Partial Least Squares Path Modeling: matching Models and Modes

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Outline

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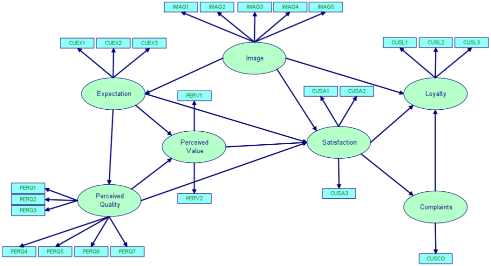
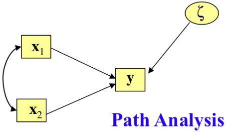
Latent Variable Path Modeling (LVPM)

Modeling a network of predictive relationships between Latent Variables measured by means of sets of items (indicators, manifest variables)

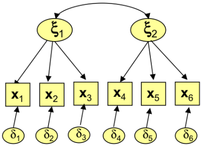


Structural model: $f_y = (I - B)^{-1}\Gamma f_x + \zeta$ Outer model: $X = \lambda f + \delta$

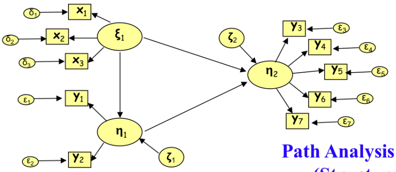
Two models in one



Path Analysis with Manifest Variables



Confirmatory Factor Model



Path Analysis with Latent Variables (Structural Equation Model)

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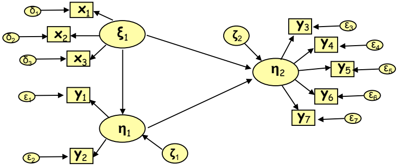
Modeling Path Analysis with Latent Variables



$$\mathbf{x} = \Lambda_x \xi + \delta$$
$$\mathbf{y} = \Lambda_y \eta + \varepsilon$$

Measurement Models

Structural Model

$$\eta = \Gamma\xi + \mathrm{B}\eta + \zeta \iff \eta = (\mathbf{I} - \mathbf{B})^{-1}(\Gamma\xi + \zeta)$$

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Jöreskog's approach to LVPM: Covariance Structure Analysis

Assuming that:

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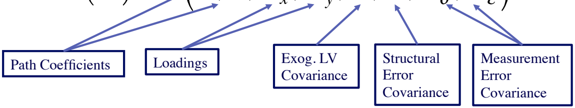
We can write the covariance matrix among the MVs in terms of model parameters (implied covariance matrix)

$$C = \Sigma(\Omega) = \Sigma\left(\Gamma, \mathbf{B}, \Lambda_x, \Lambda_y, \Phi, \Psi, \Theta_\delta, \Theta_\varepsilon\right)$$



CSA: parameter estimation

$$\Sigma = \begin{bmatrix} \Sigma_{xx} & \\ \Sigma_{yx} & \Sigma_{yy} \end{bmatrix}$$

Population Covariance matrix

$$S = \begin{bmatrix} S_{xx} & \\ S_{yx} & S_{yy} \end{bmatrix}$$

Empirical covariance matrix

$$C = \Sigma(\Omega) = \left[\begin{array}{c:c} \Lambda_x \Phi \Lambda'_x + \Theta_\delta & \\ \hdashline \Lambda_y (\mathbf{I}-\mathbf{B})^{-1} \Gamma \Phi' \Lambda'_x & \Lambda_y \left[(\mathbf{I}-\mathbf{B})^{-1} (\Gamma \Phi \Gamma' + \Psi) (\mathbf{I}-\mathbf{B})^{-1\prime} \right] \Lambda'_y + \Theta_\varepsilon \end{array}\right]$$

"Implied" covariance matrix

CSA: parameter estimation

Maximum Likelihood

$$F_{ML} = \log\left|\mathbf{C}\right| + tr\left(\mathbf{S}\mathbf{C}^{-1}\right) - \log\left|\mathbf{S}\right| - \left(P + Q\right)$$

Unweighted Least Squares

$$F_{ULS} = \frac{1}{2} tr\left[\left(\mathbf{S} - \mathbf{C}\right)^2\right]$$

Generalised Least Squares

$$F_{GLS} = \frac{1}{2} tr\left[\mathbf{W}^{-1}\left(\mathbf{S} - \mathbf{C}\right)^2\right]$$

Asymptotically Distribution Free

$$F_{ADF/WLS} = \left(\underline{\mathbf{s}} - \underline{\mathbf{c}}\right)^{\mathrm{T}} \mathbf{W}^{-1} \left(\underline{\mathbf{s}} - \underline{\mathbf{c}}\right)$$

Some observations on Jöreskog's model

Psychometrical approach to measurement theory - latent variables are modeled as common factors

