$, $J^{+(1)}$)) => Min (Precedes ($I^{-(B)}$, $K^{-(1)}$) ∧ Precedes ($K^{+(E)}$, $I^{+(1)}$)) => Contains(I, K) (8)

Equals(I, J) and Equals(J, K) => Min(Same ($I^{-(1)}$, $J^{-(1)}$) ∧ Same ($I^{-(B)}$, $J^{-(B)}$) ∧ Same ($I^{+(1)}$, $J^{+(1)}$) ∧ Same ($I^{+(E)}$, $J^{+(E)}$)) and Min(Same ($J^{-(1)}$, $K^{-(1)}$) ∧ Same ($J^{-(B)}$, $K^{-(B)}$) ∧ Same ($J^{+(1)}$, $K^{+(1)}$) ∧ Same ($J^{+(E)}$, $K^{+(E)}$)) => Min(Same ($I^{-(1)}$, $K^{-(1)}$) ∧ Same ($I^{-(B)}$, $K^{-(B)}$) ∧ Same ($I^{+(1)}$, $K^{+(1)}$) ∧ Same ($I^{+(E)}$, $K^{+(E)}$)) => Equals(I, K) (9)

$Before_{(k)}^{(\alpha, \beta)}$(I, J) and $Before_{(k)}^{(\alpha, \beta)}$(J, K) => $Precedes_{(k)}^{(\alpha, \beta)}$ ($I^{+(E)}$, $J^{-(1)}$) and $Precedes_{(k)}^{(\alpha, \beta)}$ ($J^{+(E)}$, $K^{-(1)}$) => $Precedes_{(k)}^{(\alpha, \beta)}$ ($I^{+(E)}$, $K^{-(1)}$) => $Before_{(k)}^{(\alpha, \beta)}$(I, K) (10)

$After_{(k)}^{(\alpha, \beta)}$(I, J) and $After_{(k)}^{(\alpha, \beta)}$(J, K) => $Precedes_{(k)}^{(\alpha, \beta)}$ ($J^{+(E)}$, $I^{-(1)}$) and $Precedes_{(k)}^{(\alpha, \beta)}$ ($K^{+(E)}$, $J^{-(1)}$) => $Precedes_{(k)}^{(\alpha, \beta)}$ ($K^{+(E)}$, $I^{-(1)}$) => $After_{(k)}^{(\alpha, \beta)}$(I, K) (11)

$Overlaps_{(k)}^{(\alpha, \beta)}$(I, J) and $Overlaps_{(k)}^{(\alpha, \beta)}$(J, K) => Min($Precedes^{(\alpha, \beta)}$ ($I^{-(B)}$, $J^{-(1)}$) ∧ $Precedes^{(\alpha, \beta)}$ ($J^{-(B)}$, $I^{+(1)}$) ∧ $Precedes_{(k)}^{(\alpha, \beta)}$ ($I^{+(E)}$, $J^{+(1)}$)) and Min($Precedes^{(\alpha, \beta)}$ ($J^{-(B)}$, $K^{-(1)}$) ∧ $Precedes^{(\alpha, \beta)}$ ($K^{-(B)}$, $J^{+(1)}$) ∧ $Precedes_{(k)}^{(\alpha, \beta)}$ ($J^{+(E)}$, $K^{+(1)}$)) => Min($Precedes^{(\alpha, \beta)}$ ($I^{-(B)}$, $K^{-(1)}$) ∧ $Precedes^{(\alpha, \beta)}$ ($K^{-(B)}$, $I^{+(1)}$) ∧ $Precedes_{(k)}^{(\alpha, \beta)}$ ($I^{+(E)}$, $K^{+(1)}$)) => $Overlaps_{(k)}^{(\alpha, \beta)}$(I, K) (12)

$Overlapped\text{-}by_{(k)}^{(\alpha, \beta)}$(I, J) and $Overlapped\text{-}by_{(k)}^{(\alpha, \beta)}$(J, K) => Min($Precedes^{(\alpha, \beta)}$ ($J^{-(B)}$, $I^{-(1)}$) ∧ $Precedes^{(\alpha, \beta)}$ ($I^{-(B)}$, $J^{+(1)}$) ∧ $Precedes_{(k)}^{(\alpha, \beta)}$ ($J^{+(E)}$, $I^{+(1)}$)) and Min($Precedes^{(\alpha, \beta)}$ ($K^{-(B)}$, $J^{-(1)}$) ∧ $Precedes^{(\alpha, \beta)}$ ($J^{-(B)}$, $K^{+(1)}$) ∧ $Precedes_{(k)}^{(\alpha, \beta)}$ ($K^{+(E)}$, $J^{+(1)}$)) => Min($Precedes^{(\alpha, \beta)}$ ($K^{-(B)}$, $I^{-(1)}$) ∧ $Precedes^{(\alpha, \beta)}$ ($I^{-(B)}$, $K^{+(1)}$) ∧ $Precedes_{(k)}^{(\alpha, \beta)}$ ($K^{+(E)}$, $I^{+(1)}$)) => $Overlapped\text{-}by_{(k)}^{(\alpha, \beta)}$(I, K) (13)