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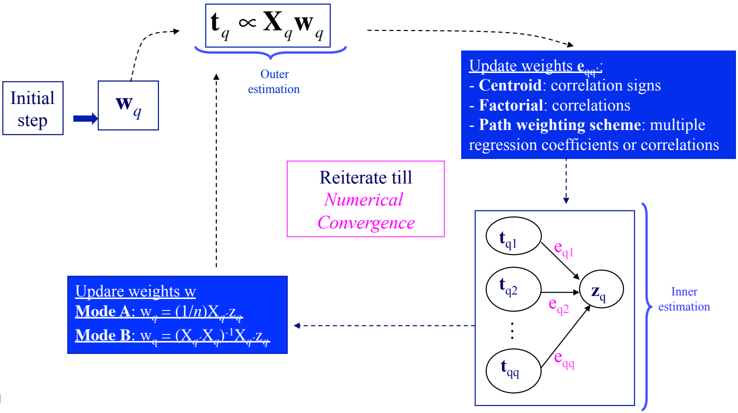
Psychometrical approach to measurement theory - latent variables are modeled as common factors

- Factor are theoretical (and random) variables
- Factor indeterminacy
- Estimation focuses on factor means and variances
- Reproduce the sample covariance matrix of the manifest variables by means of a model-implied covariance matrix which is a function of model parameters

Wold's approach to LVPM: Partial Least Squares Path Modeling algorithm

What is PLS-PM? Basically, an algorithm who provides weights for building composites (components)

MVs are centered or standardized



The three facets of Partial Least Squares Path Modeling algorithm

- Data analysis tool (Hanafi [2007], Kramer (2007), Tenenhaus and Tenenhaus [2011] among others)

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- Alternative Estimation tool for factor-based LVPMs (Wold [1977], Wold [1980], Dijkstra & Henseler [2017])

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- Data analysis tool (Hanafi [2007], Kramer (2007), Tenenhaus and Tenenhaus [2011] among others)
- Alternative Estimation tool for factor-based LVPMs (Wold [1977], Wold [1980], Dijkstra & Henseler [2017])
- Estimation tool for component-based LVPMs (Dijkstra 2017))

PLS Path Modeling: algorithm and criteria

Given Q blocks of variables $\boldsymbol{X}_q, q = \{1, \ldots, Q\}$

1. Obtain Q vectors of weights $\boldsymbol{w}_q$ through the PLS-PM iterative algorithm

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General PLS-PM criterion [Tenenhaus & Tenenhaus, 2011]

$$\underset{\forall \mathbf{w}_q}{\text{Maximize}} \sum_{q \neq q'} c_{qq'} g(\text{cov}(\mathbf{X}_q \mathbf{w}_q, \mathbf{X}_{q'} \mathbf{w}_{q'}))$$

$$\text{s. t.} \quad \tau_q ||\mathbf{w}_q||^2 + (1 - \tau_q)\text{var}(\mathbf{X}_q \mathbf{w}_q) = 1, \quad q = \{1, \dots, Q\}, \quad \tau_q = \{0, 1\}$$

$$\begin{cases} c_{qq'} = 1 & \text{if } \boldsymbol{X}_q \text{ et } \boldsymbol{X}_{q'} \text{ are connected} \\ c_{qq'} = 0 & \text{otherwise} \end{cases} \quad \begin{cases} g\{\} = & \text{square (factorial scheme)} \\ g\{\} = & \text{absolut value (centroid scheme)} \end{cases} \quad \begin{cases} \tau_q = 1 & \text{if Mode A / Mode new A} \\ \tau_q = 0 & \text{if Mode B} \end{cases}$$

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PLS as a generalized data analysis tool

Method	Criterion	PLS path model	Mode	Scheme
(1) SUMCOR (Horst 1961)	$Max \sum_{j,k} Cor(F_j, F_k)$ or $Max \sum_j Cor(F_j, \sum_k F_k)$	Hierarchical	B	Centroid
(2) MAXVAR (Horst 1961) or GCCA (Carroll 1968)	$Max\{\lambda_{first}[Cor(F_j, F_k)]\}$ (a) or $Max \sum_j Cor^2(F_j, F_{j+1})$	Hierarchical	B	Factorial
(3) SsqCor (Kettenring 1971)	$Max \sum_{j,k} Cor^2(F_j, F_k)$	Confirmatory	B	Factorial
(4) GenVar (Kettenring 1971)	$Min\{\det[Cor(F_j, F_k)]\}$			
(5) MINVAR (Kettenring 1971)	$Min\{\lambda_{last}[Cor(F_j, F_k)]\}$ (b)			
(6) Lafosse (1989)	$Max \sum_j Cor^2(F_j, \sum_k F_k)$			
(7) Mathes (1993) or Hanafi (2005)	$Max \sum_{j,k} \lvert Cor(F_j, F_k)\rvert$	Confirmatory	B	Centroid
(8) MAXDIFF (Van de Geer, 1984 & Ten Berge, 1988)	$Max_{all\ \lVert w_j \rVert = 1} \sum_{j \neq k} Cov(X_j w_j, X_k w_k)$			
(9) MAXBET (Van de Geer, 1984 & Ten Berge, 1988)	$Max_{all\ \lVert w_j \rVert = 1} \sum_{j,k} Cov(X_j w_j, X_k w_k)$			
(10) MAXDIFF B (Hanafi and Kiers 2006)	$Max_{all\ \lVert w_j \rVert = 1} \sum_{j \neq k} Cov^2(X_j w_j, X_k w_k)$			