$Starts_{(k)}^{(\alpha, \beta)}$(I, J) and $Starts_{(k)}^{(\alpha, \beta)}$(J, K) => Min(Same ($I^{-(1)}$, $J^{-(1)}$) ∧ Same ($I^{-(B)}$, $J^{-(B)}$) ∧ $Precedes_{(k)}^{(\alpha, \beta)}$ ($I^{+(E)}$, $J^{+(1)}$)) and Min(Same ($J^{-(1)}$, $K^{-(1)}$) ∧ Same ($J^{-(B)}$, $K^{-(B)}$) ∧ $Precedes_{(k)}^{(\alpha, \beta)}$ ($J^{+(E)}$, $K^{+(1)}$)) => Min(Same ($I^{-(1)}$, $K^{-(1)}$) ∧ Same ($I^{-(B)}$, $K^{-(B)}$) ∧ $Precedes_{(k)}^{(\alpha, \beta)}$ ($I^{+(E)}$, $K^{+(1)}$)) => $Starts_{(k)}^{(\alpha, \beta)}$(I, K) (14)

$Started\text{-}by_{(k)}^{(\alpha, \beta)}$(I, J) and $Started\text{-}by_{(k)}^{(\alpha, \beta)}$(J, K) => Min(Same ($J^{-(1)}$, $I^{-(1)}$) ∧ Same ($J^{-(B)}$, $I^{-(B)}$) ∧ $Precedes_{(k)}^{(\alpha, \beta)}$ ($J^{+(E)}$, $I^{+(1)}$)) and Min(Same ($K^{-(1)}$, $J^{-(1)}$) ∧ Same ($K^{-(B)}$, $J^{-(B)}$) ∧ $Precedes_{(k)}^{(\alpha, \beta)}$ ($K^{+(E)}$, $J^{+(1)}$)) => Min(Same ($K^{-(1)}$, $I^{-(1)}$) ∧ Same ($K^{-(B)}$, $I^{-(B)}$) ∧ $Precedes_{(k)}^{(\alpha, \beta)}$ ($K^{+(E)}$, $I^{+(1)}$)) => $Started\text{-}by_{(k)}^{(\alpha, \beta)}$(I, K) (15)

$During_{(k)}^{(\alpha, \beta)}$(I, J) and $During_{(k)}^{(\alpha, \beta)}$(J, K) => Min($Precedes_{(k)}^{(\alpha, \beta)}$ ($J^{-(B)}$, $I^{-(1)}$) ∧ $Precedes^{(\alpha, \beta)}$ ($I^{+(E)}$, $J^{+(1)}$)) and Min($Precedes_{(k)}^{(\alpha, \beta)}$ ($K^{-(B)}$, $J^{-(1)}$) ∧ $Precedes^{(\alpha, \beta)}$ ($J^{+(E)}$, $K^{+(1)}$)) => Min($Precedes_{(k)}^{(\alpha, \beta)}$ ($K^{-(B)}$, $I^{-(1)}$) ∧ $Precedes^{(\alpha, \beta)}$ ($I^{+(E)}$, $K^{+(1)}$)) => $During_{(k)}^{(\alpha, \beta)}$(I, K) (16)

$Contains_{(k)}^{(\alpha, \beta)}$(I, J) and $Contains_{(k)}^{(\alpha, \beta)}$(J, K) => Min($Precedes_{(k)}^{(\alpha, \beta)}$ ($I^{-(B)}$, $J^{-(1)}$) ∧ $Precedes^{(\alpha, \beta)}$ ($J^{+(E)}$, $I^{+(1)}$)) and Min($Precedes_{(k)}^{(\alpha, \beta)}$ ($J^{-(B)}$, $K^{-(1)}$) ∧ $Precedes^{(\alpha, \beta)}$ ($K^{+(E)}$, $J^{+(1)}$)) => Min($Precedes_{(k)}^{(\alpha, \beta)}$ ($I^{-(B)}$, $K^{-(1)}$) ∧ $Precedes^{(\alpha, \beta)}$ ($K^{+(E)}$, $I^{+(1)}$)) => $Contains_{(k)}^{(\alpha, \beta)}$(I, K) (17)

From the equations {(1)....(17)}, we conclude that the temporal relations {"Before", "After", "Overlaps", "Overlapped-by", "Starts", "Started-by", "During", "Contains", "Equals", "$Before_{(k)}^{(\alpha, \beta)}$", "$After_{(k)}^{(\alpha, \beta)}$", "$Overlaps_{(k)}^{(\alpha, \beta)}$", "$Overlapped\text{-}by_{(k)}^{(\alpha, \beta)}$", "$Starts_{(k)}^{(\alpha, \beta)}$", "$Started\text{-}by_{(k)}^{(\alpha, \beta)}$", "$During_{(k)}^{(\alpha, \beta)}$" and "$Contains_{(k)}^{(\alpha, \beta)}$"} are transitive.