From Tenenhaus et Hanafi (2010)

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PLS as a generalized data analysis tool

Method	Criterion	PLS path model	Mode	Scheme
(11) (Hanafi and Kiers 2006)	$Max_{all\ \Vert w_j \Vert=1} \sum_{j \neq k} \vert Cov(X_j w_j, X_k w_k) \vert$			
(12) ACOM (Chessel and Hanafi 1996) or Split PCA (Lohmöller 1989)	$Max_{all\ \Vert w_j \Vert=1} \sum_j Cov^2(X_j w_j, X_{j+1} w_{j+1})$ or $Min_{F,p_j} \sum_j \left\Vert X_j - F p_j^T \right\Vert^2$	Hierarchical	A	Path-weighting
(13) CCSWA (Hanafi et al., 2006) or HPCA (Wold et al., 1996)	$Max_{all\ \Vert w_j \Vert=1, Var(F)=1} \sum_j Cov^4(X_j w_j, F)$ or $Min_{\Vert F \Vert=1} \sum_j \left\Vert X_j X_j^T - \lambda_j F F^T \right\Vert^2$			
(14) Generalized PCA (Casin 2001)	$Max \sum_j R^2(F, X_j) \sum_h Cor^2\left(x_{jh}, \hat{F}_j\right)$ (c)			
(15) MFA (Escofier and Pagès 1994)	$Min_{F,p_j} \sum_j \left\Vert \frac{1}{\sqrt{\lambda_{first}\left[Cor(x_{jh}, x_{jl})\right]}} X_j - F p_j^T \right\Vert^2$	Hierarchical (applied to the reduced X_j) (d)	A	Path-weighting
(16) Oblique maximum variance method (Horst 1965)	$Min_{F,p_j} \sum_j \left\Vert X_j \left(\frac{1}{n} X_j^T X_j\right)^{-1/2} - F p_j^T \right\Vert^2$	Hierarchical (applied to the transformed X_j) (e)	A	Path-weighting

(c) $\hat{F}_j$ is the prediction of F in the regression of F on block X_j.

(d) The reduced block number j is obtained by dividing the block X_j by the square root of $\lambda_{first}\left[Cor(x_{jh}, x_{j\ell})\right]$.

(e) The transformed block number j is computed as $X_j[(1/n)X_j^T X_j]^{-1/2}$.

From Tenenhaus et Hanafi (2010)

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- Composites are fixed variables
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- Maximize the variances of the exogenous variables

Partial Least Squares for Factor-based LVPMs

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YES, because:

- Consistency at large, i.e. large number of cases and of indicators for each latent variable
- PLSc [Dijkstra and Henseler, 2015], PLS algorithm yield all the ingredients for obtaining CAN estimations of loadings and LVs squared correlations of a factor model where all information between the blocks is conveyed solely by the **factors**

Dijkstra & Henseler (2015) : The model

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3. Get CAN estimates of path coefficients using 2SLS or 3SLS on LV correlation matrix

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He shows that (Mode B) PLS-PM provides CAN estimators for the composite weights of this model

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- All information between the blocks is conveyed solely by the composites $r_{qq'} = r_{qq'} \lambda_q \lambda_{q'}'$

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Estimating model parameters

Dijkstra uses a step-wise approach: first the weights, then the loadings and the correlations between the composites, and finally the structural form coefficients

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"So no explicit overall fit-criterion.. The view that a lack of an overall criterion to be optimized is a major flaw is ill-founded. Estimators should be compared on the basis of their distribution functions, the extent to which they satisfy computational desiderata, and the induced quality of the predictions. There is no theorem, and their cannot be one, to the effect that estimators that optimize a function are better than those that are not so motivated."

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- GENVAR, which minimizes the product of the eigenvalues (the determinant) of $R(w)$, in order to get eigenvalues as different as possible.

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Structural coefficient can be obtained using regular regression for recursive paths and 2SLS (or 3SLS) regression for non recursive paths

Thank you for your attention!

le cnam

cedric